Math 21J **Quiz 7** **October 23, 2007** **Name:** Key

1. Find the linear and quadratic approximations to $f(x) = e^x$ near $x = 0$. Fill in the table with 4-decimal values.

x	e^x	Quadratic Approximation
0.1	1.1052	1.1050
0.2	1.2214	1.2200
0.4	1.4918	1.4800

$f(0) = 1$ $\quad f'(x) = e^x$

$f'(0) = 1$ $\quad f''(x) = e^x$

$f''(0) = 1$

Linear: $1 + 1(x-0) = \boxed{1+x}$

Quadratic: $1 + x + \frac{1}{2}(1)(x-0)^2 = \boxed{1 + x + \frac{1}{2}x^2}$

2. Find the linear and quadratic approximations to $f(x) = \sqrt[3]{x^2+7}$ near $x = 1$.

$f(1) = \sqrt[3]{8} = \boxed{2}$ $\quad f(x) = (x^2+7)^{1/3}$

$f'(x) = \frac{1}{3}(x^2+7)^{-2/3}(2x) = \frac{2}{3}x(x^2+7)^{-2/3}$

$f'(1) = \frac{1}{3}\left(\frac{1}{2^2}\right)(2) = \boxed{\frac{1}{6}}$

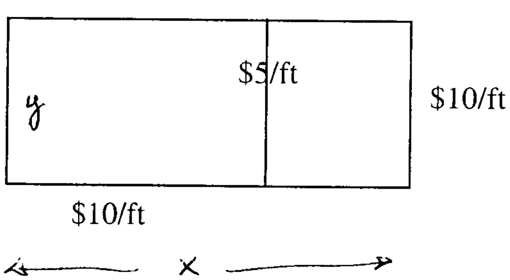
Linear: $\boxed{2 + \frac{1}{6}(x-1)}$

Quadratic: $\boxed{2 + \frac{1}{6}(x-1) + \frac{5}{72}(x-1)^2}$

$f''(x) = \frac{2}{3}x\left(-\frac{2}{3}\right)(x^2+7)^{-5/3}(2x) + \frac{2}{3}(x^2+7)^{-2/3}$

$f''(1) = \left(-\frac{4}{9}\right)\left(\frac{1}{2^5}\right)(2) + \frac{2}{3}\left(\frac{1}{2^2}\right) = -\frac{1}{36} + \frac{2}{12} = \boxed{\frac{5}{36}}$

3. A field needs to be fenced into two rectangular areas with total area of 2000ft^2. The outside fences cost \$10 per foot and the interior fence costs \$5 per foot. Find the dimensions for minimum cost.



Cost $= C = 10x + 10y + 10x + 10y + 5y$

$C = 20x + 25y$

$2000 = xy$ $\quad y = 2000x^{-1}$

$C = 20x + 25 \times 2000x^{-1}$

$C' = 20 - 25 \times 2000x^{-2} = 0$ for C.P.

$20 = \frac{25 \times 2000}{x^2}$

$x^2 = 25 \times 100$

$\boxed{x = 50 \text{ ft} \quad y = \frac{2000}{x} = 40 \text{ ft}}$