

1. (20 points) Differentiate the following functions:

$$\begin{aligned} (1) \quad (xe^x)' &= x'e^x + x(e^x)' \\ &= e^x + xe^x \\ &= (x+1)e^x \end{aligned}$$

$$\begin{aligned} (3) \quad [(2x+1)^5]' &= 5(2x+1)^4 \cdot (2x+1)' \\ &= 10(2x+1)^4 \end{aligned}$$

$$\begin{aligned} (2) \quad \left(\frac{3x-1}{2x+1}\right)' &= \frac{(3x-1)'(2x+1) - (3x-1)(2x+1)'}{(2x+1)^2} \\ &= \frac{3(2x+1) - 2(3x-1)}{(2x+1)^2} = \frac{5}{(2x+1)^2} \end{aligned}$$

$$\begin{aligned} (4) \quad (\sqrt{1+x^2})' &= [(1+x^2)^{\frac{1}{2}}]' \\ &= \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}} \end{aligned}$$

2. (20 points) Differentiate the following functions:

$$\begin{aligned} (1) \quad (e^{\sqrt{x}})' &= e^{\sqrt{x}} \cdot (\sqrt{x})' \\ &= e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}} \\ &= \frac{1}{2}x^{-\frac{1}{2}} e^{\sqrt{x}} \end{aligned}$$

$$\begin{aligned} (3) \quad [\ln(x^2+1)]' &= \frac{(x^2+1)'}{x^2+1} \\ &= \frac{2x}{x^2+1} \end{aligned}$$

$$\begin{aligned} (2) \quad \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)' &= \frac{(e^x - e^{-x})'(e^x + e^{-x}) - (e^x - e^{-x})(e^x + e^{-x})'}{(e^x + e^{-x})^2} \\ &= \frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2} \\ &= \frac{4}{(e^x + e^{-x})^2} \end{aligned}$$

$$\begin{aligned} (4) \quad [(\ln x)^2]' &= 2 \ln x \cdot (\ln x)' = 2 \ln x \cdot \frac{1}{x} \\ &= \frac{2 \ln x}{x} \end{aligned}$$

3. (15 points) Determine where each function is increasing and where it is decreasing:

$$(a) \quad f(x) = \frac{2x}{x-1}$$

$$\begin{aligned} f'(x) &= \frac{2(x-1) - 2x}{(x-1)^2} \\ &= -\frac{2}{(x-1)^2} < 0 \end{aligned}$$

critical number:  $x=1$

it decreases in  $(-\infty, 1)$  and  $(1, \infty)$

$$(b) \quad f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$$

$$\begin{aligned} f'(x) &= x^2 - x - 2 = (x+1)(x-2) = 0 \\ \text{critical numbers: } &x = -1, 2 \end{aligned}$$

$$f' \quad \begin{array}{c} + \quad - \quad + \\ \hline -1 \quad 2 \end{array}$$

increasing:  $x < -1$ ,  $x > 2$   
decreasing:  $-1 < x < 2$

4. (15 points) For the function  $f(x) = 2x^3 - 6x^2 - 18x + 1$ ,

(1) Determine its critical numbers  $f'(x) = 6x^2 - 12x - 18 = 6(x^2 - 2x - 3) = 6(x+1)(x-3) = 0$

critical numbers:  $x = -1, 3$

(2) Determine its relative maximum and minimum points, if any.

$$f''(x) = 12x - 12$$

$$f''(-1) = -24 < 0 \quad \text{relative max} \quad f(-1) = 11 \rightarrow \text{max value}$$

$$f''(3) = 24 > 0 \quad \text{relative min} \quad f(3) = -53 \rightarrow \text{min value}$$

5. (15 points) For the function  $f(x) = x^3 - 6x^2 + 9x + 10$ , find its absolute maximum and minimum on the  $x$ -interval of  $[0, 5]$ .

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$$

critical numbers:  $x = 1, 3$

$$f(1) = 14 \quad f(0) = 10$$

$$f(3) = 10 \quad f(5) = 30$$

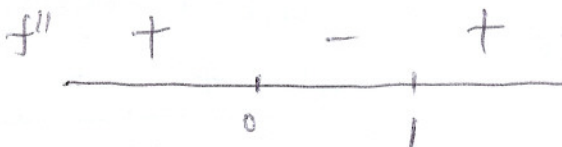
$\Rightarrow$  absolute max = 30 at  $x = 5$   
absolute min = 10 at  $x = 0, 3$

6. (15 points) For the function  $f(x) = x^4 - 2x^3 + x + 1$ , determine where its graph is concave up and where it is concave down. Also determine the inflection points.

$$f'(x) = 4x^3 - 6x^2 + 1$$

$$f''(x) = 12x^2 - 12x = 12x(x-1) = 0$$

$\Rightarrow$  inflection points:  $x = 0, 1$



$\Rightarrow$  concave up at  $x < 0$  and  $x > 1$   
concave down at  $0 < x < 1$