NAME:

Math 22 Spring 2009—QUIZ 1

1. Sketch the region enclosed by the curves $y = x^2$ and $x = y^2$ and find the area of the region.

Solution: The area is obtained by integrating the height between the two curves from $x = 0$ to $x = 1$, so is given by

$$\int_{x=0}^{1} (\sqrt{x} - x^2)\, dx = \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3} \right) \Big|_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}.$$

2. Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = x$ about the y-axis. Sketch the region, the solid, and a typical washer.

Solution: For each y from 0 to 1, the cross section of the region is a washer with outer radius given by the x-coordinate of the point on the parabola $y = x^2$ (so $x = \sqrt{y}$) and inner radius given by the x-coordinate of the point on the line $y = x$ (so $x = y$). The width of the washer is dy, so the volume of the solid is given by

$$\int_0^1 \pi \left[(\sqrt{y})^2 - (y)^2 \right] dy.$$

Evaluating the integral we obtain

$$V = \int_0^1 \pi \left[(\sqrt{y})^2 - (y)^2 \right] dy = \pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1 = \pi \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{\pi}{6}.$$