Section 8.4

**Problem:** Find the minimum value(s) for the function \( f(x, y) = xy^2 \) in the first quadrant subject to the constraint \( g(x, y) = x^2 + y^2 = 8 \).

**General Problem:** Find the maximum or minimum of a function \( f(x, y) \) subject to a constraint \( g(x, y) = 0 \)

**Step 1:** Create a new function \( F(x, y, L) = f(x, y) + L g(x, y) \)

\[
F(x, y, L) = xy^2 + L(x^2 + y^2 - 8)
\]

**Step 2:** Solve \( F_x = 0, F_y = 0, \) and \( F_L = 0 \)

a. \( F_x = y^2 + 2xL = 0 \)

\[
F_y = 2xy + 2yL = 0
\]

\[
F_L = x^2 + y^2 - 8 = 0
\]

Solve for the first 2 equations for \( L \), set them equal to each other, and solve for one of the variables:

\[
L = \frac{-y^2}{2x}
\]

\[
L = \frac{-2xy}{2y} = \frac{-y^2}{2x} = -x
\]

\[
-x = \frac{-y^2}{2x} \quad or \quad -y^2 = -2x^2 \quad or \quad y^2 = 2x^2
\]

Substitute that into \( F_L \)

\[
F_L = x^2 + y^2 - 8 = 0
\]

\[
x^2 + 2x^2 = 8
\]

\[
3x^2 = 8
\]

\[
x^2 = \frac{8}{3}
\]

\[
y^2 = \frac{16}{3}
\]

\[
x = \pm \sqrt[3]{\frac{8}{3}}, \quad y = \pm \frac{4}{\sqrt{3}}
\]