

HOMEWORK 1

Sampling (time-domain and frequency domain), Aliasing, FT, DTFT, DFT, Reconstruction.

In this homework we consider the signal

$$x(t) = \cos 2\pi 60t.$$

This signal will be windowed and sampled and the DFT obtained. The DFT will be compared to the DTFT which will then be compared with the FT.

- (a) For the given signal, determine its FT and sketch the FT amplitude and phase.
 (b) Consider the rectangular window

$$w_R(t) = \begin{cases} 1 & \text{if } 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$

and determine its FT $W_R(j\Omega)$. Show this as a *sinc* function. It may be easier to determine the Fourier spectrum of a symmetric window $[-T/2, +T/2]$ which will give you the *sinc* function and then shift it. Show and mark the zero crossings so that you see how wide is the main lobe of the spectrum.

- (c) Now construct the ‘practical’ signal $x_W(t) = x(t).w_R(t)$. Find its Fourier spectrum $X_W(j\Omega)$. Sketch it.
 (d) Now sample the signal such that

$$x_W[n] \equiv x_W(nT) = x_W(t)|_{t=nT}$$

where $T = 1\text{ms}$. Take 100 samples. Let $y[n] \equiv x_W(n)$.

- (e) Plot $Y(e^{j\omega})$.
 (f) Using MATLAB, find the DFT $Y[k]$ of $y[n]$.
 (g) Compare $Y[k]$ and $Y(e^{j\omega})$ and $X_W(j\Omega)$.
 (h) Now use a Hanning window and repeat the above. Comment.