ERRATA

Abstract Algebra, Second Edition

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If you have the Prentice-Hall version of the Second Edition you must incorporate all these errata as well as those in the separate file on these web pages. (The Prentice-Hall version is the first printing of this edition of the book.)

For the John Wiley & Sons versions there are currently three more printings—the latest version is therefore the fourth printing. To determine which printing you have, look on the bottom of your copyright page for the sequence: 10 9 8 ... n: you have the n^{th} printing (e.g., the fourth printing has the sequence 10 9 8 7 6 5 4).

Errata marked with an asterisk have been corrected in the fourth printing (a few of these errata were already corrected by the third printing); so, if you have the fourth printing, you only need to incorporate the <u>un</u>starred errata.

Individuals using any printing of the Second Edition must make corrections listed in the errata to the Third Edition as well (except for changes to text that is new in the Third Edition).

*** page 10, line 5**
from: $a_1a_2 = (b_1 + sn)(b_2 + tn) = b_1b_2 + (b_1t + b_2s + st)n$
to: $a_1a_2 = (b_1 + sn)(b_2 + tn) = b_1b_2 + (b_1t + b_2s + stn)n$

*** page 11, lines 2 and 3**
from: $\{\underline{1}, \underline{2}, \underline{4}, \underline{5}, \underline{7}, \underline{8}\} \dots \{\underline{1}, \underline{5}, \underline{7}, \underline{2}, \underline{4}, \underline{8}\}$
to: $\{\overline{1}, \overline{2}, \overline{4}, \overline{5}, \overline{7}, \overline{8}\} \dots \{\overline{1}, \overline{5}, \overline{7}, \overline{2}, \overline{4}, \overline{8}\}$

*** page 23, Exercise 33(b)**
from: Prove that ... $i \equiv k \bmod n$.
to: Prove that if $n = 2k$ and $1 \leq i < n$ then $x^i = x^{-i}$ if and only if $i = k$.

*** page 34, Definition**
from: A field is a triple $(F, +, \times)$ such that $(F, +)$ is
to: A *field* is a set F together with two binary operations $+$ and $\cdot$ on F such that $(F, +)$ is

page 39, line −4
from: $ba = ab^{-1}$
to: $ba = a^{-1}b$

*** page 50, Exercise 13**
Replace exercise with: Let H be a subgroup of the additive group of rational numbers such that $1/x \in H$ for every nonzero element x of H. Prove that $H = 0$ or $\mathbb{Q}$.

*** page 62, Exercise 26, lines 1 and 2**
from: Let $Z_n = \langle x \rangle$ be a cyclic group of order n ...
to: Let Z_n be a cyclic group of order n ...
$\sigma_a : Z_n \to Z_n$ by $\sigma_a(x) = x^a$ for all $x \in Z_n$.

*** page 86, Exercise 1**
Add: If $E \trianglelefteq H$ prove that $\varphi^{-1}(E) \trianglelefteq G$. Deduce that $\ker \varphi \trianglelefteq G$.

*** page 93, Example 2, lines 10 to 15**

from: Using the same notation ... G_i is not a normal subgroup.

to: Using the same notation let $k = \tau^{-1}(i)$, so that $\tau(k) = i$. By similar reasoning we see that

$$G_i\tau = \{\lambda \in G \mid \lambda(k) = i\},$$

i.e., the right coset $G_i\tau$ consists of the permutations in S_n which take k to i. If $n > 2$, for some nonidentity element τ we have $\tau G_i \neq G_i\tau$ since there are certainly permutations which take i to j but do not take k to i. Thus G_i is not a normal subgroup.

*** page 105, Theorem 22 (2), displayed line**

from: $M_{\pi(i)+1}/M_{\pi(i)} \cong N_{i+1}/N_i, \qquad 1 \leq i \leq r-1.$

to: $M_{\pi(i)}/M_{\pi(i)-1} \cong N_i/N_{i-1}, \qquad 1 \leq i \leq r.$

*** page 106, line −4**

from: Exercise 5

to: Exercise 8

*** page 124, Exercise 6, line 2**

from: $1N, rH, sN$

to: $1N, rN, sN$

*** page 137, Proof of Proposition 16, line 3**

from: Since φ is an

to: Since ψ_a is an

*** page 148, line 21**

from: $(5-1)\cdot 6 = 30$, accounting for 75 elements.

to: $(5-1)\cdot 6 = 24$, accounting for 69 elements.

*** page 153, Exercise 6, hint**

from: [Use the preceding exercise.]

to: [Show that every pair of elements of D lie in a finite simple subgroup of D.]

*** page 185, Example: (Groups of Order $p^3 \ldots$), line 1**

from: By Exercise 9 of the previous section the map $x \mapsto x^p$ is a homomorphism from G into $Z(G)$. Since $|Z(G)| = p$, the kernel of this homomorphism has order p^2 or p^3.

to: By Exercise 9 of the previous section the map $x \mapsto x^p$ is a homomorphism from G into $Z(G)$ and the kernel of this homomorphism has order p^2 or p^3.

*** page 188, Exercise 13**

from: where p is an odd prime.

to: where p is a prime greater than 3.

*** page 209, lines 1 and 2 of paragraph before Simple Groups of Order 168**

from: order 3159

to: order 1053

*** page 210, lines $-5, -4$**

from: but with each reflection multiplied by the transposition (6 7) to make it an even permutation

to: but multiply each odd permutation of the square by (6 7) to make it an even permutation in S_7

*** pages 214 and 215**

Move group order 351 from Exercise 13 to Exercise 4.

*** page 215, Exercise 13**

from: Add group order 3159

*** page 225, line 2 of paragraph preceding Example 5**

from: William Rowell Hamilton

to: William Rowan Hamilton

*** page 229, lines -6 and -5**

delete: If S contains an identity ... (since $a - b = a + (-1)b$).

*** page 233, Exercise 23, last line**

from: $\mathbb{Q}[\sqrt{D}]$

to: $\mathbb{Q}(\sqrt{D})$

*** page 234, Exercise 30(a), line 2**

from: . . . left inverse.

to: . . . left inverse).

*** page 236, Proposition 4**

from: Let R be an integral domain and let $p(x), q(x) \in R[x]$.

to: Let R be an integral domain and let $p(x), q(x)$ be nonzero elements of $R[x]$.

page 259, Exercise 25, line 1

from: there is a positive integer n

to: there is an integer $n > 1$

*** page 273, line 4**

from: $s = (bc - ad)(c^2 + d^2)$

to: $s = (bc - ad)/(c^2 + d^2)$

*** page 279, Exercise 8(a), hint**

from: $(1 + |D|)/16$

to: $(1 + |D|)^2/(16|D|)$

page 300, Exercise 18

Part (a) should be part of the overall hypotheses and notation for the exercise. Parts (b) to (d) should be re labeled as (a) to (c) respectively.

*** page 301, line following the proof of Corollary 4**

from: Corollary 7

to: Corollary 8

*** page 302, Exercise 3, line 2**
from: Use Proposition 6, Section 8.2
to: Use Proposition 7 in Section 8.2

*** page 305, line 5**
from: cf. Proposition 10, Section 8.3
to: cf. Proposition 12, Section 8.3

*** page 314, Proof of Proposition 15**
from: This follows from Proposition 8.6 applied to
to: This follows from Proposition 7 of Section 8.2 applied to

*** page 316, Exercise 3, line 2**
from: [Use Exercise 3 and Eisenstein's Criterion.]
to: [Use Proposition 18 in Chapter 8 and Eisenstein's Criterion.]

*** page 319, Example 4, line 3**
from: Specifically, if M is an R-module and S is a subring of R, then M is automatically an S-module as well.
to: Specifically, if M is an R-module and S is a subring of R with $1_S = 1_R$, then M is automatically an S-module as well.

*** page 325, Exercise 8(c)**
from: Show that if R has zero divisors then every nonzero R-module has torsion elements.
to: If R has zero divisors show that every nonzero R-module has nonzero torsion elements.

*** page 332, line 1 of text**
from: Let R be a ring.
to: Let R be a ring with 1.

*** page 332, paragraph following Definition**
Add: Note that these definitions do not require that the ring R contain a 1, however this condition ensures that A is contained in RA.

*** page 333, Example 2, line 1**
from: Let R be any ring with 1
to: Let R be a ring with 1

*** page 333, Example 3, line 1**
from: Let R be any ring
to: Let R be a ring with 1

*** page 333, Example 3, last line**
from: It is more difficult to show that this is a *minimal* generating set (for commutative rings this follows from Exercise 2).
to: If R is commutative then this is a *minimal* generating set (cf. Exercise 2).

*** page 335, Definition, line 4**
from: In this situation we say A is a *basis* or *set of free generators* for F and the cardinality of A is called the *rank* of F.
to: In this situation we say A is a *basis* or *set of free generators* for F. If R is a commutative ring the cardinality of A is called the *rank* of F.

*** page 339, first line of Section 10.4**

from: In this section we study the tensor product of two R-modules M and N where R is a ring (not necessarily commutative) containing 1.

to: In this section we study the tensor product of two modules M and N over a ring (not necessarily commutative) containing 1.

*** page 339, Section 10.4, second paragraph**

from: Suppose that the ring R is a subring of the ring S and that N is a left S-module. Then N can also be naturally considered as a left R-module since the elements of R (being elements of S) act on N by assumption.

to: Suppose that the ring R is a subring of the ring S. Throughout this section, we always assume that $1_R = 1_S$ (this ensures that S is a unital R-module). If N is a left S-module, then N can also be naturally considered as a left R-module since the elements of R (being elements of S) act on N by assumption.

*** page 340, line 3**

from: More generally, if $f : R \to S$ is a ring homomorphism from R into S (for example the injection map if R is a subring of S)

to: More generally, if $f : R \to S$ is a ring homomorphism from R to S with $f(1_R) = 1_S$ (for example the injection map if R is a subring of S as above)

*** page 347, Example 1**

from: Any ring S is an (S, R)-bimodule for any subring R by the associativity of the multiplication in S. More generally, if $f : R \to S$ is a ring homomorphism then

to: Any ring S is an (S, R)-bimodule for any subring R with $1_R = 1_S$ by the associativity of the multiplication in S. More generally, if $f : R \to S$ is a ring homomorphism with $f(1_R) = 1_S$ then

*** page 350, Example 6, line 1**

from: Let $f : R \to S$ be a ring homomorphism.

to: Let $f : R \to S$ be a ring homomorphism with $f(1_R) = 1_S$.

page 353, Corollary 16(2), top line of commutative diagram

from: $M \times \cdots \times M_n \xrightarrow{\iota} M \otimes \cdots \otimes M_n$

to: $M_1 \times \cdots \times M_n \xrightarrow{\iota} M_1 \otimes \cdots \otimes M_n$

*** page 356, Exercise 1**

from: Let $f : R \to S$ be a ring homomorphism from the ring R to the ring S.

to: Let $f : R \to S$ be a ring homomorphism from the ring R to the ring S with $f(1_R) = 1_S$.

*** page 356, Exercise 7, last line**

from: $d \in Q$

to: $d \in R$

*** page 358, second line before Exercise 27**

from: via Exercise 24 and

to: via Exercise 26 and

*** page 359, line 3**
Add: As in the previous section, throughout this section all rings contain a 1.

*** page 366, line 5**
from: Let R be a ring and suppose
to: Let R be a ring with 1 and suppose

*** page 383, at beginning of Exercises**
Add: Let R be a ring with 1.

*** page 386, Exercise 23, line 2**
from: $f : R \to S$,
to: $f : R \to S$ with $f(1_R) = 1_S$,

*** page 386, Exercise 25(c)**
Replace: R with K in each displayed equation, not including tensor subs (5 instances)

page 390, Theorem 3, line 1
from: $\mathcal{A} = \{a_1, a_2, \ldots, a_m\}$
to: $\mathcal{A} = \{a_1, a_2, \ldots, a_n\}$

*** page 403, Exercise 9, lines 3 and 4**
from: Prove that φ ... are both nonsingular.
to: Prove that if φ restricted to W and $\widetilde{\varphi}$ on V/W are both nonsingular then so is φ. Prove the converse to the latter result holds when W is finite dimensional, and give a counterexample when W is infinite dimensional.

page 413, proof of Theorem 19, line 3
from: $= E_v(f) + \alpha E_g(v)$
to: $= E_v(f) + \alpha E_v(g)$

*** page 416, proof of Proposition 22, line 10**
from: Exercise 6 of Section 3.5
to: Exercise 3 of Section 3.5

*** page 499, Exercise 6, line 2**
from: monic polynomial $x^n + a_n a_{n-1} x^{n-1} + \cdots + a_n^{n-1} a_1 x + a_n^n a_0$.
to: monic polynomial $x^n + a_{n-1} x^{n-1} + a_n a_{n-2} x^{n-2} + \cdots + a_n^{n-2} a_1 x + a_n^{n-1} a_0$.

page 546, Example 7, first line after second display
from: we see that $\sigma_p^{p^n} = 1$
to: we see that $\sigma_p^n = 1$

*** page 561, middle row of the displayed field diagram**
from: $\mathbb{Q}((i - i)\sqrt[4]{2}\,)$
to: $\mathbb{Q}((1 - i)\sqrt[4]{2}\,)$

page 581, two lines preceding display (14.12)
from: if and only if the field $K = \mathbb{Q}(\alpha)$ is obtained
to: if and only if $\mathbb{Q}(\alpha)$ is contained in a field K which is obtained

*** page 646, Exercise 10**

from: Prove that the subring $k[x, x^2y, x^3y^2, \ldots, x^iy^{i-1}, \ldots]$ of the polynomial ring $k[x, y]$ is *not* a finitely generated k-algebra. (Thus a subalgebra of a finitely generated k-algebra need not be finitely generated nor Noetherian.)

to: Prove that the subring $k[x, x^2y, x^3y^2, \ldots, x^iy^{i-1}, \ldots]$ of the polynomial ring $k[x, y]$ is *not* a Noetherian ring, hence not a finitely generated k-algebra. (Thus subrings of Noetherian rings need not be Noetherian and subalgebras of finitely generated k-algebras need not be finitely generated.)

*** page 663, Exercise 32, line 2**

from: ideas

to: ideals

*** page 664, Exercise 41(b), line 3**

from: $\cup_{a \in R}(0)_a = \cup_{a \in R}\text{rad }((0)_a)$.]

to: $\cup_{a \in R-\{0\}}\,(0)_a = \cup_{a \in R-\{0\}}\,\text{rad }((0)_a)$.]

*** page 665, Exercise 45**

from: $A = \text{rad } I$.

to: $A = \text{rad } ((F))$.

*** page 665, Exercise 45(f), line 1**

from: $Q \neq P$.

to: $Q \neq P, M$.

*** page 665, Definition**

from: $1_S \in R$.

to: $1 = 1_S \in R$.

*** page 670, line −5**

from: $da_0, \ldots, da_{k-1} \in \mathbb{Z}$

to: $d^k a_0, d^{k-1}a_1, \ldots, da_{k-1} \in \mathbb{Z}$

*** page 675, end of line 2**

from: i.e., hence

to: i.e.,

*** page 676, Exercise 10, hint**

from: the conjugates of α are

to: the conjugates of α, i.e., the roots of $m_{\alpha,k}(x)$, are

*** page 676, Exercise 12**

from: Suppose that S is an integral domain, that R is integrally closed in S, and that P is a prime ideal in R. Let s be any element in the ideal PS generated by P in S.

to: Suppose S is an integral domain that is integral over a ring R as in the previous exercise. If P is a prime ideal in R, let s be any element in the ideal PS generated by P in S.

*** page 692, line −8**

from: Corollary 10

to: Corollary 13

page 694, Exercise 13, line 1

from: Suppose $\varphi : R \to S$ is a ring homomorphism

to: Suppose $\varphi : R \to S$ is a ring homomorphism with $\varphi(1_R) = 1_S$

page 694, Exercise 16(a)

from: Prove that

to: Prove that $P_2 S_{Q_1}$ is contained in a prime ideal M in S_{Q_1} such that $M \cap R = P_2$. [Localize S_{Q_1} with respect to $D = R - P_2$ and consider a maximal ideal in this ring intersected with S_{Q_1}.]

*** page 698, Proposition 43(1)**

from: $\mathcal{Z}(I) = \mathcal{Z}(\text{rad }(I)) = \mathcal{Z}(\mathcal{I}(I))$

to: $\mathcal{Z}(I) = \mathcal{Z}(\text{rad }(I)) = \mathcal{Z}(\mathcal{I}(\mathcal{Z}(I)))$

*** page 711, second last line of the Example**

from: a map on from

to: a map from

*** page 717, Proposition 1(3)**

from: belongs to J

to: belongs to $\mathcal{J}$

*** page 746, line 10 after the first Definition**

from: cf. the proof of Proposition 3 below

to: cf. the proof of Proposition 4 following

*** page 746, Definition of Ext**

from: Then

$$\text{Ext}^n_R(A, D) = \ker d_{n+1}/\text{image}\, d_n$$

where $\text{Ext}^0_R(A, D) = \ker d_1$.

to: Define

$$\text{Ext}^n_R(A, D) = \ker d_{n+1}/\text{image}\, d_n, \quad n \geq 1,$$

and $\text{Ext}^0_R(A, D) = \ker d_1$.

*** page 755, Definition of Tor**

from: Then

$$\text{Tor}^R_n(D, B) = \ker(1 \otimes d_n)/\text{image}\,(1 \otimes d_{n+1})$$

where $\text{Tor}^R_0(D, B) = (D \otimes P_0)/\text{image}\,(1 \otimes d_1)$.

to: Define

$$\text{Tor}^R_n(D, B) = \ker(1 \otimes d_n)/\text{image}\,(1 \otimes d_{n+1}), \quad n \geq 1,$$

and $\text{Tor}^R_0(D, B) = (D \otimes P_0)/\text{image}\,(1 \otimes d_1)$.

*** page 760, Exercise 15, line 2**

from: $I \otimes_R M \to M$ that maps $i \otimes m$ to im

to: $M \otimes_R I \to M$ that maps $m \otimes i$ to mi

*** page 761, Exercise 22**

from: Suppose S is a commutative ring and $f : R \to S$
to: Suppose R and S are commutative rings and $f : R \to S$

*** page 762, Exercise 28(b), line 6**
from: $N \otimes R/\mathfrak{m} \cong N/\mathfrak{m}$
to: $N \otimes R/\mathfrak{m} \cong N/\mathfrak{m}N$

*** page 770, last line before first display in proof of Proposition 23**
from: we us
to: we use

*** page 779, Exercise 21, line 1**
from: let $G = \sigma$
to: let $G = \langle \sigma \rangle$

*** page 798, line -14**
from: cf. Theorem 40 below
to: cf. Theorem 42 below

*** page 803, line 4**
from: $B(F)$
to: $Br(F)$

*** page 850, character table, χ_7 row**
from: $\chi_7((1\ 2\ 3\ 4\ 5)) = 0$
to: $\chi_7((1\ 2\ 3\ 4\ 5)) = 1$

*** page 850, last line, displayed equation**
from: $\chi_7 = \dfrac{\rho - (\chi_1 - \chi_2 - 4\chi_3 - 4\chi_4 - 5\chi_5 - 5\chi_6)}{6}$
to: $\chi_7 = \dfrac{\rho - \chi_1 - \chi_2 - 4\chi_3 - 4\chi_4 - 5\chi_5 - 5\chi_6}{6}$

page 889, Index entry: free rank
from: 161, 219, 336, 440, 444
to: 161, 219, 336, 339, 440, 444

page 892, Index entry: Krull dimension
from: 710, 720
to: 716, 720

page 893
Add Index entry: Nakayama's Lemma, 717

page 895, Index entry: rank of a free module
from: 319, 335, 439
to: 319, 335, 339, 439