April 27, 2007

Test #3, Question 6

\[ y' = 2xe^{x^2}, \; y(0) = 3 \]

\[ y = \int 2xe^{x^2} \, dx = \int e^u \, du = e^u + C \]

\[ u = x^2 \]

\[ du = 2x \, dx \]

\[ y = e^{x^2} + C \]

\[ x = 0, \; y = 3 \]

\[ 3 = e^0 + C \]

\[ C = 2 \]

\[ y = e^{x^2} + 2 \]

Question #7

\[ y' = \sqrt{\frac{2y}{3x}} \]

\[ \frac{1}{\sqrt{2y}} \cdot y' = \frac{1}{\sqrt{3x}} \]

\[ \frac{1}{\sqrt{2}} \cdot y^{-1/2} \cdot y' = \frac{1}{\sqrt{3}} \cdot x^{-1/2} \]

\[ \int \frac{1}{\sqrt{2}} \cdot y^{-1/2} \, dy = \int \frac{1}{\sqrt{3}} \cdot x^{-1/2} \, dx \]

\[ \frac{1}{\sqrt{2}} \left( 2y^{1/2} \right) = \frac{1}{\sqrt{3}} \left( 2x^{1/2} \right) + c \]

\[ y^{1/2} = \frac{\sqrt{2}}{\sqrt{3}} x^{1/2} + C \left( C = \frac{\sqrt{2}}{2} c \right) \]

\[ y = \left( \frac{\sqrt{2}}{\sqrt{3}} x^{1/2} + C \right)^2 \]

Problem #5

Verify that \( y = Ce^{4x} \) is a solution to \( y' = 4y \)

We want to substitute \( y \) and \( y' \) into the differential equation and see if we get a true statement.

\( y = Ce^{4x}, \; y' = 4Ce^{4x} \)

Substitution gives \( 4Ce^{4x} = 4(Ce^{4x}) \) which is true.

Problem #2
<table>
<thead>
<tr>
<th>Random Variable</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X = x)</td>
<td>0.3</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Part a: Expected value or mean

\[0(0.3) + 1(0.2) + 3(0.2) + 4(0.3) = 0 + 0.2 + 0.6 + 1.2 = 2\]

Problem #4

uniform probability distribution function on [0, 3]

So, the pdf \( f(x) = \frac{1}{3-0} = \frac{1}{3} \)

Part a:

\[
\mu = \int_{0}^{3} x f(x) \, dx = \int_{0}^{3} \frac{1}{3} x \, dx = \left[ \frac{1}{3} \left( \frac{1}{2} \right) x^2 \right]_{0}^{3} = \frac{1}{6} (3)^2 - \frac{1}{6} (0)^2 = \frac{9}{6} = \frac{3}{2}
\]