LAB 2

CONVOLUTION, FILTERING, STEADY-STATE AND TRANSIENT ANALYSIS

Recall that for a digital filter $h[n]$ with frequency response $H(e^{j\omega})$ defined as

$$H(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} h[n]e^{j\omega n}$$

the output $y[n]$ to an input $x[n] = e^{j\omega_1 n}$ is

$$y[n] = H(e^{j\omega_1})e^{j\omega_1 n}$$

that is, after a long time (when transients have died) we get back the input signal (sinusoid) which is changed only in magnitude and phase (i.e. delayed!).

**Design problem of simple digital filter**

Let the input consist of the sum of two cosine sequences with angular frequencies 0.1 rad/sec and 0.4 rad/sec respectively. We need to design a highpass filter that will pass the high-frequency component and block the low-frequency part.

For simplicity, we consider a (FIR) filter of length 3, that is

$$h[0] = h[2] = \alpha, \quad h[1] = \beta.$$ 

Hence we know from convolution that the input-output relation is


where $x[n]$ and $y[n]$ represent the input and output signals respectively.

Do the following:

(i) Show that the frequency response of the digital filter is

$$H(e^{j\omega}) = (2\alpha \cos \omega + \beta)e^{-j\omega}$$
and that (assuming \(2\alpha \cos \omega + \beta\) is positive)

\[
|H(e^{j\omega})| = 2\alpha \cos \omega + \beta, \\
\theta(\omega) = -\omega.
\]

(ii) Show that to stop the low frequency and pass the high frequency we must have

\[
\alpha = -0.676195, \beta = 13.456335.
\]

(iii) Now verify using Matlab that the filter

\[
y[n] = -6.76195(x[n] + x[n - 2]) + 13.456335x[n - 2]
\]

filters

\[
x[n] = \cos(0.1n) + \cos(0.4n)u[n]
\]
in the way that you have designed it. You can use the attached program. (Under Matlab, do 'help filter' to see the implicit convolution.)

(iv) In the results, observe
(a) the transient part of the output.
(b) the steady-state part.
(c) When does the steady-state start and is it consistent with the phase (and hence the signal time delay) of the frequency response?

Program

```matlab
b=[-6.76195 13.456335 -6.76195];
zi = [0 0];
n = 0:99;
x1 = cos(0.1*n);
x2 = cos(0.4*n);
y = filter(b, 1, x1+x2, zi);
plot(n,y,'r-',n,x2,'b-',n,x1,'g-'); grid
axis([0 100 -1.2 4]);
ylabel('Amplitude'); xlabel('Time index n');
legend('r-', 'y[n]', 'b-', 'x2[n]', 'g-', 'x1[n]')
```