

NAME:

Math 22 Spring 2009—QUIZ 6

1. Evaluate the integral $\int \frac{2x + 1}{x^2 + 2x + 10} dx$.

Solution:

$$\begin{aligned}\int \frac{2x + 1}{x^2 + 2x + 10} dx &= \int \frac{2x + 2}{x^2 + 2x + 10} dx - \int \frac{1}{x^2 + 2x + 10} dx \\ &= \ln(x^2 + 2x + 10) - \int \frac{1}{(x + 1)^2 + 9} dx \\ &= \ln(x^2 + 2x + 10) - \frac{1}{3} \arctan \frac{x + 1}{3} + C.\end{aligned}$$

2. Determine whether each of the following integrals is convergent or divergent. Evaluate those that are convergent.

$$(a) \int_1^{\infty} \frac{1}{x} dx = \lim_{t \rightarrow \infty} \ln x \Big|_1^t = \lim_{t \rightarrow \infty} (\ln t) = \text{divergent} .$$

$$(b) \int_1^{\infty} \frac{1}{x^2} dx = \lim_{t \rightarrow \infty} \frac{-1}{x} \Big|_1^t = \lim_{t \rightarrow \infty} (1 - \frac{1}{t}) = 1 = \text{convergent}.$$

$$(c) \int_0^{\infty} \frac{1}{x^2 + 1} dx = \lim_{t \rightarrow \infty} \arctan x \Big|_1^t = \lim_{t \rightarrow \infty} \arctan t = \pi/2 = \text{convergent}.$$