NAME:

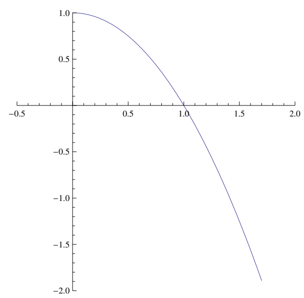
Math 22 Spring 2009—QUIZ 9

1. Sketch the curve defined by the parametric equations $x = \sqrt{t}$, $y = 1 - t$.

Solution: The curve can be sketched by plotting some points for various values of t, or by eliminating t to obtain the Cartesian equation for the curve:

$$y = 1 - t = 1 - x^2,$$

noting that $x = \sqrt{t}$ is always positive, which gives the portion of the parabola shown:



2. Find the (exact) length of the curve defined parametrically by $x = 1 + 3t^2$, $y = 4 + 2t^3$, for $0 \le t \le 1$.

Solution: The arclength is given by

$$s = \int ds = \int_{t=0}^{t=1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

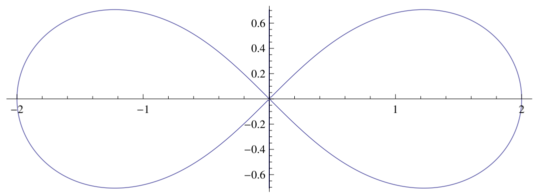
where here

$$\frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 6t^2$$

so

$$s = \int_0^1 \sqrt{36t^2 + 36t^4}\, dt = 6\int_0^1 t\sqrt{1+t^2}\, dt = 6\frac{(1+t^2)^{3/2}}{3}\Bigg|_0^1 = 2(2^{3/2} - 1) = 4\sqrt{2} - 2.$$

3. Find the area enclosed by the curve defined in polar coordinates by $r^2 = 4\cos 2\theta$:



(This figure is called a *lemniscate* from the French (and Latin and Greek...) for a pendant ribbon.)

Solution: By symmetry, the area A enclosed by the figure is 4 times the area in the first quadrant, so

$$A = 4\int_{\theta=0}^{\theta=\pi/4} \frac{1}{2}r^2\,d\theta = 4\int_0^{\pi/4} \frac{1}{2}(4\cos 2\theta)\,d\theta = 8\int_0^{\pi/4} \cos 2\theta\,d\theta = 8\frac{\sin 2\theta}{2}\Big|_0^{\pi/4} = 4.$$