

NAME:

Math 22 Spring 2009—QUIZ 14

1. Find the integrating factor and use it to solve the initial value problem $y' = x + y$, $y(0) = 2$.

Solution: Writing the linear equation in standard form gives $y' - y = x$, so the integrating factor is

$$e^{\int (-1) dx} = e^{-x}.$$

Hence the equation becomes

$$(e^{-x}y)' = xe^{-x}.$$

Integrating gives

$$e^{-x}y = \int xe^{-x} dx.$$

Making the substitution $u = -x$, so $du = -dx$ in the integral on the right hand side (or you can integrate directly by parts) we have

$$\int xe^{-x} dx = \int (-u)e^u(-du) = \int ue^u du = e^u(u - 1) = e^{-x}(-x - 1).$$

Then the equation becomes

$$e^{-x}y = e^{-x}(-x - 1) + C$$

so

$$y = -x - 1 + Ce^x.$$

The initial condition is that $y = 2$ when $x = 0$, so substituting these values gives

$$2 = -1 + C,$$

which gives $C = 3$ and the solution is

$$y = -x - 1 + 3e^x.$$

2. In the predator-prey system of equations

$$\begin{aligned}\frac{dx}{dt} &= -0.05x + 0.0001xy \\ \frac{dy}{dt} &= 0.1y - 0.005xy\end{aligned}$$

determine which of x or y represents the prey population and which represents the predator population.

Solution: Without interaction (i.e., without the xy terms), the population for y increases and the population for x decreases. Hence x represents the predator and y represents the prey populations.

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3. Find all solutions to the differential equation $y'' + 3y' + 2y = 0$.

Solution: The equation is $(D^2 + 3D + 2)y = 0$. Since $D^2 + 3D + 2 = (D + 1)(D + 2)$, the complete set of solutions to the equation is given by

$$y = C_1e^{-x} + C_2e^{-2x}.$$

4. Your homework assignment included an Exercise to read (without doing any of the exercises) page 590 in Section 9.3. In ten words or less, what was the topic on this page?

Solution: The page considered [whether it takes a ball longer to go up or come down](#) .