HOMEWORK 9

2-CHANNEL PERFECT RECONSTRUCTION FILTER BANKS-3

1. **Polyphase**

Let $x[n]$ be the sequence $x(0), x(1), x(2), x(3), x(4), \ldots$. We have $x[n] \Leftrightarrow X(z)$. Hence,

$$X(z) = x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3}.....$$

(a) Express $X(z)$ in terms of $X_E(z^2)$ and $X_O(z^2)$.

(b) Consider

$$y = (\downarrow 2)x \tag{1}$$

Write the first few terms of $y[n]$.

(c) Now do part (b) using z-transforms. First, express equation (1) in z-transforms. (You can find $Y(e^{j\omega})$ and then substitute $z = e^{j\omega}$). Then use the expression for $X(z)$ in part (a) to find $Y(z)$ in terms of $X_E(z)$. This of course, must be consistent with $y[n]$ of part (b).

(d) Now consider

$$y = (\downarrow 2)Cx$$

Using the concepts of part (c), show that

$$Y(z) = X_E(z)C_E(z) + X_O(z)C_O(z)$$

Then show the block diagram of the polyphase realization of one arm of the analysis part of a 2-channel PRFB.

2. **2-channel orthogonal PRFB**

Consider the 2-channel orthogonal filter bank shown above. The synthesis lowpass and highpass filters are labelled as $h_0(n)$ and $h_1(n)$ respectively. Consider them as N-point filters.

(a) For the orthogonal filter bank, the analysis lowpass and highpass filters are related to the synthesis lowpass and highpass filters. Hence give the analysis lowpass and highpass filters in terms of the corresponding synthesis filters. Make the former causal.

(b) Recalling that we have a LTI system, use the analysis filters to express wavelet coefficients $c_1(n)$ and $c_2(n)$ in terms of the analysis filters and $x(n)$. What does this operation correspond to? Hence, when finding wavelet coefficients, what are you doing?

3. **Wavelets**

Assume a signal $\underline{\mathbf{x}}$ of size 8. The signal is analyzed with a 2-channel Haar filter bank. Then the coefficients are synthesized with the corresponding 2-channel synthesis filter bank $\underline{\mathbf{x}} = \underline{\mathbf{A}}^{\mathbf{T}}\ \underline{\mathbf{c}}$. Sketch all the (discrete) wavelets involved.