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Math 21F Quiz VII October 28, 2014

As always, show your method for solving each problem

1a. For the function \( f(x) = 3x^{1/3} - x \), show how to use calculus to find the absolute maximum and minimum values of \( f(x) \) for \( x \) in the interval \([-8, 27]\).

\[
f'(x) = \frac{-2x^{2/3}}{x^{2/3}} - 1 = \frac{1 - x^{2/3}}{x^{2/3}} \quad \text{Critical values at}
\]

\( f'(x) = 0 \) and \( f'(x) \) undefined. \( f'(x) = 0 \) for \( x^{2/3} = 1 \) so \( x = 1 \).

\( f'(x) \) undefined for \( x = 0 \). We test \( f(x) \) at end points and critical points.

\[
f(-8) = 3 \cdot (-2) - (-8) = 2 \quad f(-1) = -3 + 1 = -2
\]

\[
f(0) = 0 \quad f(1) = 3 - 1 = 2 \quad f(27) = 3 \cdot 3 - 27 = -18
\]

Comparing, we see [\( \text{max of } f \) at \( x = -8 \) and \( x = 1 \)]

\[
\text{min of } -18 \text{ at } x = 27
\]

b. Where does \( f(x) \) have a local max or min in this interval? (These are not possible at endpoints)

\[
\text{local min of } -2 \text{ at } x = -1
\]

\[
\text{local max of } 2 \text{ at } x = 1
\]

2a. State the Mean Value Theorem in full detail.

If \( f(x) \) is continuous on \([a, b]\) and differentiable on \((a, b)\),

\[
\text{there is a } c \in (a, b) \text{ so that}
\]

\[
\frac{f(b) - f(a)}{b - a} = f'(c)
\]

b. State one of the important theorems or facts that is proved by using the Mean Value Theorem.

10  Constant Thm: If \( f'(x) = 0 \) on an interval, then \( f(x) \) is constant.

\( \text{or: Same Derivative Thm}: \text{ If } f'(x) = g'(x) \text{ on an interval, then } f(x) = g(x) + C \).