NAME:

Math 22 Spring 2009—QUIZ 11

1. Find the sum of the series $\frac{2}{3} - \frac{1}{2} + \frac{3}{8} - \frac{9}{32} + - \ldots$

Solution: This is a geometric series with first term $2/3$ and common ratio $-3/4$, so the sum is

$$\frac{2/3}{1-(-3/4)} = \frac{2/3}{7/4} = \frac{8}{21}.$$

2. A sequence is defined recursively by $a_1 = \sqrt{3}$ and $a_{n+1} = \sqrt{3a_n + 4}$ for $n = 1, 2, 3, \ldots$. Assume the sequence converges to a limit $\lim_{n\to\infty} a_n = s$. Find s.

Solution: Since $a_{n+1} = \sqrt{3a_n + 4}$, we have

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{3a_n + 4},$$

so

$$s = \sqrt{3s + 4}.$$

This is the equation $s^2 = 3s + 4$, or $s^2 - 3s - 4 = 0$, i.e., $(s-4)(s+1) = 0$, so $s = 4$ or $s = -1$. Since the terms of the sequence are positive,

$$s = \lim_{n\to\infty} a_n = 4.$$

3. Determine whether the following series converge or diverge:

Solution:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{n^4-1}$ converges diverges

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\sin n}{n^2}$ converges diverges

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{2n^2+n+1}$ converges diverges

(d) $\displaystyle\sum_{n=1}^{\infty} (1+\frac{1}{n})^2 e^{-n}$ converges diverges

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n}$ converges diverges

(f) $\displaystyle\sum_{n=1}^{\infty} \cos(1/n)$ converges diverges

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges diverges

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges diverges

(i) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \sin(1/n)$ converges diverges

(j) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}$ converges diverges