Problem #1
\[ f(x, y) = x^2 + xy + 2y^2 - 8x + 3y \]
We want to find all critical points
\[ f_x = 2x + y - 8 \]
\[ f_y = x + 4y + 3 \]
Set each of these equal to 0 and solve
\[ 2x + y = 8 \]
\[ x + 4y = -3 \]
\[ 2x + y = 8 \]
\[ x + 4y = -3 \]
\[ -7y = 14 \text{ or } y = -2 \]
\[ 2x - 2 = 8 \text{ or } 2x = 10 \text{ or } x = 5 \]
Critical point is (5, -2)
Determine the nature of this point: Is it a relative maximum, a relative minimum, or a saddle point?
\[ D(x, y) = f_{xx}(x, y) f_{yy}(x, y) - (f_{xy}(x, y))^2 \]
\[ f_{xx} = 2; \quad f_{yy} = 4; \quad f_{xy} = 1 \]
\[ D(x, y) = (2)(4) - 1^2 = 7 \]
Look at \( f_{xx} = 2 \)
Relative minimum at (5, -2)

Problem #17
\[ f_x = 3 \left( -1x^{-2} \left( \frac{1}{y} \right) \right) - \left( 2x^{-2} \right) \left( \frac{1}{y} \right) = -\frac{3}{x^2 y} + \frac{2}{x^3 y} + \frac{1}{x^2 y^2} \]
\[ f_y = -\frac{3}{xy^2} + \frac{1}{x^2 y^2} + \frac{2}{xy^3} \]
Solve
\[-3 \frac{x^2 y}{x^3 y} + \frac{2}{x^2 y^2} + \frac{1}{x^3 y^2} = 0 \quad (\text{multiply by } x^3 y^2)\]

\[-3 \frac{x^2 y^2}{xy^2} + \frac{1}{x^2 y} + \frac{2}{xy^3} = 0 \quad (\text{multiply by } x^2 y^3)\]

\[x \neq 0, y \neq 0\]

\[-3xy + 2y + x = 0 \implies y(-3x + 2) = -x \implies y = \frac{-x}{2 - 3x}\]

\[-3xy + y + 2x = 0 \implies -3x\left(\frac{-x}{2 - 3x}\right) + \frac{-x}{2 - 3x} + 2x = 0\]

\[3x^2 - x + 2x(2 - 3x) = 0\]

\[3x^2 - x + 4x - 6x^2 = 0\]

\[-3x^2 + 3x = 0\]

\[-3x(x - 1) = 0\]

\[x = 0, x = 1\]

\[x = 0, y = \frac{-x}{2 - 3x} = 0\]

\[x = 1, y = \frac{-x}{2 - 3x} = \frac{-1}{2 - 3} = 1\]

Critical points \((0, 0)\) [Can’t be!!!] and \((1, 1)\)

The only one is \((1, 1)\)