Do the first 4 problems.
All 4 problems have same number of points.

NAME ________________________________________________

1. ___________________

2. ___________________

3. ___________________

4. ___________________
1. FFT

Using pencil and paper, sample \( x(t) = \cos 2\pi 10t \) at \( t = nT_o \) where \( f_o = 1/T_o = 100 \text{ hz} \) to get the discrete signal \( x[n] \). Generate a total of 100 samples, starting from \( t = 0 \).

Do the following:

(a) What is the frequency \( f_o \) (hertz) of the signal \( x(t) \)? Also the frequency \( \Omega_o \) (radians/sec). What is the sampling frequency? Therefore do you expect aliasing?

(b) Now that you have \( x[n] \), determine its period \( N \). Therefore determine its frequency \( \omega_o \). Does this \( \omega_o \) lie in the appropriate range to preclude aliasing? Explain. Given that you got \( x[n] \) from sampling the signal \( x(t) \), translate \( \omega_o \) to \( \Omega_o \) and then to \( f_o \).

(c) Still using pencil and paper, calculate the DFT of \( x[n] \) you generated.

(d) Now we find the DFT of \( x[n] \) using MATLAB. Note that you can use the command window to look at the signal and do its fft there, or you can go to the sptool GUI and do things from there. Both have their advantages.

(e) Import your signal using sptool. Note that in SPTool:startup.spt, you have under Edit, the capability of setting your sampling frequency. Note also that in the Spectrum Viewer you have under Options the ability to set the magnitude scale to linear or db. This allows you to look at signals from different perspectives. You also have under Options, the ability to set the frequency range. (Try for instance, \( \{-f_s/2, f_s/2\}\).)

Create the corresponding spectra of your signal and look at a 100-point FFT. Use the frequency range \( \{-f_s/2, f_s/2\}\). Use both linear and db magnitude scales. In magnitude, how small is -350 db? Is the spectrum what you expect? Comment. (You can use the mouse zoom icon (3rd from the left on the top left-hand side). Also the “data cursor” when trying to determine \( x - y \) coordinates of points on a Figure.

(f) Now for that same 100-point signal, do for example, a 187-point fft. Look at both db and linear magnitude scales. Spectra using db magnitude looks different. Give the reasons here.

(g) Do a 1870-pt fft. Look at spectra using both linear and db scales. Comment.

(h) Now apply a 100-pt Hamming window to the original 100-point data. Take the fft and see if the spectrum is “better” than the one without the window.

(i) Generate the same sampled signal, but now generate 2000 samples. Take its 2000-pt fft without and with a Hamming window. Comment.
2. FFT of windowed data
Here we repeat the spectral analysis of the signal in (1) above, but we window the data. SPTool does not have the capability to apply a window to your data, so you have to do it in the Command window. Generate a 100-pt Hamming window,

\[
\gg w=\text{hamming}(100);
\]

and multiply point wise your 100-pt signal \(x[n]\) with the 100-pt window \(w[n]\) to get \(xw[n] = x[n].w[n]\). Plot \(xw[n]\) and observe how it has changed from \(x[n]\).

(a) How has \(xw[n]\) changed from \(x[n]\)?

(b) Do a 100-pt fft on \(xw[n]\). You might wish to use the frequency range \((-f_s/2, f_s/2)\). Compare the 100-pt spectra of both \(x[n]\) and \(xw[n]\) using both linear and db magnitudes. Comment on the differences. In the linear magnitude case for example, how are the two magnitude spectra different?

(Note that if you have both the SPTool:startup.spt window and the Spectrum Viewer window on display, you can compare spectra easily by highlighting the desired one in the Spectra window).

(c) Repeat (b) using a 187-pt fft.
3. Analysis of cardiogram data - heart1

Go to http://www.cems.uvm.edu/mirchand/classes/EE275/images/ and copy cardio.mat somewhere in your computer. Go to MATLAB and do “load cardio”. This is real data. You will see three files: heart1, heart2 and heart3. While typically these may be many minutes of data, you have here only 2048 points. The original continuous signal was sampled at 1 msec (1000 hz), so you have 2.048 seconds of data for each signal. You will be working with heart1, heart2 and heart3 in problems 3, 4 and 5 respectively.

The goal is to determine as well as possible, the period of the spikes in the data. The frequency of these spikes will be referred to as the “dominant” frequency. We will try to determine this frequency using the \( \text{fft} \).

Do the following: (You can import the data in SPTool and use its graphical capabilities to do the measurements).

(a) Plot the time series \( x[n] = \text{heart1} \) and using the data cursor, determine the period of the spikes. Is it the same for the whole data set? Determine the average frequency \( \omega_o \) of the spikes in the signal \( x[n] \) and the average frequency in hz of the corresponding analog signal.

(b) Do a 1000 pt. \( \text{fft} \) and obtain the DFT \( X[k] \). Given that you measured the average period in part (a), determine the corresponding value of \( k \) where you should typically expect a peak. Is that what happens when you do the \( \text{fft} \)? If not, is there any information in the spectrum that identifies the frequency of the spikes.

(c) Apply a Hamming window and see if that helps.

(d) Do a 2048 pt. \( \text{fft} \) and see what that does.

(e) Comment.

4. Analysis of cardiogram data- heart2

Have a look at heart2 and the 2048 points in the data. Call it \( y[n] \). Identify the period of the spikes. The goal again is to determine the “dominant” frequency in the data using the \( \text{fft} \). The data has been sampled at 1000 hz.

Do the following:

(a) Determine the frequency \( \omega_o \) of the signal \( y[n] \) and the frequency in hz of the corresponding analog signal.

(b) Do a 1000 pt. \( \text{fft} \) on \( x[n] \) and check to see if it identifies the period of the spikes.

(c) Apply a window and see if that helps.

(d) Do a 2048 pt. \( \text{fft} \) and see what that does.

(e) Comment
5. Analysis of cardiogram data- heart3

Nothing to do here. Now you are seeing data which is a bit more complex. Like electrocardiogram (ECG) data, electroencephalogram (EEG) date also exhibits non-stationary behavior. Also the activations (the peaks) can get fragmented and thus the average frequency difficult to resolve, even manually.