NAME:

Math 22 Spring 2009—QUIZ 7

1. The approximate values of the function $f(x) = \sin x$ for x between 0 and π are the following:

$$f(0) = 0 \quad f(\pi/8) = 0.38 \quad f(\pi/4) = 0.71 \quad f(3\pi/8) = 0.92 \quad f(\pi/2) = 1.00$$

$$f(5\pi/8) = 0.92 \quad f(3\pi/4) = 0.71 \quad f(7\pi/8) = 0.38 \quad f(\pi) = 0$$

Use Simpson's Rule with $n = 8$ to determine an approximate value of $\int_0^\pi \sin x \, dx$ (set up the sum – you need not evaluate the sum).

Solution: Using $n = 8$, $a = 0$, $b = \pi$ in Simpson's Rule gives

$$\begin{aligned}\int_0^\pi \sin x \, dx \sim \frac{\pi}{3(8)}\big[f(0) + 4f(\pi/8) + 2f(\pi/4) + 4f(3\pi/8) + 2f(\pi/2) + \\ 4f(5\pi/8) + 2f(3\pi/4) + 4f(7\pi/8) + f(\pi)\big] \\ = \frac{\pi}{24}\big[0 + 4(0.38) + 2(0.71) + 4(0.92) + 2(1.00) + \\ 4(0.92) + 2(0.71) + 4(0.38) + 0\big] \\ = 1.99\end{aligned}$$

2. Find the length of the curve $y = \ln(\sec x)$, $0 \le x \le \pi/4$.

Solution: The arclength differential is given by $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$. Here

$$\frac{dy}{dx} = \frac{1}{\sec x} \sec x \tan x = \tan x$$

so

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \sqrt{1 + (\tan x)^2}\, dx = \sec x \, dx.$$

Hence the arclength is given by

$$\begin{aligned}\int_0^{\pi/4} \sec x \, dx &= \ln(\sec x + \tan x)\Big|_0^{\pi/4} \\ &= \ln(\sqrt{2} + 1) - \ln(1 + 0) \\ &= \ln(1 + \sqrt{2}).\end{aligned}$$

3. Set up, but do not evaluate, an integral giving the surface area obtained by rotating the curve $y = x^4$, $0 \leq x \leq 1$ (a) about the x-axis, and (b) about the y-axis.

Solution: The surface area is given by integrating $2\pi \ell \, ds$ where ℓ is the distance from the arclength differential ds to the axis of rotation. In (a), the distance to the axis of rotation is $\ell = y$ and in (b), the distance to the axis of rotation is $\ell = x$.

Since $y = x^4$, $dy/dx = 4x^3$, and the arclength differential is given by

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx = \sqrt{1 + 16x^6} \, dx.$$

Hence

(a) The surface area is given by

$$\int_0^1 2\pi y \sqrt{1 + 16x^6} \, dx = 2\pi \int_0^1 x^4 \sqrt{1 + 16x^6} \, dx.$$

(a) The surface area is given by

$$\int_0^1 2\pi x \sqrt{1 + 16x^6} \, dx = 2\pi \int_0^1 x \sqrt{1 + 16x^6} \, dx.$$