Study guide for Test 3

1. Know how to use the following rules to differentiate functions: (Two problems)

- The power rule: \((x^n)' = nx^{n-1}\)
  
  Example: \((x^5)' = 5x^4\), \\
  \((x^{-1})' = -x^{-2}\), \\
  \((\sqrt{x})' = (x^{1/2})' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\)

- The product rule: \((fg)' = f'g + fg'\)
  
  Example: \((xe^x)' = e^x + xe^x\)

- The quotient rule: \(\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}\)
  
  Example: \(\left(\frac{1}{x^2+1}\right)' = \frac{0 \cdot (x^2+1) - 1 \cdot (2x)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}\)

- The exponential-function rule: \((e^x)' = e^x\), \((a^x)' = a^x \ln a\)
  
  Example: \((10^x)' = 10^x \ln 10\)

- The logarithmic function rule: \((\ln x)' = \frac{1}{x}\), \((\log_a x)' = \frac{1}{x \ln a}\)
  
  Example: \((\log x)' = \frac{1}{x \ln 10}\)

- The generalized power rule: \([f^n(x)]' = nf^{n-1}(x)f'(x)\)
  
  Example: \([(1+2x)^5]' = 5(1+2x)^4 \cdot 2 = 10(1+2x)^4\)

  \[((x+e^x)^3)' = 3(x+e^x)^2 \cdot (1+e^x) = 3(1+e^x)(x+e^x)^2\]

- The generalized exponential rule: \((e^{f(x)})' = e^{f(x)}f'(x)\), \((a^{f(x)})' = a^{f(x)}f'(x)\ln a\)
  
  Example: \([e^{x^2+1}]' = e^{x^2+1} \cdot (2x) = 2xe^{x^2+1}\)
\[ [5^{x^{x^1}}]' = 5^{x^{x^1}} \ln 5 \cdot (2x) = 2(\ln 5)x5^{x^{x^1}} \]

- The generalized log rule: \([\ln f(x)]' = \frac{f'(x)}{f(x)}\), \([\log_a f(x)]' = \frac{f'(x)}{f(x) \ln a}\)

Example: \([\ln(3x^2 + x)]' = \frac{6x + 1}{3x^2 + x}\)

\([\log(3x + 1)]' = \frac{3}{(3x + 1) \ln 10}\)

- The chain rule

Example: \(y = \sqrt{1+\sqrt{x}}\)

Set \(y = \sqrt{u}\), \(\frac{dy}{du} = \frac{1}{2\sqrt{u}}\)

\(u = 1 + \sqrt{x}\), \(\frac{du}{dx} = \frac{1}{2\sqrt{x}}\)

Thus, \(\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{4\sqrt{x}\sqrt{1+\sqrt{x}}}\)

2. Know how to determine equations of tangent lines if a function \(f(x)\) and a point \(x=x_0\) is given

Example: \(y = \sqrt{x^2 + 5}\) at \(x=2\), determine the equation of the tangent line.

3. Know how to determine on what intervals a function is increasing or decreasing

Example: Determine where the function \(y = 4x^3 - 9x^2 - 30x + 6\) is increasing and where it is decreasing.

(a) \(y'(x) = 12x^2 - 18x - 30\)

(b) critical numbers: \(12x^2 - 18x - 30 = 0 \Rightarrow x = -1, \frac{5}{2}\)

(c) \(y'(x) < 0\) when \(-1 < x < \frac{5}{2}\), thus the function is decreasing when \(-1 < x < \frac{5}{2}\)
(d) \( y'(x) > 0 \) when \( x < -1 \) and \( x > \frac{5}{2} \), thus the function is increasing when \( x < -1 \) and \( x > \frac{5}{2} \).

4. Know how to determine relative maxima and minima of a given function using the first derivative test and the second derivative test; know how to obtain critical numbers and critical points (two problems).

Example 1: For the function \( y = 4x^3 - 9x^2 - 30x + 6 \)

(1) Determine its critical numbers

(2) Determine its relative maximum and minimum points (if any) using the first derivative test.

Example 2: For the function \( y = 4x^3 - 9x^2 - 30x + 6 \)

(1) Determine its critical numbers

(2) Determine its relative maximum and minimum points (if any) using the second derivative test.

5. Know how to determine on what intervals a function is concave up or concave down and where are the inflection points.

Example: For the function \( y = 4x^3 - 9x^2 - 30x + 6 \), determine where it is concave up and where it is concave down. Where are the inflection points?