Problem #9

\[ y = \sin^3 x \cos^4 x = (\sin x)^3 (\cos x)^4 \]
\[ y' = (\sin x)^3 \left[ 4 (\cos x)^3 (-\sin x) \right] + (\cos x)^4 \left[ 3 (\sin x)^2 (\cos x) \right] \]
\[ = -4 \sin^4 x \cos^3 x + 3 \sin^2 x \cos^5 x \]

Problem #15

Find the equation of the line tangent to the curve \( y = \sin 3x \) at \( x = \frac{\pi}{12} \)

Need:
1. Need a point
   \[ x = \frac{\pi}{12} \]
   \[ y = \sin \left( 3 \left( \frac{\pi}{12} \right) \right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \]
2. Need slope
   \[ m = \frac{dy}{dx} \bigg|_{x=\frac{\pi}{12}} = 3 \cos 3x \bigg|_{x=\frac{\pi}{12}} = 3 \cos \left( 3 \left( \frac{\pi}{12} \right) \right) = 3 \cos \frac{\pi}{4} = 3 \left( \frac{\sqrt{2}}{2} \right) \]

Equation
\[ y - \frac{\sqrt{2}}{2} = 3 \frac{\sqrt{2}}{2} \left( x - \frac{\pi}{12} \right) \]

Problem #17

Find max and min of \( y = \sin x + \cos x \) for \( x \) in the interval \([0, 2\pi]\)

Need critical point(s); solve \( y' = 0 \)
\[ \frac{dy}{dx} = \cos x - \sin x \]
\[ \cos x - \sin x = 0 \]
\[ \cos x = \sin x \]
\[ x = \frac{\pi}{4}, \frac{5\pi}{4} \]

Max or min?
\[
\frac{d^2 y}{dx^2} = -\sin x - \cos x
\]

\[
x = \frac{\pi}{4}, \quad \frac{d^2 y}{dx^2} = -\sin \frac{\pi}{4} - \cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}
\]

max

\[
x = \frac{5\pi}{4}, \quad \frac{d^2 y}{dx^2} = -\sin \frac{5\pi}{4} - \cos \frac{5\pi}{4} = -\left( -\frac{\sqrt{2}}{2} \right) - \left( -\frac{\sqrt{2}}{2} \right) = +\sqrt{2}
\]

min

Problem #19
max and min of

\[
f(x) = x + \sin x, \quad [0, 2\pi]
\]

\[
f'(x) = 1 + \cos x
\]

\[
f'(x) = 0 \text{ when } 1 + \cos x = 0
\]

\[
\cos x = -1
\]

\[
x = \pi
\]

\[
f''(x) = 0 - \sin x
\]

\[
f''(\pi) = -\sin \pi = 0
\]

\[
f'(\frac{5\pi}{6}) = 1 + \cos \frac{5\pi}{6} = 1 - \frac{\sqrt{3}}{2} > 0
\]

\[
f'(\frac{7\pi}{6}) = 1 + \cos \frac{7\pi}{6} = 1 - \frac{\sqrt{3}}{2} > 0
\]

inflection point