0. Define the terms: group, subgroup, homomorphism, isomorphism, order of a group, order of an element, abelian group, cyclic group, cycle notation, symmetric group of degree \( n \), equivalence relation, equivalence class, conjugate, well-defined function.

1. Multiple Choice: Underline the correct wording inside the parentheses.
   a. If \( g^N = 1 \) then \( N \) must be (the order of \( g \), a multiple of the order of \( g \), a factor of the order of \( g \)).
   b. A cyclic group of order 6 is isomorphic to ((\( \mathbb{Z}/7\mathbb{Z} \))\(^\times \), \( S_3 \), \( D_{12} \)).
   c. The group \( D_8 \) is (cyclic, abelian, non-abelian).
   d. The exponent rule \( (g^c)^d = g^{cd} \), \( (gh)^c = g^ch^c \) for \( c > 0 \), \( (gh)^{-1} = g^{-1}h^{-1} \) is valid in any group.
   e. An example of a subgroup of \( \mathbb{Q} \) is (the set of rational numbers in lowest terms whose denominator is odd, the set of rational numbers in lowest terms whose denominator is even, the set of all non-zero rational numbers).
   f. The group operation in \( (\mathbb{Z}/n\mathbb{Z})^\times \) is (ordinary multiplication, multiplication of equivalence classes, addition of equivalence classes, composition of functions).
   g. The order of \( \mathbb{T} \) in \( \mathbb{Z}/6\mathbb{Z} \) is (4, 6, 3, 3/2, infinite, 0, undefined).

2. Let \( \sigma = (1 \ 4 \ 2 \ 5)(3 \ 6 \ 7 \ 9) \) in \( S_9 \), and \( \tau = (2 \ 4 \ 5 \ 7 \ 9) \).
   a. Find \( \sigma^{-1} \) in standard cycle notation.
   b. Find the value of \( \tau(3) \) and \( \tau(5) \).
   c. What is the order of \( \sigma \)?
   d. Write down (in cycle notation) the elements of the subgroup generated by \( \sigma \).
   e. Show how to compute \( \sigma \tau \sigma^{-1} \) in two different ways.

3a. Suppose that \( A \) and \( B \) are groups. Show that \( A \times B \) is a group with componentwise operations.
   b. Show that \( A \times \{e_B\} \) is a subgroup of \( A \times B \).
   c. Show that \( A \times \{e_B\} \cong A \).

4. Suppose that \( G \) is a group and \( g \) is a specific element in \( G \). We define \( \varphi : \mathbb{Z} \to G \) by \( \varphi(k) = g^k \).
   a. Prove that \( \varphi \) is a homomorphism.
   b. If \( g \) generates \( G \) show that \( \varphi \) is onto.
   c. If \( g \) has infinite order, show that \( \varphi \) is one-to-one.
   d. If \( G = \langle g \rangle \), show that \( \phi \) is an isomorphism.

5. Let \( H = \left\{ \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix} : ad \neq 0 \right\} \) be the set of diagonal matrices in \( \text{Gl}_2(\mathbb{R}) \).
   a. Show that \( H \) is a subgroup.
   b. Show that \( H \) is abelian.
   c. Write down an element of order 2 in \( H \).
   d. Write down an element of infinite order in \( H \).
   e. Show that \( H \) is isomorphic to \( \mathbb{R}^\times \times \mathbb{R}^\times \).