

In the tables that follow, definite exceptions are given in **bold**.

Values not known to belong to $B(Q_{\geq 8})$

10	12	14	15	18	20	21	22	24	26	28	30
33	34	35	36	38	39	40	42	44	45	46	48
50	51	52	54	55	56	58	60	62	63	66	68
69	70	74	75	76	77	78	82	84	85	86	87
90	92	93	94	95	98	100	102	106	108	110	111
114	116	118	119	122	124	126	130	132	134	138	140
142	146	148	150	154	159	162	164	166	170	172	174
175	178	180	182	183	186	188	190	194	196	198	202
204	206	210	212	214	218	220	222	226	228	230	234
236	238	242	244	246	250	252	258	260	262	266	268
270	274	276	278	282	284	286	290	292	294	298	300
302	303	306	308	310	314	318	322	324	326	330	332
334	335	338	340	346	348	350	354	356	358	362	364
366	370	372	374	378	380	382	386	388	390	394	396
398	402	406	410	412	414	418	420	422	426	428	430
434	436	438	442	444	446	450	452	454	458	460	462
466	468	470	474	476	478	482	484	486	490	492	494
498	500	502	506	508	510	514	516	518	526	530	532
534	542	546	548	550	554	556	558	562	564	566	570
572	578	580	582	586	588	594	596	598	602	604	606
610	612	614	618	620	622	626	628	630	634	636	638
642	644	646	650	652	654	658	660	662	666	668	670
674	676	678	682	684	686	690	692	694	698	700	702
706	708	710	716	718	722	724	726	730	732	734	738
740	742	746	748	750	754	756	758	762	764	766	770
772	774	778	780	786	788	794	796	798	802	804	806
810	812	814	818	820	822	826	828	830	834	836	842
844	846	850	852	854	858	860	862	866	868	874	876
878	882	884	886	890	892	894	898	902	906	908	910
914	916	918	922	926	930	934	938	940	942	946	948
950	954	956	958	964	966	982	994	996	998	1002	1004
1006	1010	1012	1014	1018	1020	1022	1026	1030	1034	1036	1038
1042	1044	1046	1050	1052	1054	1058	1060	1062	1066	1068	1070
1074	1076	1078	1082	1084	1086	1090	1092	1094	1098	1100	1106
1108	1110	1114	1116	1118	1122	1124	1126	1130	1132	1138	1140

1146	1150	1154	1162	1186	1188	1190	1194	1206	1210	1218	1226
1234	1242	1250	1252	1258	1266	1274	1306	1314	1322	1330	1354
1378	1386	1394	1402	1410	1418	1426	1434	1442	1450	1490	1506
1514	1522	1554	1570	1578	1586	1594	1602	1610	1618	1626	1650
1658	1682	1698	1714	1722	1730	1754	1762	1786	1802		

Theorem 0.1 *If $v \equiv 0, 1 \pmod{8}$ then a $(v, \{8, 9\})$ -PBD exists except possibly for $v \in \{16-56, 88-113, 144-225, 248-281, 304-337, 360-393, 416, 417, 448, 472-497, 528-560, 600-616, 624, 808-840, 888-896, 952, 1064, 1384-1408, 1456-1465, 1496-1505, 1528-1624, 1664-1680, 1720, 1721, 1776-1793, 1840-1848, 2408, 3080, 3136, 3192, 3296, 3297, 3352-3353\}$.*

Theorem 0.2 *Let $Q_{0,1(8)}$ denote the set of prime powers $\equiv 0$ or $1 \pmod{8}$. If $v \equiv 0$ or $1 \pmod{8}$, then there exists a $(v, Q_{0,1(8)})$ -PBD except possibly when $v \in \{24, 33, 40, 48, 56, 88, 96, 104, 105, 112, 160, 161, 168, 176, 177, 184, 185, 192, 224, 312, 368, 377, 384, 448, 888, 896\}$.*

Theorem 0.3 *Let $Q_{0,1,5(8)}$ denote the set of prime powers $\equiv 0, 1$ or $5 \pmod{8}$ and ≥ 8 . Then a $(v, Q_{0,1,5(8)})$ -PBD exists in the following cases:*

(1) $v \equiv 0, 1$ or $5 \pmod{8}$ and $v \notin E$ where $E = \{21, 24, 33, 40, 45, 48, 56, 69, 77, 85, 88, 93, 96, 160, 161, 165, 168, 176, 177, 184, 185, 192, 224, 368, 384\}$.

(2) $v \equiv 4 \pmod{8}$, and either $v \geq 2612$ or $v \in \{316, 404, 900, 924, 932, 1028, 1340, 1348, 1652, 1660, 1668, 1964, 2060, 2068, 2292, 2332, 2388, 2492, 2556, 2564, 2580, 2588, 2596\}$.

Theorem 0.4 *Let $E_{57} = \{9, 11, 13, 15, 17, 19, 23, 27, 29, 31, 33, 37, 39\}$ and $P_{57} = \{43, 47, 51, 53, 55, 57, 59, 63, 69, 71, 73, 75, 77, 79, 83, 87, 89, 93, 95, 97, 99, 107, 109, 111, 113, 115, 119, 133, 135, 137, 139, 153, 157, 159, 173, 177, 179, 191, 193, 195, 199, 211, 219, 231, 233, 235, 237, 239, 253, 255, 263, 279, 291, 299, 303, 347, 351, 353, 355, 359, 363, 383, 399, 407, 413, 419, 423, 431, 435, 439, 443, 447, 453, 459, 471, 473, 475, 477, 479, 483, 503, 507, 519, 531, 533, 535, 559, 639\}$. If v is odd and ≥ 5 , then $v \in B(\{5, 7\})$ except for $v \in E_{57}$ and possibly for $v \in P_{57}$.*

Theorem 0.5 *Let $E_{58} = \{9, 12, 13, 16, 17, 20, 24, 28, 29, 32, 33, 37, 44, 49, 52\}$ and $P_{58} = \{53, 60, 68, 69, 72, 73, 76, 77, 84, 89, 92, 93, 96, 97, 100, 104, 108, 109, 112, 113, 116, 124, 129, 132, 136, 140, 149, 152, 153, 156, 164, 169, 172, 189, 192, 209, 244, 268, 272, 273, 284, 292, 300, 308, 312, 313, 324, 332, 333, 348, 352, 353, 364, 373, 404, 412, 444, 452, 468, 492, 508, 524, 532, 552, 564, 572, 580, 588, 628, 644, 724, 732, 772, 812, 852\}$. If $v \equiv 0, 1 \pmod{4}$, then $v \in B(\{5, 8\})$ except for $v \in E_{58}$ and possibly for $v \in P_{58}$.*

Values ≥ 5 and $\equiv 0, 1 \pmod{5}$ not known to belong to $B(\{5, 6\})$

10 11 15 16 20 35 40 50 51 80

Values ≥ 5 and $\equiv 1 \pmod{4}$ not known to belong to $B(\{5, 9\})$

13 17 29 33 113

Values ≥ 5 not known to belong to $B(\{5, 6, 7\})$

8 9 10 11 12 13 14 15 16 17 18 19 20 22 23 24
27 28 29 32 33 34 68 69 93 94 98 99 104 108 109 114
 124

Values ≥ 5 not known to belong to $B(\{5, 6, 8\})$

7 9 10 11 12 13 14 15 16 17 18 19 20 22 23 24
27 28 29 32 33 34 35 37 38 39 42 44 47 50 51 52
 53 58 59 62 63 67 68 69 72 74 77 78 79 82 83 84
 87 89 92 94 97 98 99 103 104 107 109 112 114 118 119 122
 123 124 129 139 142 149 152 154 159 169 172 174 179 182 189 194
 199 202 214 219 239 242 244 247 259 262 267 272 312

Values ≥ 5 not known to belong to $B(\{5, 6, 9\})$

7 8 10 11 12 13 14 15 16 17 18 19 20 22 23 24
27 28 29 32 33 34 35 38 39 40 42 43 44 47 48 52
 58 59 62 64 67 68 72 79 80 82 83 84 87 88 92 98
 99 102 103 104 107 108 112 113 118 119 122 123 124 127 128 132
 138 142 147 148 152 158 162 167 172 178 179 182 188 199 207 208
 218 219 227 238 248 288

Values ≥ 5 not known to belong to $B(\{5, 7, 8\})$

6 9 10 11 12 13 14 15 16 17 18 19 20 22 23 24
26 27 28 29 30 31 32 33 34 37 38 39 42 43 44 46
 47 51 52 53 58 59 60 62 66 68 69 70 71 72 73 74
 75 76 77 78 79 82 83 84 86 87 89 90 93 94 95 96
 97 98 99 100 102 104 106 107 108 109 110 111 114 115 116 118
 122 124 126 130 132 134 135 136 138 140 142 146 150 152 153 154
 156 158 162 164 166 170 172 174 178 186 190 191 192 194 195 198
 202 206 210 211 214 226 230 234 244 258 262 268 272 274 278 282
 284 298 300 338 359 422 443 471 478 562

Values ≥ 5 and $\equiv 1 \pmod{2}$ not known to belong to $B(\{5, 7, 9\})$

11 13 15 17 19 23 27 29 31 33 39 43 51 59 71 75
 83 87 95 99 107 111 113 115 119 139 179

Values ≥ 5 and $\equiv 0, 1 \pmod{4}$ not known to belong to $B(\{5, 8, 9\})$

12 13 16 17 20 24 28 29 32 33 44 52 60 68 84 92
 96 100 104 108 112 113 116 124 132 140 156 172 244 268 300 308

Values ≥ 5 not known to belong to $B(\{5, 6, 7, 8\})$

9 10 11 12 13 14 15 16 17 18 19 20 22 23 24 27
28 29 32 33 34 94 124

Values ≥ 5 not known to belong to $B(\{5, 6, 7, 9\})$

8 10 11 12 13 14 15 16 17 18 19 20 22 23 24 27
28 29 32 33 34 68 98 99 104 108 124

Values ≥ 5 not known to belong to $B(\{5, 6, 8, 9\})$

7 10 11 12 13 14 15 16 17 18 19 20 22 23 24 27
28 29 32 33 34 35 38 39 42 47 52 67 79 82 83 84
 87 92 98 99 103 107 118 119 122 123 124 142 172 182

Values ≥ 5 not known to belong to $B(\{5, 7, 8, 9\})$

6 10 11 12 13 14 15 16 17 18 19 20 22 23 24 26
27 28 29 30 31 32 33 34 38 39 42 43 44 46 51 52
 60 94 95 96 98 99 100 102 104 106 107 108 110 111 116 138
 139 140 142 146 150 154 156 158 162 166 170 172 174 206

Values ≥ 5 not known to belong to $B(\{5, 6, 7, 8, 9\})$

10 11 12 13 14 15 16 17 18 19 20 22 23 24 27 28
29 30 31 32 33 34

Values ≥ 5 and $\equiv 1 \pmod{2}$ not known to belong to $B(OQ_{\geq 5})$

15 33 39 51 75 87