

Name:

Math 20 Exam 4
Closed books and notes

1. (10 points) Change the angle measure as requested:

(1) 18° to radians

$$= 18^\circ \times \frac{\pi}{180^\circ}$$
$$= \frac{\pi}{10}$$

(2) $\frac{\pi}{10}$ to degrees

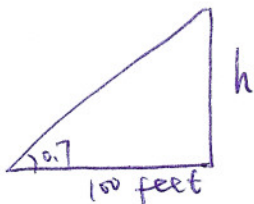
$$\frac{\pi}{10} \times \frac{180^\circ}{\pi}$$
$$= 18^\circ$$

2. (15 points) Use your calculator to find the following values

(1) $\sin \frac{\pi}{8} = 0.3827$

(2) $\cos 15^\circ = 0.9659$

3. (15 points) An instrument located 100 feet from a tree measures the indicated angle in radians as 0.7. Find the height of the tree.



$$\frac{\sin 0.7}{\cos 0.7} = \frac{h}{100}$$

$$h = 100 \cdot \frac{\sin 0.7}{\cos 0.7} = 84.2 \text{ feet}$$

4. (30 points) Calculate the following derivatives:

(1) $\sin(5x)$

$$= 5 \cos 5x$$

(2) $\frac{\sin x}{\cos x}$

$$\frac{(\sin x)' \cos x - \sin x (\cos x)'}{\cos^2 x}$$

$$= \frac{\cos x \cdot \cos x - \sin x (-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$(3) \ln(\cos x)$$

$$= \frac{-\sin x}{\cos x}$$

$$(4) \sin^2 x \cos^2 x$$

$$\begin{aligned} & (\sin^2 x)' \cos^2 x + \sin^2 x \cdot (\cos^2 x)' \\ &= 2 \sin x \cos x \cdot \cos^2 x + \sin^2 x \cdot 2 \cos x \cdot (-\sin x) \\ &= 2 \sin x \cos^3 x - 2 \sin^3 x \cos x \\ &= 2 \sin x \cos x (\cos^2 x - \sin^2 x) \end{aligned}$$

5. (30 points) Evaluate each integral:

$$(1) \int \sin 5x dx$$

$$= -\frac{1}{5} \cos 5x + c$$

$$(2) \int x^2 \cos x^3 dx = \frac{1}{3} \int \cos u \cdot du$$

$$u = x^3 \quad = \frac{1}{3} \sin u + c$$

$$du = 3x^2 dx$$

$$= \frac{1}{3} \sin x^3 + c$$

$$(3) \int \frac{\cos x}{\sin x + 2} dx$$

$$u = \sin x + 2$$

$$du = \cos x dx$$

$$= \int \frac{du}{u} = \ln u + c$$

$$= \ln(\sin x + 2) + c$$

$$(4) \int_0^{\frac{\pi}{3}} \sin x dx$$

$$= -\cos x \Big|_0^{\frac{\pi}{3}}$$

$$= -\cos \frac{\pi}{3} + \cos 0$$

$$= 1 - \cos \frac{\pi}{3}$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$