Homework 2, Due Tuesday, Feb. 21

1. Use the pseudo-spectral method to solve the KdV equation

$$u_t + u_{xxx} + 6uu_x = 0,$$

on the x interval $[-40,\ 40]$ for $0 \le t \le 10$ with the following initial conditions

(1) $u(x,0) = 2\text{sech}^2(x+20)$;

(2) $u(x,0) = 2\text{sech}^2(x+20) + \frac{1}{2}\text{sech}^2\frac{x+5}{2}$.

Choose your Δx and Δt values so that your numerical solution has accuracy no less than 10^{-5}. Turn in your numerical code as well as the numerical results.

2. In this question, we want to verify the spectral accuracy of the pseudo-spectral method for the KdV equation

$$u_t + u_{xxx} + 6uu_x = 0.$$

You can choose the x interval $[-40,\ 40]$, $0 \le t \le 10$ and the initial condition

$$u(x,0) = 2\text{sech}^2(x+20).$$

Use a very small fixed time step (so that temporal error is negligible), and obtain the numerical solution's error at $t = 10$ for various Δx values (note that the above initial condition is a soliton so its solution at $t = 10$ can be analytically obtained from that in your Homework 1). Then plot this error versus $1/\Delta x$ and report how this error decays with $1/\Delta x$. This process is a repetition of that in my book (see Fig. 7.1), except that it is for the KdV equation rather than the NLS equation now.