Solutions

Math 19 Test #4

Name:

Differentiate the following functions:

(1) $f(x) = 2x^2 - x$

$$f'(x) = 4x - 1$$

(2) $f(x) = e^x + \sqrt{x}$

$$f'(x) = e^x + \frac{1}{2\sqrt{x}}$$

(3) $f(x) = \ln x - \frac{1}{x}$

$$f'(x) = \frac{1}{x} + \frac{1}{x^2}$$

(4) $f(x) = \frac{1+x}{1-x}$

$$f'(x) = \frac{(1+x)'(1-x) - (1+x)(1-x)'}{(1-x)^2} = \frac{1-x+(1+x)}{(1-x)^2}$$

$$= \frac{2}{(1-x)^2}$$

(5) $f(t) = t^2 \ln t$

$$f'(t) = 2t \ln t + t^2 \cdot \frac{1}{t}$$

$$= 2t \ln t + t$$

$$= t(2\ln t + 1)$$

(6) $f(x) = \frac{3e^x}{1+x^2}$

$$f'(x) = \frac{3e^x(1+x^2) - 3e^x(1+x^2)'}{(1+x^2)^2}$$

$$= \frac{3e^x(1+x^2) - 3e^x \cdot 2x}{(1+x^2)^2} = \frac{3e^x(x-1)^2}{(1+x^2)^2}$$

(7) $f(x) = \frac{1}{(x^2+x+1)^5}$

$$f'(x) = -5(x^2+x+1)^{-6} \cdot (2x+1)$$

$$= -\frac{5(2x+1)}{(x^2+x+1)^6}$$

(8) $f(t) = (t^2+1)(t^4+t+1)^3$

$$f'(t) = 2t(t^4+t+1)^3 + (t^2+1) \cdot 3(t^4+t+1)^2 \cdot (4t^3+1)$$

$$= (t^4+t+1)^2 \left[2t(t^4+t+1) + 3(t^2+1)(4t^3+1) \right.$$

$$= (t^4+t+1)^2 \left[2t^5 + 2t^2 + 2t + 3(4t^5 + 4t^3 + t^2 + 1) \right.$$

$$= (t^4+t+1)^2 \left[14t^5 + 12t^3 + 5t^2 + 2t + 3 \right.$$

(9) $f(x)=\ln(x^2+1)+e^{3x}$

$$f'(x)=\frac{2x}{x^2+1}+3e^{3x}$$

(10) $f(t)=\sqrt[3]{t^2} = t^{\frac{2}{3}}$

$$f'(t)=\frac{2}{3}t^{-\frac{1}{3}}=\frac{2}{3}\cdot\frac{1}{\sqrt[3]{t}}$$

(11) $f(x)=\dfrac{e^{2x}}{1+x}$

$$f'(x)=\frac{2e^{2x}(1+x)-e^{2x}}{(1+x)^2}$$

$$=\frac{e^{2x}(2x+1)}{(1+x)^2}$$

(12) $f(x)=\ln\left(x+\ln(x^2+1)\right)$

$$f'(x)=\frac{1+\frac{2x}{x^2+1}}{x+\ln(x^2+1)}$$

$$=\frac{(x+1)^2}{[x+\ln(x^2+1)][x^2+1]}$$