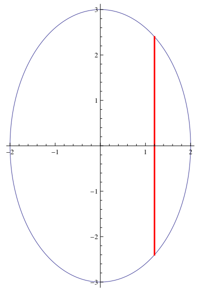
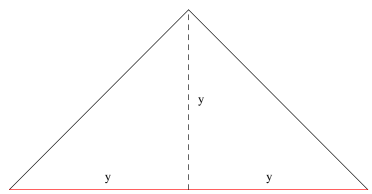
NAME:

Math 22 Spring 2009—QUIZ 2

1. The base S of a solid is an elliptical region with boundary curve $9x^2 + 4y^2 = 36$. Cross sections perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base. Find the volume V of the solid.

Solution: The x-coordinate of points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ varies from $x = -2$ to $x = 2$. For each value of x between -2 and 2 the cross section is an isosceles right triangle with hypotenuse $2y$.





The area of such a triangle is y^2, so the volume of each cross section is $y^2 dx$. By symmetry, the volume of the solid is twice the volume of the portion when $x \geq 0$, so

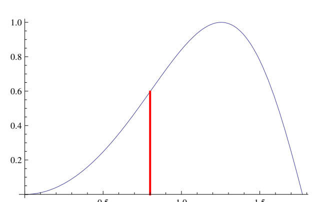
$$V = 2\int_0^2 y^2\,dx.$$

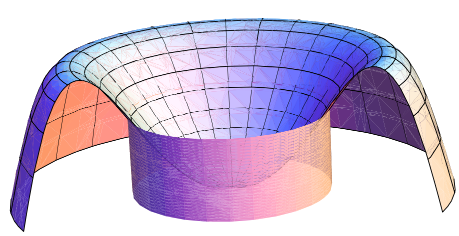
From the equation of the ellipse, $y^2 = 9(1 - x^2/4)$, so

$$V = 2\int_0^2 9(1 - x^2/4)\,dx = 18\left(x - \frac{x^3}{12}\right)\Big|_0^2 = 18(2 - \frac{8}{12}) = 18(4/3) = 24.$$

2. Let S be the solid obtained by rotating the region under the curve $y = \sin(x^2)$ between $x = 0$ and $x = \sqrt{\pi}$ about the y-axis. Write down an integral for the volume of S using cylindrical shells (you need not evaluate the integral).

Solution:





For each x from 0 to $x = \sqrt{\pi}$, the cylindrical shell obtained by rotating the line segment from $y = 0$ to $y = \sin(x^2)$ has volume $dV = 2\pi x y dx = 2\pi x \sin(x^2) dx$, so the volume of the solid is given by

$$\int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\,dx.$$

While not required, this integral can be easily evaluated, since the derivative of $\cos(x^2)$ is $-2x\sin(x^2)$, so

$$\int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\,dx = \pi(-\cos(x^2))\Big|_0^{\sqrt{\pi}} = \pi(-\cos(\pi) - (-\cos 0)) = \pi(1 + 1) = 2\pi.$$

3. Find the average value of the function $f(x) = 4x - x^2$ on the interval $[0, 4]$.

Solution: The average value is given by

$$\frac{1}{4}\int_0^4 (4x - x^2)\, dx = \frac{1}{4}\left(2x^2 - \frac{x^3}{3}\right)\Big|_0^4 = \frac{1}{4}(32 - \frac{64}{3}) = \frac{8}{3}.$$