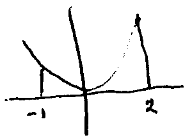
Math 21J Quiz 8 October 30, 2007 Name: Key

1. (a) Find the area under $f(x) = x^4$ from x = -1 to x = 2. (b) Find the area under $f(x) = x^3$ from x = 1 to x = 3.

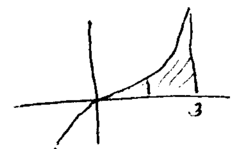
(a) Area under x^4 from 0 to b is $\frac{b^5}{5}$ so



area 0 to 2 is $\boxed{\frac{2^5}{5}}$

area 0 to 1 is the same as area -1 to 0 and is $\boxed{\frac{1^5}{5}}$ total is $\frac{32}{5} + \frac{1}{5} = \boxed{\frac{33}{5}}$

(b) area under x^3 from 0 to b is $\frac{b^4}{4}$.



Area 0 to 3 is $\frac{3^4}{4}$

Area 0 to 1 is $\frac{1^4}{4}$

Difference is $\frac{81}{4} - \frac{1}{4} = \frac{80}{4} = \boxed{20}$

2. Evaluate: (a) $\sum_{k=2}^{5} k^2$ (b) $\sum_{k=1}^{5} k(k+1)$ (c) $\sum_{m=1}^{5} m(m+1)$

(a) $= 2^2 + 3^2 + 4^2 + 5^2$
$= \boxed{54}$

(b) $= 1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + 5\cdot 6$
$= \boxed{70}$

(c) Same as (b)
$\boxed{70}$

3. Write in sigma (Σ) notation:

(a) $1^3 + 2^3 + 3^3 + \cdots + n^3$

$= \sum_{k=1}^{n} k^3$

(b) $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots + \frac{n}{n+1}$

$= \sum_{k=2}^{n} \frac{k}{k+1}$

4. We know that $\sum_{k=1}^{n} k^2 = \frac{1}{3}n^3 + an^2 + bn$ for all $n \geq 1$. Find a and b.

$n=1$: $\sum_{k=1}^{1} k^2 = \frac{1}{3} + a + b$

$1 = \frac{1}{3} + a + b$

$n=2$: $\sum_{k=1}^{2} k^2 = \frac{8}{3} + 4a + 2b$

$5 = \frac{8}{3} + 4a + 2b$

So
$a + b = 2/3$ $(1 - 1/3)$
$4a + 2b = 7/3$ $(5 - 8/3)$
$\times 2$: $2a + 2b = 4/3$

subt $2a = 3/3 = 1$

$\boxed{a = 1/2}$

$b = \frac{2}{3} - a = \frac{2}{3} - \frac{1}{2} = \boxed{\frac{1}{6} = b}$