BOUND STATES IN PERIODIC LATTICES

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Abstract:

The bifurcation of gap solitary waves near edges of Bloch bands in 1-D periodic media is studied in terms of the nonlinear Schrödinger (NLS) equation with a periodic potential. Based on standard multiple-scale perturbation theory, in the small-amplitude limit, solitary waves bifurcating from band edges are in the form of wavepackets, modulated by a “sech” envelope whose position relative to the underlying periodic Bloch mode remains undetermined. It is shown by means of exponential asymptotics that, out of this one-parameter solution family, only two branches of truly locally confined solutions bifurcate from band edges; these fundamental branches correspond to “on-site” and “off-site” gap solitons, the former being stable and the other unstable. In addition, there exist an infinite number of solution branches that comprise more than one fundamental gap solitons; such bound states, however, bifurcate in the interior of the band gap at small but finite amplitude. The predictions of the asymptotic theory are compared against numerical results.