NAME:

Math 22 Spring 2009—QUIZ 5

1. For each of the following, write out the form of the partial fraction decomposition of the rational function. Do not determine the numerical values of the coefficients.

(a) $\dfrac{2x}{(x+2)(3x+1)} = \dfrac{A}{x+2} + \dfrac{B}{x+1/3}$ or $\dfrac{A}{x+2} + \dfrac{B}{3x+1}$.

(b) $\dfrac{1}{x^3+2x^2+x} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

(c) $\dfrac{x^4+1}{x^5+4x^3} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{Dx+E}{x^2+4}$

(d) $\dfrac{t^4+t^2+1}{(t^2+1)(t^2+4)^2} = \dfrac{At+B}{t^2+1} + \dfrac{Ct+D}{t^2+4} + \dfrac{Et+F}{(t^2+4)^2}$

2. Evaluate the integral $\int_0^1 \dfrac{x-1}{x^2+3x+2}dx$.

Solution: Decomposing the partial fraction:

$$\frac{x-1}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2}$$

which gives

$$x-1 = A(x+2) + B(x+1).$$

Substituting $x=-1$ gives $-2=A$ and substituting $x=-2$ gives $-3=B(-1)$. Hence

$$\begin{aligned}\int_0^1 \frac{x-1}{x^2+3x+2}dx &= \int_0^1 \left[\frac{-2}{x+1} + \frac{3}{x+2}\right] dx \\ &= -2\ln(x+1) + 3\ln(x+2)\Big|_0^1 \\ &= (-2\ln 2 + 3\ln 3) - (-2\ln 1 + 3\ln 2) \\ &= -5\ln 2 + 3\ln 3 = \ln(27/32).\end{aligned}$$

3. Use integration by parts and partial fractions to evaluate the integral $\int 2x \arctan x \; dx$.

Solution: First integrate by parts, letting $u = \arctan x$ (so $du = \frac{1}{x^2+1} dx$) and $dv = 2x \; dx$ (so $v = x^2$), to give

$$\int 2x \arctan x \; dx = x^2 \arctan x - \int x^2 \frac{1}{x^2+1} dx.$$

Dividing x^2 by x^2+1 gives

$$\frac{x^2}{x^2+1} = 1 - \frac{1}{x^2+1}$$

so

$$\int x^2 \frac{1}{x^2+1} dx = \int \left(1 - \frac{1}{x^2+1}\right) dx = x - \arctan x.$$

Hence

$$\int 2x \arctan x \; dx = x^2 \arctan x - x + \arctan x + C.$$