This will be a closed book, closed notes exam. No calculators or other devices are allowed. Just writing utensils, and scrap paper if needed.

**KNOW THE FOLLOWING CONCEPTS**

union, intersection, complement, cartesian product, image of a set under a function, inverse image of a set under a function, field properties of the real numbers, order properties of the real numbers, least upper bound property of the real numbers, sup, inf, greatest lower bound, metric or distance function, metric space, open and closed balls, open and closed sets and their properties, sequence in a metric space, convergent sequence, Cauchy sequence, limit of a sequence, subsequence, complete, compact, connected, monotone convergence theorem, Heine-Borel theorem, Bolzano-Weierstrass theorem for sets and for sequences, completeness of $E^m$, closed sets in terms of sequences, bounded set.

**PRACTICE PROBLEMS:**

1. Suppose that $A$ and $B$ are both connected sets in a metric space $X$, and that the intersection $A \cap B$ is not empty. Show that the union $A \cup B$ is a connected set.

2. Show that the "sup metric" defined on $\mathbb{R}^2$ by $d_{\infty}((a, b), (c, d)) = \max\{|c - a|, |d - b|\}$ is truly a metric.

3. Suppose that $\{p_n\}$ is a sequence of points in a metric space and $p$ is a point such that for each $n$, $d(p, p_n) < 1/n$. Show that $\lim_{n \to \infty} p_n = p$.

4. Fill in the details of the following proof of the Lebesque covering lemma (p. 64. #33 in our text).

Let $E$ be a compact metric space and $U_i$ be an open covering of $E$. Then there exists a number $\delta > 0$ (known as a Lebesque number for the covering) such that any ball in $E$ of radius $\delta$ is contained in $U_i$ for some $i$.

Proof by contradiction: Suppose there does not exist such a number. Then for each positive integer $n$, $\delta = 1/n$ does not have the desired property. Thus there is a point $p_n$ in $E$ such that $B(p_n, 1/n)$ is not contained in any of the sets $U_i$. The sequence $\{p_n\}$ has a convergent subsequence $\{p_{n_k}\}$ by...

Let $p \in E$ be the limit of the subsequence. Then $p$ is in $U_{i_0}$ for some $i_0$ because....

Then there is a ball $B(p, r) \subset U_{i_0}$ for some $r > 0$ by......

However, we can choose $N$ such that $p_{n_k} \in B(p, r/2)$ for all $k > N$ by....

Now fix $k > N + 2/r$. We have $d(p, p_{n_k}) < r/2)$ and $1/n_k \leq 1/k < r/2$ because...

Thus if $x \in B(p_{n_k}, 1/n_k)$, we have $d(p_{n_k}, x) < r/2$ because...

Then $d(p, x) \leq d(p, p_{n_k}) + d(p_{n_k}, x) < r/2 + r/2$ by....
Thus $d(p, x) < r$, which means that $x \in B(p, r)$. We have shown that $B(p_{nk}, 1/n_k) \subset B(p, r)$ because...

Now notice that $B(p_{nk}, 1/n_k) \subset B(p, r) \subset U_{i_0}$, so $B(p_{nk}, 1/n_k) \subset U_{i_0}$. Now we have a contradiction, because....