HOMEWORK 1

Sampling (time-domain and frequency domain), Aliasing, FT, DTFT, DFT, Reconstruction.

In this homework we consider the signal,

\[ x(t) = \cos 2\pi 60t. \]

This signal will be windowed and sampled and the Discrete Fourier Transform (DFT) obtained. The DFT will be compared to the Discrete-time Fourier Transform (DTFT) which will then be compared with the Fourier Transform (FT).

(a) For the given signal, determine its FT and sketch the FT amplitude and phase.

(b) Consider the “rectangular window” \( w_R(t) \),

\[ w_R(t) = \begin{cases} 1 & \text{if } 0 < t < T \\ 0 & \text{otherwise} \end{cases} \]

and determine its FT \( W_R(j\Omega) \). Show this as a sinc function. It may be easier to determine the Fourier spectrum of a symmetric window \([-T/2, +T/2]\) which will give you the sinc function and then shift it. Show and mark the zero crossings so that you see how wide is the main lobe of the spectrum.

(c) Now construct the ‘practical’ signal \( x_W(t) = x(t).w_R(t) \). Find its Fourier Transform \( X_W(j\Omega) \). Sketch it.

(d) Do this in MATLAB: Sample the signal such that you get,

\[ x_W[n] \equiv x_W(nT) = x_W(t)|_{t=nT} = \cos(2\pi \frac{60}{1000} n) \equiv y[n] \quad (1) \]

where \( T = 1ms \). Take 100 samples. Plot \( y[n] \). You can consider \( y[n] \) as the real part of signal \( x[n] = e^{j2\pi(\frac{60}{1000})n} \).

(e) Sketch \( Y(e^{j\omega}) \). (Use the fact that \( Y(e^{j\omega}) \) is the (scaled) periodic repetition of \( Y(j\Omega) \).)

(f) Assume that you are going to be doing 100-point DFT of \( y[n] \):

\[ Y[k] = \sum_{n=0}^{99} y[n](e^{-j(\frac{2\pi}{100}kn)}) \quad k = 0, 1, \ldots 99 \]

(2).

So you are expressing the periodic signal \( y[n] \) as

\[ y[n] = \frac{1}{100} \sum_{k=0}^{99} Y[k](e^{j(\frac{2\pi}{100}kn)}) \quad n = 0, 1, \ldots 99 \]
where the orthogonal basis functions being used are \( \{ e^{j \frac{2\pi}{N} kn} \} \quad n = 0, 1, \ldots 99 \).

We know of course, that for orthogonal basis functions, the coefficients of expansion are the “Fourier coefficients”. That is, as given in equation (2),

\[
Y[k] = \langle e^{j \frac{2\pi}{N} kn}, y[n] \rangle \quad k = 0, 1, \ldots , 99
\]

Confirm that the signal \( y[n] \) in (1) corresponds to one of the basis functions for a certain \( k \). Hence, given orthogonality of basis functions, what do you conclude about values of \( Y[k] \) for \( k = 0, 1, \ldots , 99 \)?

(f) Using MATLAB, find the 100-point DFT \( Y[k] \) of \( y[n] \). Does it conform with part (e)?

(g) Now do a 1000-point DFT of \( y[n] \). Comment. Recall \( Y[k] \) is related to \( Y(e^{j\omega}) \) which is related to \( X_W(j\Omega) \).

(h) Now use a 100-point Hanning window \( w_H[n] \) on the 100-point signal \( x_W[n] \) to get

\[
y[n] = x_W[n] * w_H[n]
\]

Now find the 1000-point DFT of \( y[n] \) and compare it with the 1000-point DFT of \( y[n] \) when you used a rectangular window. Comment on the difference.