Lecture: Sec. 8.1

**Definition:** A function of two variables has two input variable, often $x$ and $y$, and one output variable, often $z$. The input variables are the independent variables and the output variable is the dependent variable.

The domain of the function is all inputs which make sense, collectively.

**Example:** $f(x, y) = x^2 + 2y^2 + 3xy$
Find $f(1, 3) = 1^2 + 2(3)^2 + 3(1)(3) = 28$

**Example:** Find the domain of the function
$f(x, y) = \sqrt{x + y}$
domain: $x + y \geq 0$
$
\{(x, y) | x + y \geq 0\}$

**Graph of a function of two input variables:**
**Example:**
$f(x, y) = \sqrt{x^2 + y^2 + 1}$

**Example:**
$f(x, y) = x^2 - y^2 + 1$
Level curves:
Dark is a low point, light is a high point.

Find the equation of the level curves for $k = 1, 2, 3, 4$

Level curve for $k = 1$ (This means the level curve for a height 1 unit up; so $z = 1$) would be the curve

Want to solve

$$f(x, y) = \sqrt{x^2 + y^2 + 1}$$

$$\sqrt{x^2 + y^2 + 1} = 1$$

$$x^2 + y^2 + 1 = 1$$

$$x^2 + y^2 = 0$$

$$x = 0, y = 0$$

This would just be a point at $(0, 0, 1)$
For $k = 2$,
\[
\sqrt{x^2 + y^2} + 1 = 2 \\
x^2 + y^2 + 1 = 4 \\
x^2 + y^2 = 3
\]
So this is a circle, centered at the origin, radius $\sqrt{3}$.

If $k = 4$,
\[
\sqrt{x^2 + y^2} + 1 = 4 \\
x^2 + y^2 = 15
\]
circle, centered at the origin, radius $\sqrt{15}$

Find the equation of the level curves for $k = 2$
\[x^2 - y^2 + 1 = 2\]
\[x^2 - y^2 = 1\]

Suppose we want the level curve for \(k = 4\)
\[x^2 - y^2 + 1 = 4\]
\[x^2 - y^2 = 3\]

What about \(k = -4\)?
\[x^2 - y^2 + 1 = -4\]
\[x^2 - y^2 = -5\]
\[y^2 - x^2 = 5\]