Lecture: Sec. 8.1

**Definition:** A function of two variables has two input variables, often x and y, and one output variable, often \( z \). The input variables are the independent variables and the output variable is the dependent variable.

The domain of the function is all inputs which make sense, collectively.

**Example:** \( f(x, y) = x^2 + 2y^2 + 3xy \)
Find \( f(1, 3) = 1^2 + 2(3)^2 + 3(1)(3) = 28 \)

**Example:** Find the domain of the function
\[
f(x, y) = \sqrt{x + y}
\]
\( x + y \geq 0 \)
\( \{(x, y) \mid x + y \geq 0\} \)

**Graph of a function of two variables:**

**Example:**
\[
f(x, y) = \sqrt{x^2 + y^2 + 1}
\]

**Example:**
\[
f'(x, y) = x^2 - y^2 + 1
\]
Level curves:
Dark is a low point, light is a high point.

Find the equation of the level curves for \( k = 1, 2, 3, 4 \)

Level curve for \( k = 1 \) (This means the level curve for a height 1 unit up; so \( z = 1 \)) would be the curve

\[
\sqrt{x^2 + y^2} + 1 = 1
\]

\[
x^2 + y^2 + 1 = 1
\]

\[
x^2 + y^2 = 0
\]

This would just be a point at \((0, 0, 1)\)

For \( k = 2 \),

\[
f(x, y) = \sqrt{x^2 + y^2 + 1}
\]
\[ \sqrt{x^2 + y^2 + 1} = 2 \]
\[ x^2 + y^2 + 1 = 4 \]
\[ x^2 + y^2 = 3 \]

So this is a circle, centered at the origin, radius \( \sqrt{3} \)

If \( k = 3 \), we would have \( x^2 + y^2 = 8 \) circle with radius \( \sqrt{8} \)

\[ f(x, y) = x^2 - y^2 + 1 \]

Find the equation of the level curves for \( k = 2 \)

Suppose we want the level curve for \( k = 2 \)