Section 9.2

We need to define two new functions, sine and cosine

![Diagram of unit circle with point P(x, y)](image)

This defines the functions for all real numbers. That is, the domain of each function is all real numbers. The range (y values) is -1 to +1, inclusive

Pythagorean Identity

Equation of the unit circle is \(x^2 + y^2 = 1\)

\[(\cos \theta)^2 + (\sin \theta)^2 = 1\]
\[\cos^2 \theta + \sin^2 \theta = 1\]

Trig values of different angles

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(0)</th>
<th>(\pi/2)</th>
<th>(\pi)</th>
<th>(3\pi/2)</th>
<th>(2\pi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin \theta)</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\cos \theta)</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\theta)</th>
<th>(0)</th>
<th>(\pi/6)</th>
<th>(\pi/4)</th>
<th>(\pi/3)</th>
<th>(\pi/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sin \theta)</td>
<td>(\sqrt{3}/2)</td>
<td>(\sqrt{3}/2)</td>
<td>(\sqrt{3}/2)</td>
<td>(\sqrt{3}/2)</td>
<td>(1)</td>
</tr>
<tr>
<td>(\cos \theta)</td>
<td>(1)</td>
<td>(\sqrt{3}/2)</td>
<td>(\sqrt{3}/2)</td>
<td>(\sqrt{3}/2)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

Signs of sine and cosine in the various quadrants

\[
\begin{align*}
\text{Sin } t & > 0 & \text{All } > 0 \\
\frac{\text{Tan } t}{\text{Cos } t} & > 0 & \text{Cos } t & > 0
\end{align*}
\]

To find \(\sin\) and \(\cos\) (and cosine) whose terminal side is in a quadrant other than the first:
Next find the reference angle, which is the smallest positive angle between the x-axis and the terminal side of the angle.

The angle of interest has sine and cosine of the same magnitude as the reference angle.

**Example:**

\[ t = 4 \pi/3, \sin 4 \pi/3 = - \sin \pi/3 = -\frac{\sqrt{3}}{2} \]
\[ \cos 4 \pi/3 = - \cos \pi/3 = -1/2 \]

Right triangle definitions:

\[ \sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{b}{c} \]
\[ \cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{a}{c} \]
\[ \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}} = \frac{b}{a} \]