The ten numbered problems are worth 10 points each and the two extra credit problems are worth 20 points each. (Notice that problem 8 has TWO PARTS.) **Show your work!**

\[(\text{ANSWER}) + (\text{NO WORK}) = (\text{ZERO CREDIT})\].

Work that doesn’t make any sense counts as ‘no work’.

1. What is the effective annual interest rate of 11% annual interest, compounded continuously? Round your answer to the nearest tenth of a percent.

   **Solution.** It’s \(e^{11} - 1 \approx .116278\), which rounds to 11.6%.

2. What is the present value of $10,000, to be received 5 years from now, if the interest rate is 6%, compounded continuously? Round your answer to the nearest dollar.

   **Solution.** It’s \(10,000e^{-(.06)5} = 10,000e^{-3} \approx 7408.18\) dollars, which rounds to $7408.

3. The population of Ocean View, Illinois, is growing exponentially. In 1999 it was 250,000 and this year (2009) it is 350,000. If this trend continues, what will its population be in 2014? Round your answer to the nearest thousand.

   **Solution.** The population is \(P(t) = 250000e^{kt}\), where \(t\) is the number of years since 1999. Since \(P(10) = 350000\), \(k = (\ln(7/5))/10 \approx .0336472\). The population in 2014 is \(P(15) = 250000e^{15k} \approx 414126\), which rounds to 414,000.

4. The half-life of upsidaisium (atomic symbol \(∩\)) is 17 years. How much of a 100 gram sample of \(∩\) will remain after 28 years have passed? Round your answer to the nearest gram.

   **Solution.** After \(t\) years, \(100e^{-kt}\) grams will remain, where \(k = (\ln(2))/17\) (because 17 is the half-life). Thus \(k \approx .0407734\), implying the amount remaining after 28 years is \(100e^{-k(28)} \approx 31.929\) grams, which rounds to 32.

5. Find the limit:

   \[
   \lim_{x \to 3} \frac{x^2 - 2x - 3}{\sqrt{x+1} - 2}.
   \]

   **Solution.** We factor the numerator and multiply top and bottom by the conjugate of the denominator. When \(x \neq 3\),

   \[
   \frac{x^2 - 2x - 3}{\sqrt{x+1} - 2} = \frac{(x-3)(x+1)}{\sqrt{x+1}-2} \cdot \frac{\sqrt{x+1}+2}{\sqrt{x+1}+2} = \frac{(x-3)(x+1)(\sqrt{x+1}+2)}{x-3} = (x+1)(\sqrt{x+1}+2),
   \]

   which equals \(4 \times 4 = 16\) when \(x = 3\), and that is the value of the limit.
6. Consider the following piecewise defined function:

\[ f(x) = \begin{cases} 
2x + c & \text{if } x \leq 2; \\
x^3 + cx & \text{if } x > 2.
\end{cases} \]

What value of \( c \) will make \( f \) continuous everywhere?

**Solution.** The two pieces must match at \( x = 2 \), which yields the equation \( 4 + c = 8 + 2c \), which has solution \( c = -4 \).

7. What is the average rate of change of \( x^2 + 3x \) on the interval \([1, 4]\). State your answer as a whole number or as a fraction in lowest terms.

**Solution.** It’s

\[
\frac{(16 + 12) - (1 + 3)}{4 - 1} = \frac{24}{3} = 8.
\]

8a) Find the derivative of \( x^3 + 5x^2 + 7x - 2 \).

8b) Find the derivative of \( 3\sqrt{x} + \frac{2}{x} \).

**Solutiona.** a) \( 3x^2 + 10x + 7 \). b) \( \frac{3}{2\sqrt{x}} - \frac{2}{x^2} \).

9. Suppose \( h(x) = (x^2 + 3x + 1)(2x^2 - 5x + 2) \). Find \( h'(0) \).

**Solution.** Write the function as \( f(x)g(x) \), where \( f \) and \( g \) have the obvious meanings. Then \( f(0) = 1, f'(0) = 3, g(0) = 2, \) and \( g'(0) = -5 \). By the product rule, \( h'(0) = 3 \cdot 2 + 1 \cdot (-5) = 1 \). Or you can multiply \( h \) out and get \( h(x) = 2x^4 + x^3 - 11x^2 + x + 2 \), which yields the same result.

10. Find the derivative of

\[ \frac{3x^2 + x + 2}{7x - 9}. \]

For full credit, simplify the numerator (top) by collecting like terms.

**Solution.** It’s

\[
\frac{(6x + 1)(7x - 9) - (3x^2 + x + 2)(7)}{(7x - 9)^2},
\]

which simplifies to

\[ \frac{21x^2 - 54x - 23}{(7x - 9)^2}. \]

**Extra Credit 1.** You have the choice of two ways to be paid for a job: $5000 right now, or $3000 a year from now, followed by another $3000 two years from now. Assume the interest rate is 8%, compounded continuously. Which is the better deal?

**Solution.** The present value of the delayed payments is \( 3000(e^{-0.08} + e^{-0.16}) \approx 5325.78 \) dollars. The SECOND option is better.
Extra Credit 2. (See problem #4.) Years ago, a sample of upsidalium was placed under a rock. When the rock was finally lifted up, 88% of the sample had decayed away. How long was the sample under the rock? Round your answer to the nearest year.

Solution. We seek a time $\tilde{t}$ such that $A(\tilde{t}) = 0.12A(0)$, where $A(t) = A(0)e^{-kt}$, and the $k$ is the one we found in problem #4. Using natural logs, we get

$$-k\tilde{t} = \ln(0.12),$$

which implies

$$\tilde{t} = -(\ln(0.12))/k \approx 52.0011$$

years; and this rounds to 52 years.