NAME:

Math 22 Spring 2009—QUIZ 12

1. Find the sum of the series $\frac{3}{4} - \frac{1}{2} + \frac{1}{3} - \frac{2}{9} + \frac{4}{27} - + \ldots$

Solution: This is a geometric series with first term $3/4$ and common ratio $-2/3$, so the sum is

$$\frac{3/4}{1-(-2/3)} = \frac{3/4}{5/3} = \frac{9}{20}.$$

2. Determine whether the following series are absolutely convergent conditionally convergent, or divergent:

Solution:

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n e^{1/n}}{n^3}$ absolutely convergent conditionally convergent divergent

(b) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$ absolutely convergent conditionally convergent divergent

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}$ absolutely convergent conditionally convergent divergent

3. Find the radius of convergence and the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$.

Solution: Using the ratio test:

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x|^{n+1}}{\sqrt{n+1}} \frac{\sqrt{n}}{|x|^n} = \lim_{n\to\infty} |x| \frac{\sqrt{n}}{\sqrt{n+1}} = |x|$$

so the series converges for $-1 < x < 1$ and diverges if $|x| > 1$. Hence the radius of convergence is 1 . For the interval of convergence, it remains to check the endpoints $x = 1$ and $x = -1$.

When $x = 1$, the series is $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, which diverges, and when $x = -1$, the series is $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$, which is a convergent alternating series. Hence the interval of convergence is

$$-1 \le x < 1 \qquad \text{or} \qquad [-1, 1).$$

4. Write down the Maclaurin series (i.e., the Taylor series at $x = 0$) expansion for each of the following functions:

Solution:

(a) $e^{2x} = 1 + \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \ldots + \frac{(2x)^n}{n!} + \ldots = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \ldots + \frac{2^n}{n!}x^n + \ldots$

(b) $\sin x^2 = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - + \ldots = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - + \ldots + (-1)^n \frac{x^{2(2n+1)}}{(2n+1)!} + \ldots$

(c) $\cos x/2 = 1 - \frac{(x/2)^2}{2!} + \frac{(x/2)^4}{4!} - + \ldots = 1 - \frac{1}{8}x^2 + \frac{1}{2^4 4!}x^4 + \ldots + (-1)^n \frac{1}{2^{2n}(2n)!}x^{2n} + \ldots$

(d) $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - + \ldots$