NAME:

Math 22 Spring 2009—QUIZ 8

1. Find the centroid of the region bounded by $y = e^x$, $y = 0$, $x = 0$, $x = 1$.

Solution: First, the area of the region is given by

$$A = \int_0^1 e^x \, dx = e^x \Big|_0^1 = e - 1.$$

Then the coordinates of the centroid are given by

$$\bar{x} = \frac{1}{A}\int_0^1 xe^x \, dx = \frac{1}{e-1} e^x (x-1)\Big|_0^1 = \frac{1}{e-1}$$

and

$$\bar{y} = \frac{1}{A}\int_0^1 \frac{1}{2}(e^x)^2 \, dx = \frac{1}{A}\int_0^1 \frac{1}{2}e^{2x} \, dx = \frac{1}{e-1}\frac{1}{4}e^{2x}\Big|_0^1 = \frac{1}{e-1}\frac{1}{4}(e^2-1) = \frac{e+1}{4},$$

so the centroid of the region is the point $(\bar{x}, \bar{y}) = (\frac{1}{e-1}, \frac{e+1}{4})$.

2. Use Pappus' Theorem to find the volume of the solid obtained by rotating the region in problem 1 above (a) about the y-axis, and (b) about the x-axis.

Solution: The area of the region is $A = e - 1$, so by Pappus' Theorem,

(a) the volume of the solid rotated around the y-axis is $2\pi \bar{x} A = 2\pi \frac{1}{e-1}(e-1) = 2\pi$.

(b) the volume of the solid rotated around the x-axis is $2\pi \bar{y} A = 2\pi \frac{e+1}{4}(e-1) = \frac{\pi(e^2-1)}{2}$.