1. Let $K \subset \mathbb{R}$ be compact, and let $f_n : K \rightarrow \mathbb{R}$ be a sequence of continuous functions such that, for all $x$ and $n$, $f_n(x) \geq f_{n+1}(x)$. Suppose furthermore that $f_n \rightarrow 0$ pointwise on $K$. Show that $f_n \rightarrow 0$ uniformly on $K$. Show that the uniform convergence can fail if the set $K$ is not compact.

2. Let $g : [0,1] \rightarrow \mathbb{R}$ be continuous and suppose that $g(1) = 0$. Show that the sequence $\{x^n g(x)\}_{i=1}^{\infty}$ converges uniformly on $[0,1]$.

3. Find the radii of convergence of:
   a) $\sum_{n=1}^{\infty} n^n x^n$;
   b) $\sum_{n=1}^{\infty} n^n x^n (n^2)$.

4. Let $c \in \mathbb{R}$ and define $f_n : [0,1] \rightarrow \mathbb{R}$ by $f_n(x) = n^c x (1 - x^2)^n$. Show that $f_n \rightarrow 0$ pointwise on $[0,1]$, no matter what $c$ is. For what values of $c$ is this convergence uniform? For what values of $c$ do the integrals $\int_{0}^{1} f_n \, dx$ converge to 0 as $n \rightarrow \infty$? You may use familiar facts from calculus to solve this problem.

5. Find a sequence of positive numbers $\{a_n\}_{i=1}^{\infty}$ such that the real power series $\sum_{n=1}^{\infty} a_n x^n$ has radius of convergence equal to 1, but, for all $r > 0$,
   $$\sum_{n=1}^{\infty} n^r a_n$$ is finite.