April 20, 2007
Answer questions, Section 12.1

Problem #5

\( y(x) = Ce^{x^2} \)

Claim Solution to \( y' = 2xy \)
Is it indeed a solution?

\( y(x) = Ce^{x^2} \)
\( y'(x) = Ce^{x^2} (2x) \)

Substitution
\( y' = 2xy \)
\( Ce^{x^2} (2x) = 2x \left( Ce^{x^2} \right) \)

Since this is a true statement, the function \( y(x) \) is a solution

Problem #9

Given the differential equation \( y' = \frac{y}{x^2} \), is \( y = Ce^{-1/x} \) a solution?

\( y = Ce^{-1/x} \)
\( y' = Ce^{-1/x} \left( \frac{1}{x^2} \right) \)

If we substitute \( y \) and \( y' \) into the differential equation, we have
\( y' = \frac{y}{x^2} \)
\( Ce^{-1/x} \left( \frac{1}{x^2} \right) = \frac{Ce^{-1/x}}{x^2} \)

Since these are equal, this is a solution

Problem #13

Given \( y_p = e^x + C \). Claim this is a solution to the boundary value problem
\( y' = e^x, y(0) = 0 \).

The fact that \( y(0) = 0 \) tells us when \( x = 0 \), then \( y = 0 \). This allows us to find \( C \).
Plug these values for \( x \) and \( y \) into the solution, I have \( 0 = e^0 + C \) or \( 0 = 1 + C \); so \( C = -1 \).
The solution to the boundary value problem \( (y' = e^x, y(0) = 0) \) is \( y = e^x - 1 \).