NAME:

Math 22 Spring 2009—QUIZ 4

1. For each of the following, write down a trigonometric substitution that can be used to evaluate the integral (you need not evaluate any integrals).

(a) $\int \frac{1}{x^2\sqrt{x^2-9}}dx \qquad x = 3\sec\theta$

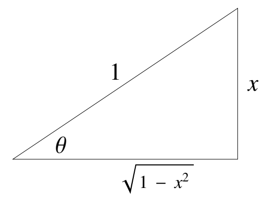
(b) $\int x^3\sqrt{25-x^2}dx \qquad x = 5\sin\theta$

(c) $\int \frac{x^3}{\sqrt{x^2+9}}dx \qquad x = 3\tan\theta$

2. Evaluate the integral $\int x^3\sqrt{1-x^2}dx$.

Solution: Make the substitution $x = \sin\theta$. Then $dx = \cos\theta\, d\theta$ and $\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \cos\theta$, so

$$\begin{aligned}\int x^3\sqrt{1-x^2}dx &= \int \sin^3\theta\cos\theta(\cos\theta\, d\theta)\\ &= \int \sin^3\theta\cos^2\theta\, d\theta\\ &= \int \sin\theta(1-\cos^2\theta)\cos^2\theta\, d\theta\\ &= \int \sin\theta\cos^2\theta\, d\theta - \int \sin\theta\cos^4\theta\, d\theta\\ &= -\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5}.\end{aligned}$$



Since $\cos\theta = \sqrt{1-x^2}$ this gives

$$\begin{aligned}\int x^3\sqrt{1-x^2}dx &= -\frac{(\sqrt{1-x^2})^3}{3} + \frac{(\sqrt{1-x^2})^5}{5} + C\\ &= (1-x^2)^{3/2}\left(-\frac{1}{3} + \frac{1-x^2}{5}\right)\\ &= \frac{-(1-x^2)^{3/2}(2+3x^2)}{15} + C\end{aligned}$$