Section 6.6

Problem: Given two functions, \( f(x) \) and \( g(x) \), find the area bounded by the two curves with \( x \) in the interval from \( a \) to \( b \).

Answer: 
\[
\int_{a}^{b} |f(x) - g(x)| \, dx
\]

Example 1: \( f(x) = x^2, y = 0, x = 1 \) to \( x = 3 \)

NOTE: \( y = 0 \) (x-axis), also \( g(x) = 0 \)

Area = 
\[
\int_{1}^{3} x^2 \, dx = \frac{1}{3} x^3 \bigg|_{1}^{3} = \frac{1}{3} (3)^3 - \frac{1}{3} (1)^3 = 9 - \frac{1}{3} = \frac{26}{3}
\]

Example 2: \( f(x) = x^2, g(x) = x, x \) from -1 to 1

\( f(x) = x^2 \) is turquoise and \( g(x) = x \) is yellow. They intersect at \((0,0)\)

\[
\int_{-1}^{0} (x^2 - x) \, dx - \int_{0}^{1} (x - x^2) \, dx = \left[ \frac{1}{3} x^3 - \frac{1}{2} x^2 \right]_{-1}^{0} - \left[ \frac{1}{2} x^2 - \frac{1}{3} x^3 \right]_{0}^{1}
\]

\[
= 0 - \left[ \frac{1}{3} (-1)^3 - \frac{1}{2} (-1)^2 \right] - \left[ \left( \frac{1}{2} \cdot \frac{1}{2} \right) - \left( \frac{1}{3} \cdot \frac{1}{3} \right) \right] = - \left[ -\frac{1}{3} - \frac{1}{2} \right] - \left[ \frac{1}{2} - \frac{1}{3} \right] = \frac{1}{3} + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} = \frac{2}{3}
\]