

Math 330 HW 1

1. Plot the phase portrait (preferably using computers) and classify the fixed point of the following linear systems. In each case, also indicate the directions of the trajectories.

$$(1) \begin{pmatrix} x \\ y \end{pmatrix}_t = \begin{pmatrix} 5 & 10 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}; \quad (2) \begin{pmatrix} x \\ y \end{pmatrix}_t = \begin{pmatrix} 5 & 2 \\ -17 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix};$$

$$(3) \begin{pmatrix} x \\ y \end{pmatrix}_t = \begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}; \quad (4) \begin{pmatrix} x \\ y \end{pmatrix}_t = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix};$$

2. The motion of a pendulum with linear damping is modeled by the equation

$$\theta'' + \alpha\theta' + \sin \theta = 0$$

where α is a positive constant.

- (1) Determine all its critical points and their type and stability.
- (2) Sketch the whole phase portraits.