Power Grid Pad Assignment & Design

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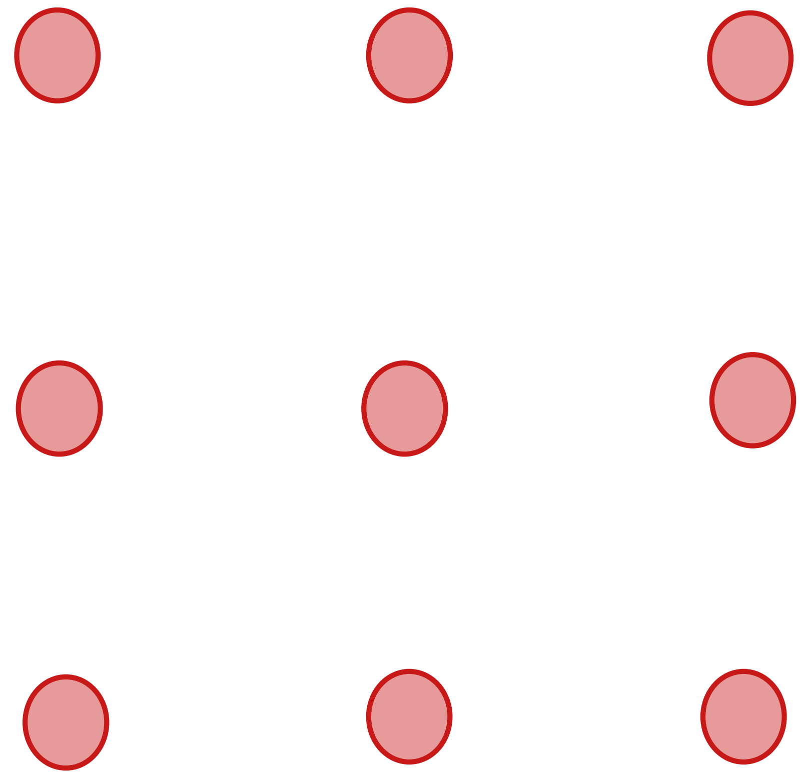
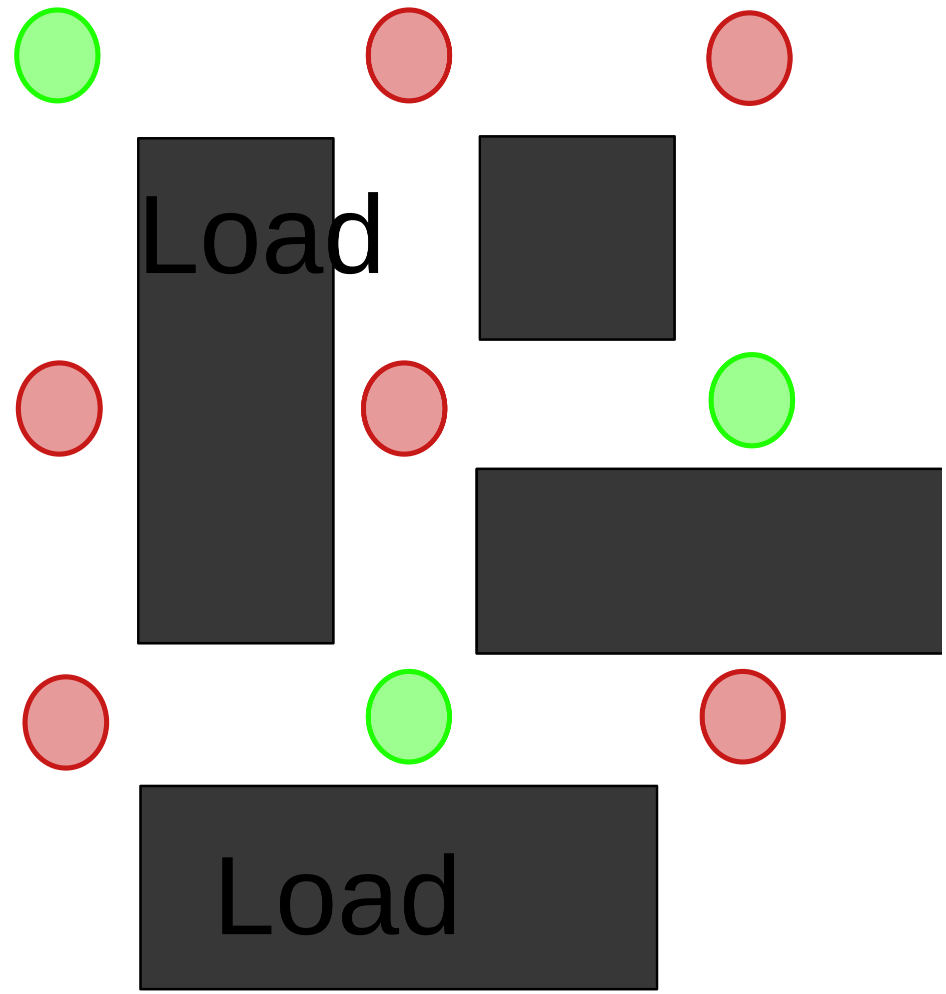
Power Distribution: C4 Pads

- Interface to the external World
- Connect to Pins on the back of a chip



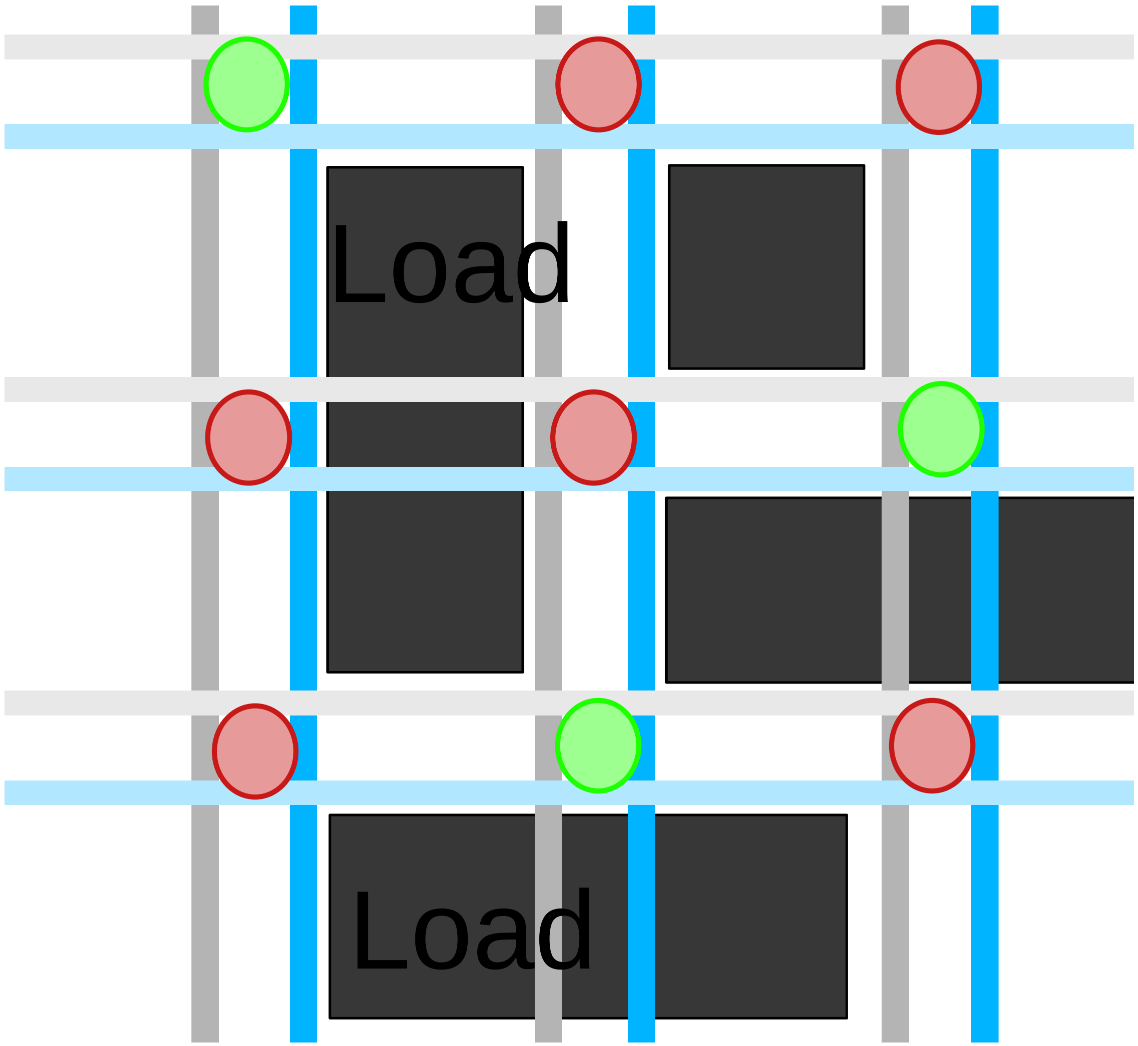
Power Distribution: Loads

- Perform various logical operations
- Pads assigned for Inputs & Outputs (green)
- Need power supply for operation



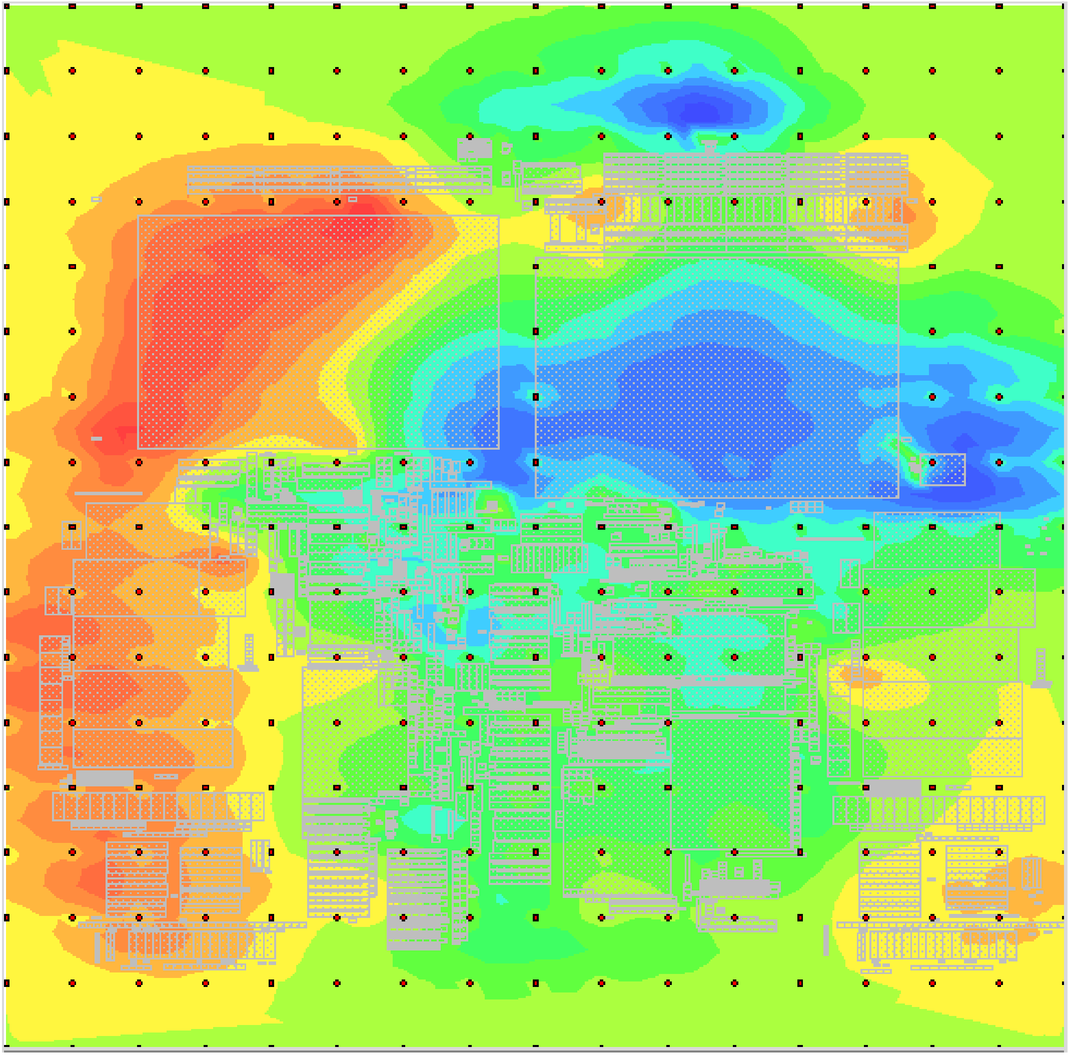
Power Distribution: Power Grid

- Grey & Blue different power levels (say GND & VDD)
- Different metal levels for Horizontal & Vertical,
- Load connects to lowest metal, pads to the highest
- Load contributes "demand" to both levels



Motivation

- Shows thermal map of VDD power
- Red areas show drop in voltage
- Blue areas have adequate voltage supply

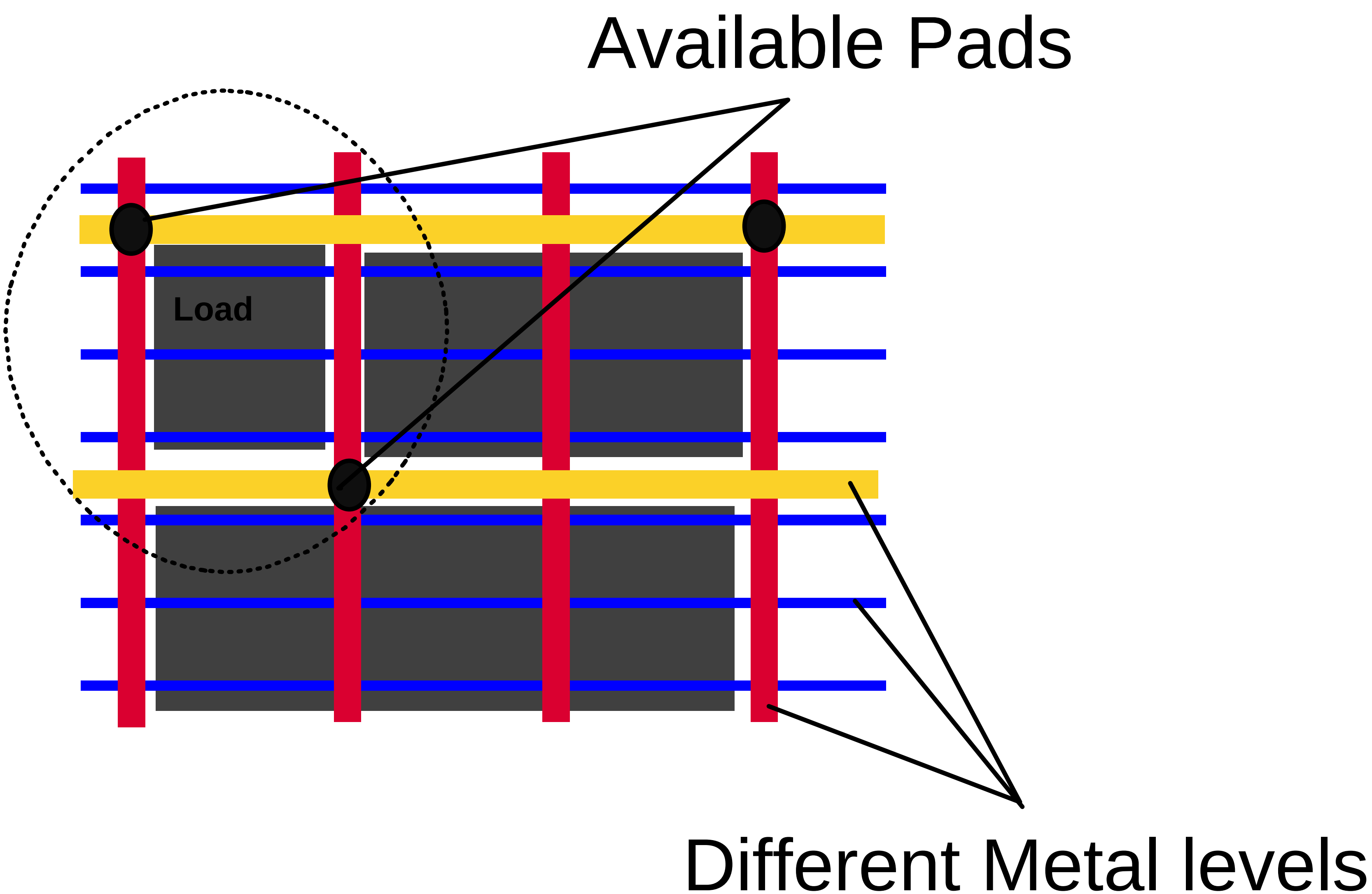


Problem Statement

Assign the available C4 pads, such that each C4 pad has an unique voltage level associated with it, or is unassigned, such that the demands from all the nodes is satisfied

Modelling

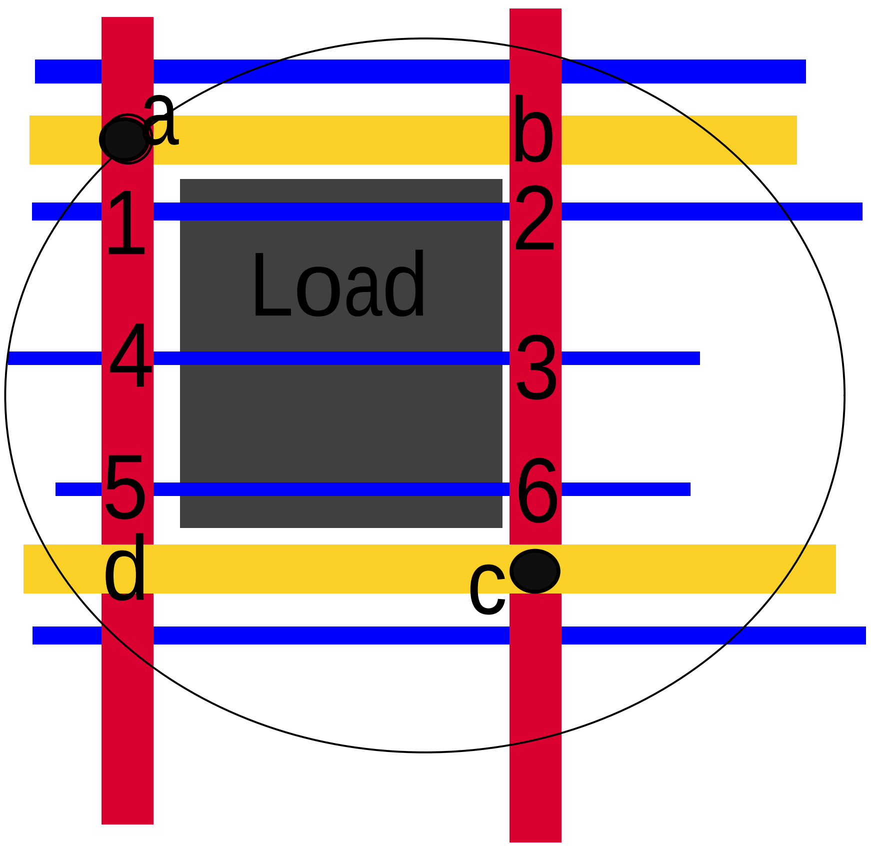
- Decoupling the different power levels

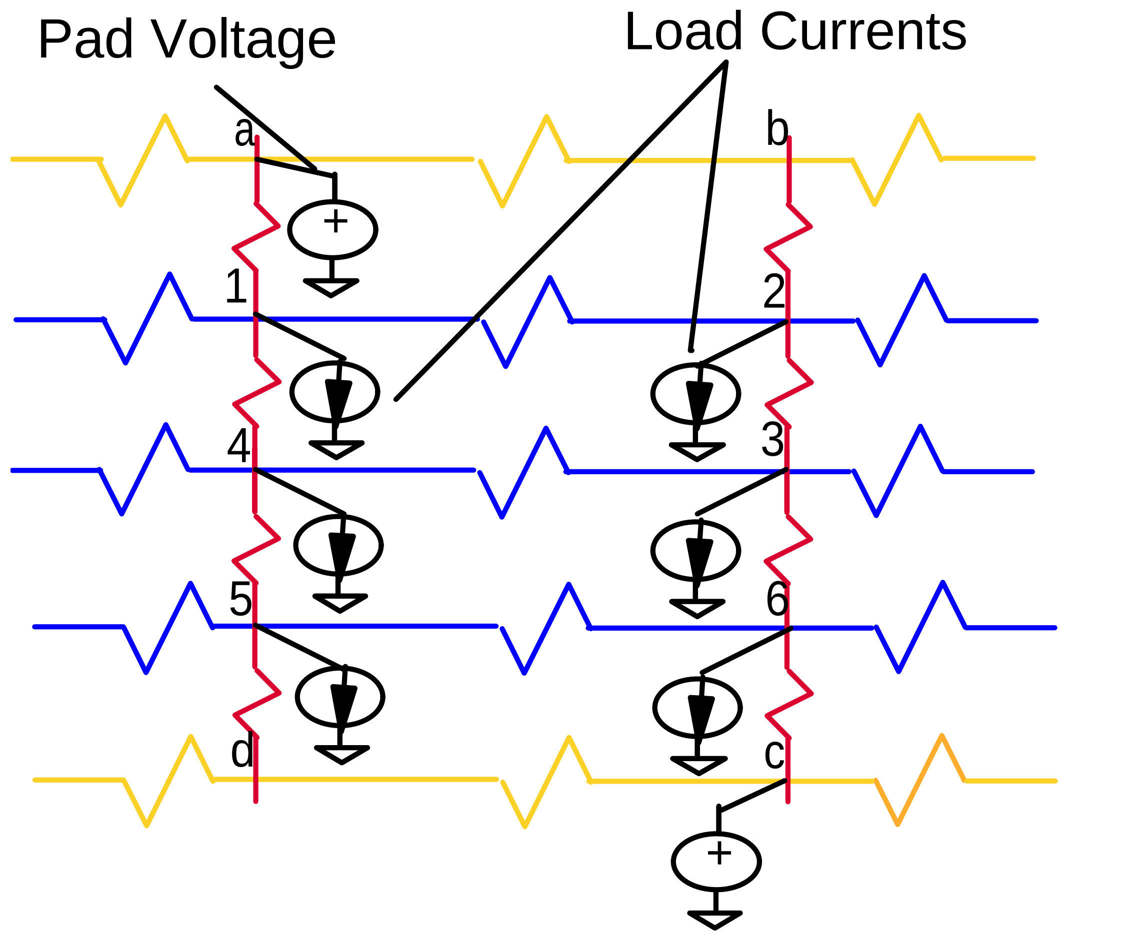


Modelling

- Metal Wires: Linear elements (resistors)
  - Value depends on size of the wire
- Load Demands: Current Sources
- Pads: Constant Voltage Sources
- Intersection between metal wires: vertices in resistor grid

Modelling



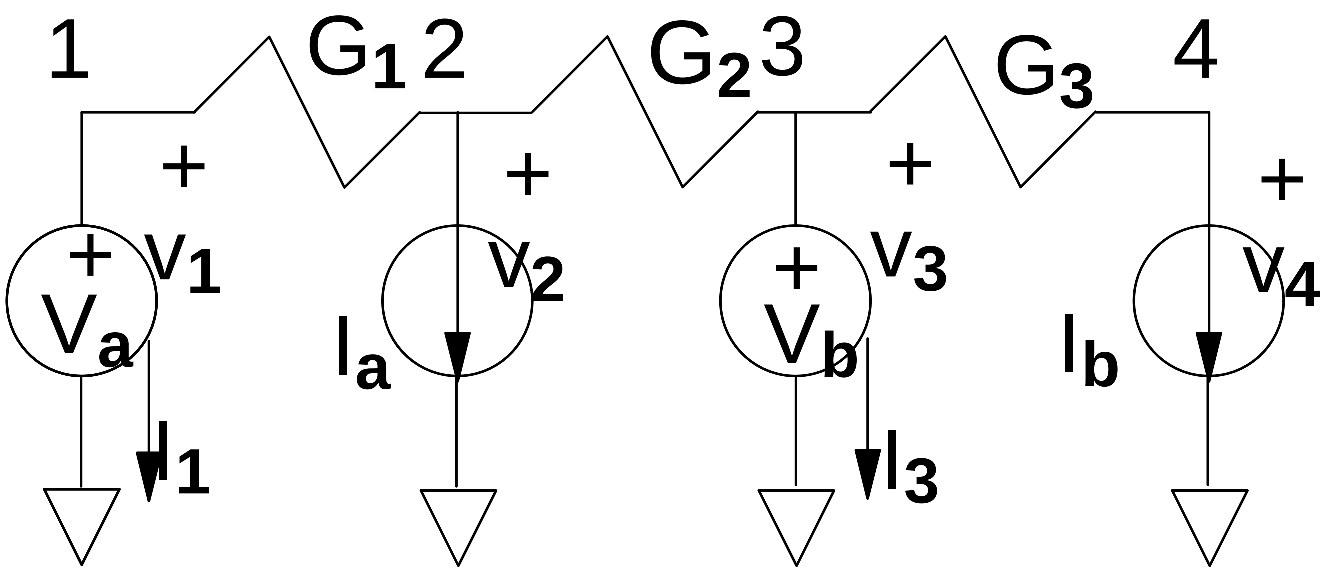


Circuit Analysis: Review

- Kirchoff's Voltage Law: $\Sigma V_i = 0$
  - Sum of voltages around a loop is zero
- Ohm's law: $GV = I$
  - Current through a linear element proportional to voltage across it
- Kirchoff's Current Law: $\Sigma I_i = 0$
  - Total current at any node is zero

Circuit Analysis: Example

- Circuit with 4 unknowns: $V_1, V_2, V_3, V_4,$
- Current sources add unknowns: I_1 & I_3

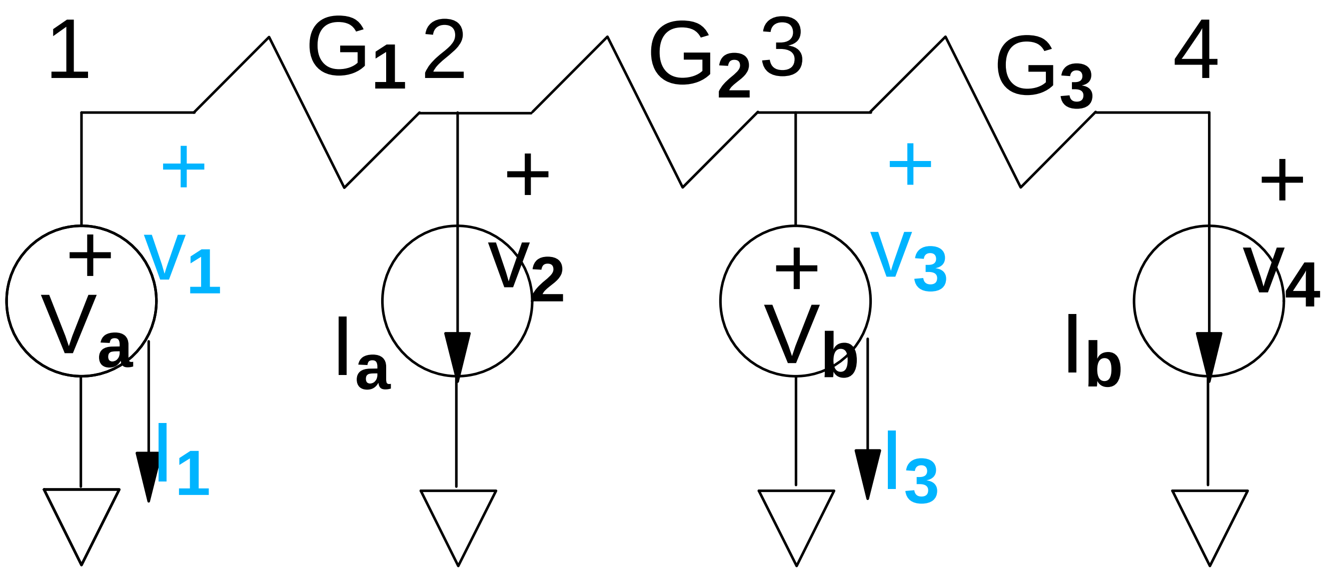


Circuit Analysis: Example

- KVL gives us

$V_1 = V_a$

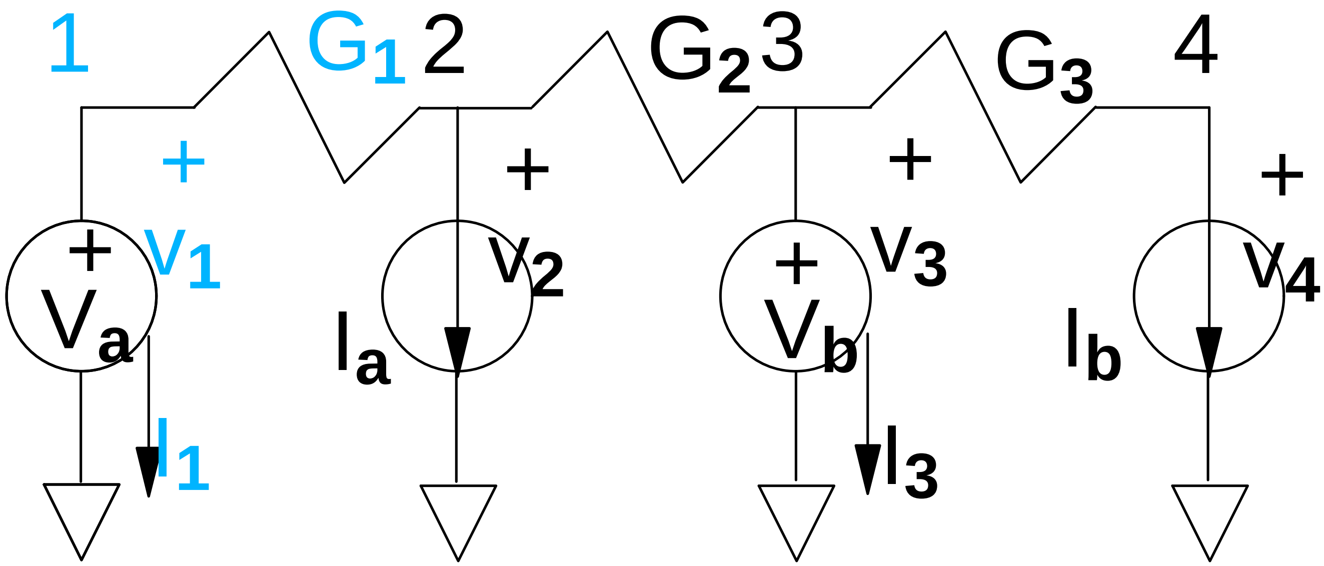
$V_3 = V_b$



Circuit Analysis: Example

- Applying KCL at node 1:

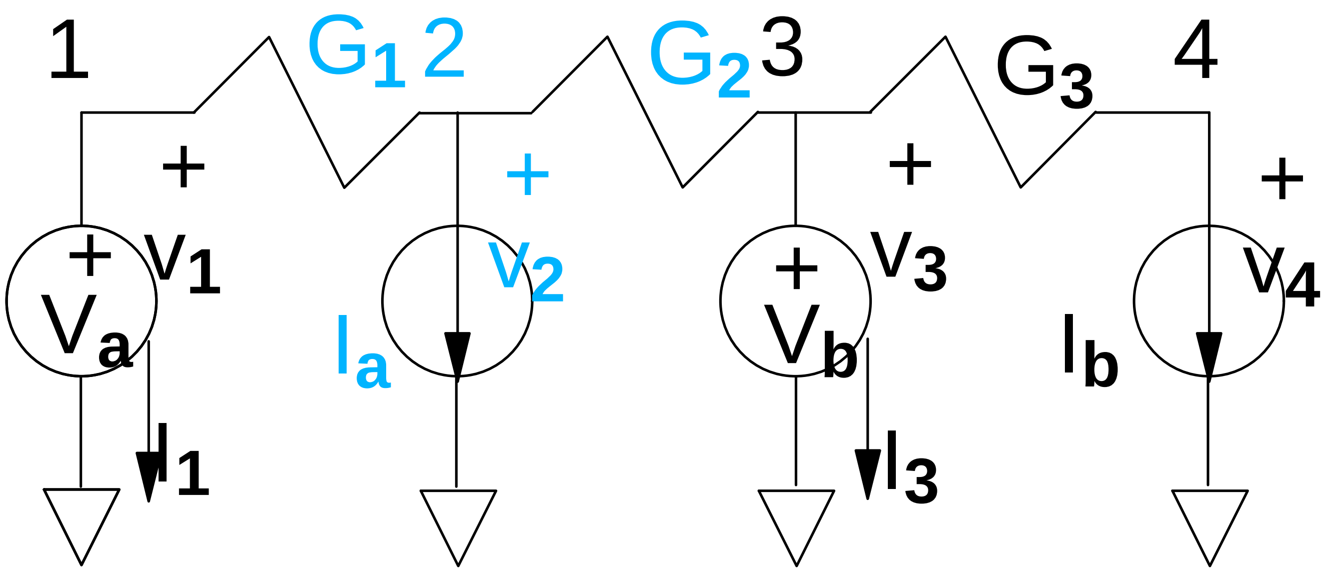
  $+I_1 + (V_1 - V_2)G_1 = 0$



Circuit Analysis: Example

- Applying KCL at node 2

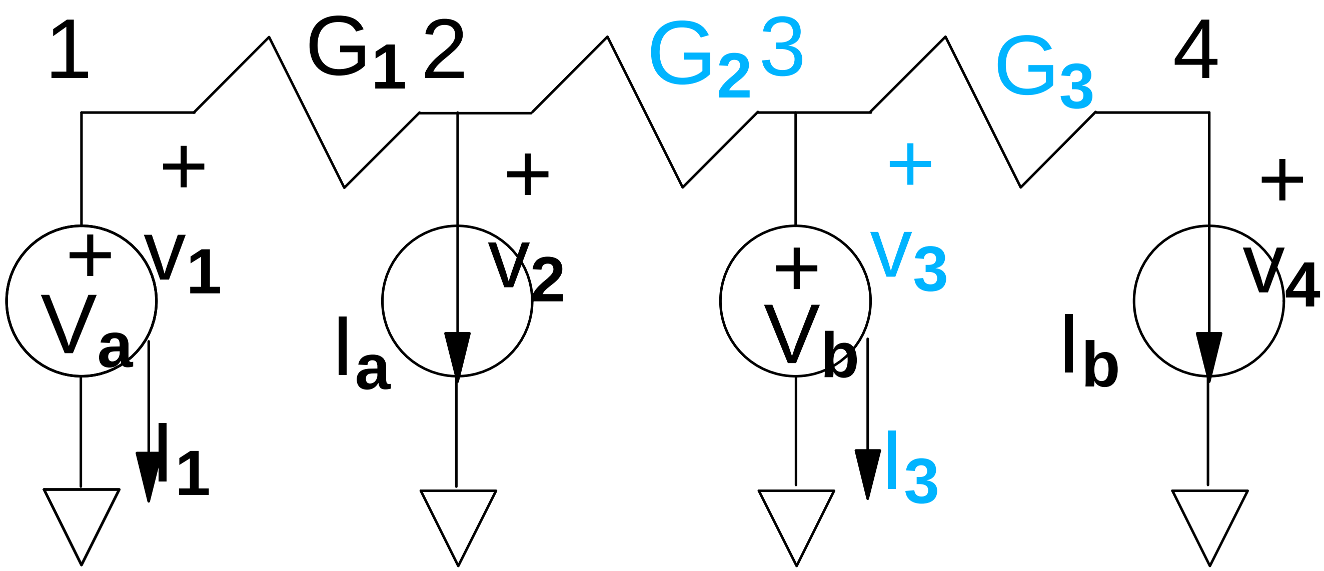
  $(V_2 - V_1)G_1 + (V_2 - V_3)G_2 + I_a = 0$



Circuit Analysis: Example

- Applying KCL at node 3

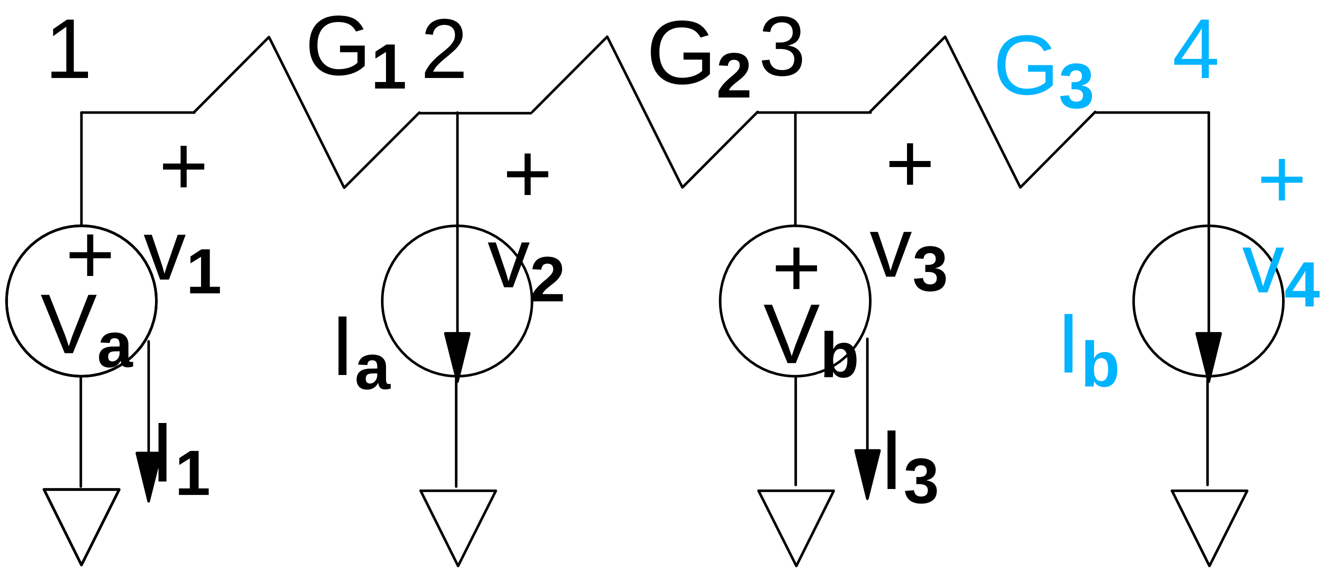
  $(V_3 - V_2)G_2 + (V_3 - V_4)G_3 + I_3 = 0$



Circuit Analysis: Example

- Applying KCL at node 4

$(V_4 - V_3)G_3 + I_b = 0$



Circuit Example

- Collecting these equations, we get

$$
\begin{bmatrix}
G1 & -G1 & 0 & 0 & 1 & 0 \\
-G2 & G1+G2 & -G2 & 0 & 0 & 0 \\
0 & -G2 & G2+G3 & -G3 & 0 & 1 \\
0 & 0 & -G3 & G3 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_1 \\ I_2
\end{bmatrix}
=
\begin{bmatrix}
0 \\ I_a \\ 0 \\ I_b \\ V_1 \\ V_2
\end{bmatrix}
$$

Voltage at load

Pad voltages

Circuit Analysis: Example

- In general, these circuits have the form:

  GV = I

  - **G** is called the "Admittance" matrix
  - pad voltage sources appear are elements in **I**
  - Node voltages are solved as elements of **V**
- Solvable in linear time & memory
- Effect of adding or subtracting voltage sources calculated in constant time

Quality of Assignment

- Node Voltages are within a particular range

  $V_{min} < V_i < V_{max}$

- Currents on power grid must not exceed their capacity to carry it

  $I_{max} > G_{ik}(V_i - V_k)$

Formulation

Given Pads $\{C_1 \ldots C_n\}$ and power levels $\{0, P_1 \ldots P_k\}$ derive an unique assignment of P_i for every C_u, such that the demand requirements on P_i

$$V_{minPi} < D_{Pi}V < V_{maxPi}$$

are met, where D_{Pi} is the "demand" matrix with V satisfying the relationship

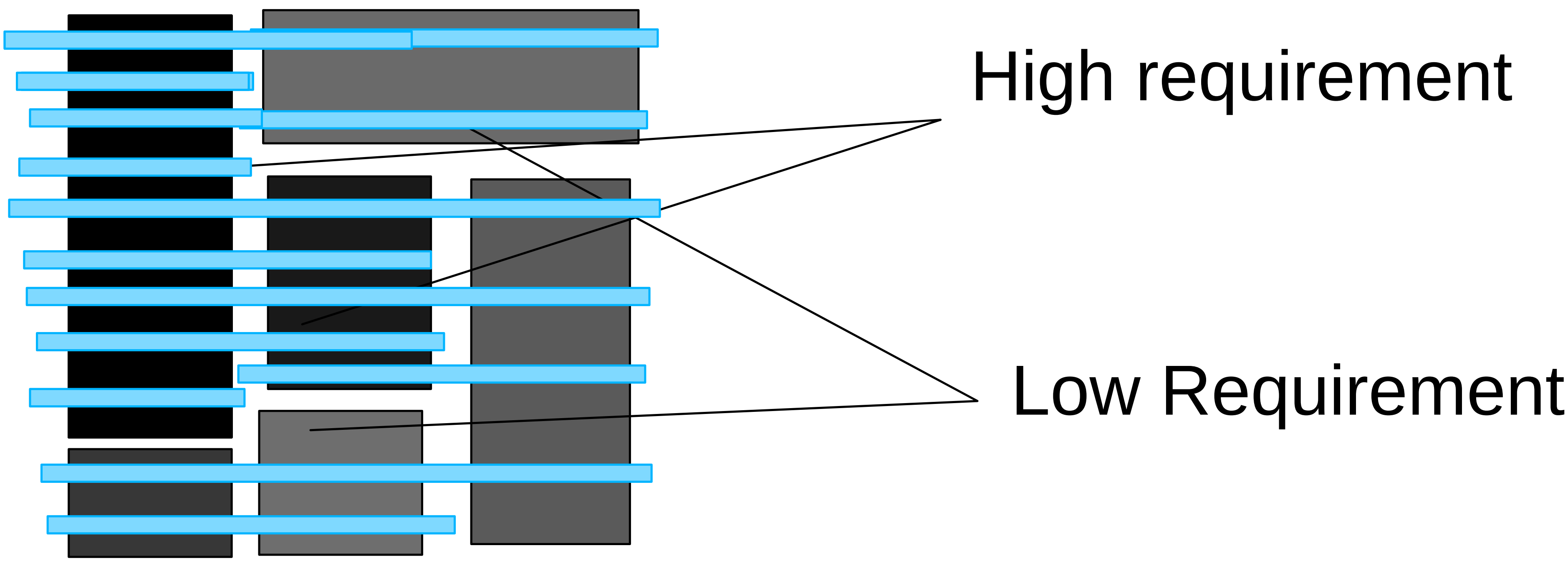
$$G_{Pi}V = I_{Pi}$$

Dimension of the Problem

- Number of Pads : 1000s
- Number of Voltage Levels : 2-4
- Number of Load Nodes: ~10-30 Million
- Dimension of Demand matrix: constant factor (~4-5) in number of load nodes

Power Grid Design

- Different parts have different load requirements
- Desirable to minimise metal usage in low demand regions



Power Grid Requirements

- Different Metal Usage strategies for a given load region possible
- Leads to multiple possibilities for the "Admittance" matrix **G**

Power Grid Requirements

In addition to Pad assignment, optimise the grid's metal usage by selecting a **G** matrix from $\{G_{1P_i} \ldots G_{mP_i}\}$ for each Power level P_i

Other Issues: Time Dependency

- Current model is static, in reality need voltages over multiple time points
  - Causes the relationship to become

  $GV + CV' = I$ (**V'** is the derivative)
- Solved by converting to difference equations and using trapezoidal methods
- Require current through Pads (I_1 & I_3 in example) to not exceed Pad's capacity