E E 2 7 6 February 16, 2006

LAB 4 - Generation & Testing of Multivariate Gaussian Process and Gauss-Markov Process

1. Generation of Gaussian Random Variables Most computer software libraries include a *uniform random number generator*. Such a generator generates random numbers between 0 and 1 with equal probability. With the finite precision computation, it is not possible to represent a continuum of numbers in the interval $0 \leq x \leq 1$. Theoretically, we have a random variable X whose values $x \in [0, 1]$ with a uniform pdf $f_X(x)$.

To generate a random variable with a pdf other than uniform, we start with a uniformly distributed RV and use it to generate the RV with the desired pdf. Assume X is uniform on $[0, 1]$ with cdf $F_X(x)$. Suppose we wish to generate a RV Y with cdf $F_Y(y)$. Call it g(Y). Since the range of $F_Y(y)$ is in $[0, 1]$, we begin by generating a uniformly distributed RV X in the range $[0, 1]$. If we set

$$F_Y(y) \equiv g(y) = x$$

then the transformation $Y = g^{-1}(X)$ will give us a RV Y with the desired cdf $g(y)$.

Example: Want $F_Y(y) = y^2/4, y \in [0, 2]$. Consider $F_X(x) = x \in [0, 1]$. Let $Y^2/4 = X$. Therefore $Y = 2\sqrt{(X)}$.
Hence,

$$P(X \leq x) = x, \quad 0 \leq x \leq 1; \tag{1}$$
$$P(Y^2/4 \leq x) = x \quad 0 \leq x \leq 1; \tag{2}$$
$$P(Y \leq 2\sqrt{x}) = x \quad 0 \leq x \leq 1; \quad let 2\sqrt{x} = y; \tag{3}$$
$$P(Y \leq y) = y^2/4 \quad 0 \leq y \leq 2 \tag{4}$$

With this technique we have problems when *inverses do not exist.* That is the case for the Gaussian RV. Noise encountered in physical systems is often characterized by Gaussian or normal distributions. That is

$$f(Y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-y^2/2\sigma^2} \quad -\infty < y < +\infty$$

Then we have

$$F(y) = \int_{-\infty}^{y} f(x)dx \tag{5}$$

The inverse of the integral in (5) is difficult to achieve. A way to circumvent this problem is to use 2 transformation using a Rayleigh distribution (see reference below). A Rayleigh cdf of random variable R is

$$F(R) = \begin{cases} 0, & R < 0 \\ 1 - e^{-R^2/2\sigma^2}, & R \geq 0 \end{cases}$$

Letting $F(R) = X$ where X uniformly distributed in $[0, 1]$, we get,

$$R = \sqrt{2\sigma^2 ln \frac{1}{1 - A}}$$

Now for a second uniformly distributed RV θ we know that a pair of Gaussian RVs C and D are related to R and θ as:

$$C = Rcos\theta \quad (6)$$

$$D = Rcos\theta \quad (7)$$

The parameter σ^2 is the variance of C and D.

The MATLAB script file gngauss.m generates independent gaussian RVs with mean 'm' and standard deviation σ. Note that when you invoke that function, it only generates *one* value for each of the random variables.

2. Gaussian and Gauss-Markov Process

We generate samples of a 2-D GRP with given mean and covariance matrix.

Def: A *Gauss-Markov process* $X(t)$ is a Markov process whose pdf is Gaussian. The simplest way of generating a Markov process is as follows:

$$X(n) = \rho X(n - 1) + w(n)$$

where $w(n)$ is a sequence of zero-mean i.i.d. (white) random variables and ρ is a parameter that determines the degree of correlation between $X(n)$ and $X(n - 1)$. That is,

$$E[X(n)X(n - 1)] = \rho E[X^2(n - 1) = \rho\sigma^2(n - 1)$$

If the sequence {w(n)} is Gaussian, then the resulting process is a Gauss-Markov process.

EXPERIMENT

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1. Run the function [gsrv1,gsrv2]=gngauss. Read the code and figure out the algorithm for generating two *independent Gaussian random variables* with given mean (say 0) and given standard deviation (say 1). How would you go about generating more than 2 independent GRVs? Repeat the function, say 100 times, to generate 100 values of each of the 2 random variables. Now check to see that indeed each of them have a Gaussian pdf and that the 2 random variables are uncorrelated.

2. Now run the script file IP_02_02. This does 2 things. First, it has a built in function that generates 2 Gaussian RVs with zero mean and a pre-defined C_x (just one sample for each). Then it simply generates and plots a 2-D Gaussian pdf for the mean and C_x defined above. Run this and understand the code.

3. Now run IP_02_03. This uses a white *Gaussian* noise process 'noise' to generate a Gauss-Markov process 'X' with a given ρ.
Read and understand IP_02_03.m and gaus_mar.m.
Get outputs X (GM process), noise (GWRP) and $R_x(m)$. (You have to adjust gaus_mar.m function to output the noise process). Determine that the noise is indeed Gaussian, that it has the desired mean and variance and that it white. (hist, mean, var, xcorr (try Rx_est)).
Examine the G-M process and check its autocorrelation function using Rx_est.
Check autocorrelation function of X using xcorr(X). Why (how) does that differ from Rx_est? Observations?

4. Autocorrelation and PSD.
Run IP_02_04.
Look at the i.i.d process.
Look at Rx_av and Sx_av.
Look at any other aspects of interest.
Turn 'echo off' if you wish.

REPORT

1. Comment on 1.
2. Comment on 2.
3. Comment on 3.
4. Comment on 4.

Reference: Chapter 2, Contemporary Communication Systems- Using MATLAB
PWS Publishing Co. 1998. Also, same title, Second Edition, Bookware Companion Series, 2004.