Fractal scattering in weak interactions of solitary waves

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Abstract:

Weak interactions of solitary waves refer to the interactions where two solitary waves initially are well separated and having almost the same velocities. These interactions occur due to the tail overlap between the two waves. Previous work has shown that in a large class of generalized NLS equations, weak interactions of solitary waves exhibit a universal fractal scattering phenomenon, i.e., the interaction outcome depends on the initial conditions in a universal fractal manner [1]. Through two successive asymptotic reductions, this fractal scattering was found to be governed by a universal second-order map which does not contain any free parameters [2,3]. In this talk, we analyze the properties of this second-order map. We show that this map exhibits a fractal structure of its own. More importantly, this fractal of the map gives a complete characterization of universal fractal structures arising in the original generalized NLS equations. The physical mechanism for fractal scatterings is also investigated. This scattering is linked to second-order solitons in the integrable NLS equation whose zero in the spectral plane is double-fold. Due to the degeneracy of this double zero, when these second-order NLS solitons are perturbed, whether they will stay together or break apart depends sensitively on the perturbations. This sensitivity is ultimately responsible for fractal scatterings in the generalized NLS equations. This physical mechanism is very different from the previous mechanism of resonant energy exchange between translational motion and internal oscillations, which was responsible for fractal scatterings in solitary wave collisions [4]. This talk is based in part on the materials in Ref. [5].

References: