

Math 330 HW3

1. Consider the rabbits-sheep and the lions-sheep models discussed in class:

$$(a) \begin{cases} x' = x(\alpha_{11} - \alpha_{12}x - \alpha_{13}y) \\ y' = y(\alpha_{21} - \alpha_{22}x - \alpha_{23}y) \end{cases} \quad (b) \begin{cases} x' = x(\alpha_{11} - \alpha_{12}x - \alpha_{13}y) \\ y' = y(-\alpha_{21} + \alpha_{22}x) \end{cases}$$

For each model, choose different sets of parameter values α_{ij} as you like (as long as they are all positive) and explore their phase portraits. Then turn in five most interesting phase portraits you have explored. If you see phase portraits qualitatively different from those given in class, explain them theoretically.

2. The motion of a pendulum with linear damping is modeled by the equation

$$\theta'' + \alpha\theta' + \sin\theta = 0$$

where α is a positive constant.

- (1) Determine all its critical points and their type and stability.
- (2) Sketch the whole phase portraits.