Math 19 Exam 3 Name

Show work on all questions

1. (20 points) Differentiate the following functions:

(1) $(xe^x)'$

$= x'e^x + x(e^x)'$

$= e^x + xe^x$

$= (x+1)e^x$

(2) $\left(\frac{3x-1}{2x+1}\right)' = \frac{(3x-1)'(2x+1)-(3x-1)(2x+1)'}{(2x+1)^2}$

$= \frac{3(2x+1)-2(3x-1)}{(2x+1)^2} = +\frac{5}{(2x+1)^2}$

(3) $\left[(2x+1)^5\right]'$

$= 5(2x+1)^4 \cdot (2x+1)'$

$= 10(2x+1)^4$

(4) $\left(\sqrt{1+x^2}\right)' = \left[(1+x^2)^{\frac{1}{2}}\right]'$

$= \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{1+x^2}}$

2. (20 points) Differentiate the following functions:

(1) $\left(e^{\sqrt{x}}\right)'$

$= e^{\sqrt{x}} \cdot (\sqrt{x})'$

$= e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2}x^{-\frac{1}{2}}e^{\sqrt{x}}$

(2) $\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)' = \frac{(e^x-e^{-x})'(e^x+e^{-x})-(e^x-e^{-x})(e^x+e^{-x})'}{(e^x+e^{-x})^2}$

$= \frac{(e^x+e^{-x})(e^x+e^{-x})-(e^x-e^{-x})(e^x-e^{-x})}{(e^x+e^{-x})^2}$

$= \frac{4}{(e^x+e^{-x})^2}$

(3) $\left[\ln(x^2+1)\right]'$

$= \frac{(x^2+1)'}{x^2+1}$

$= \frac{2x}{x^2+1}$

(4) $\left[(\ln x)^2\right]'$

$= 2\ln x \cdot (\ln x)' = 2\ln x \cdot \frac{1}{x}$

$= \frac{2\ln x}{x}$

3. (15 points) Determine where each function is increasing and where it is decreasing:

(a) $f(x) = \frac{2x}{x-1}$

$f'(x) = \frac{2(x-1)-2x}{(x-1)^2}$

$= -\frac{2}{(x-1)^2} < 0$

critical number: $x = 1$

it decreases in

$(-\infty, 1)$ and $(1, \infty)$

(b) $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$

$f'(x) = x^2 - x - 2 = (x+1)(x-2) = 0$

critical numbers: $x = -1, 2$

f'	$+$	-1	$-$	2	$+$

increasing: $x < -1$, $x > 2$

decreasing: $-1 < x < 2$

4. (15 points) For the function $f(x) = 2x^3 - 6x^2 - 18x + 1$,

(1) Determine its critical numbers

$f'(x) = 6x^2 - 12x - 18 = 6(x^2 - 2x - 3) = 6(x+1)(x-3) = 0$

critical numbers: $x = -1, 3$.

(2) Determine its relative maximum and minimum points, if any.

$f''(x) = 12x - 12$.

$f''(-1) = -24 < 0$. relative max. $f(-1) = 11 \rightarrow$ max value

$f''(3) = 24 > 0$ relative min $f(3) = -53 \rightarrow$ min value

5. (15 points) For the function $f(x) = x^3 - 6x^2 + 9x + 10$, find its absolute maximum and minimum on the x-interval of [0 5].

$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3)$

critical numbers: $x = 1, 3$

$f(1) = 14$ $f(0) = 10$ $\Rightarrow$ absolute max $= 30$ at $x = 5$

$f(3) = 10$ $f(5) = 30$ absolute min $= 10$ at $x = 0, 3$

6. (15 points) For the function $f(x) = x^4 - 2x^3 + x + 1$, determine where its graph is concave up and where it is concave down. Also determine the inflection points.

$f'(x) = 4x^3 - 6x^2 + 1$

$f''(x) = 12x^2 - 12x = 12x(x-1) = 0$

$\Rightarrow$ inflection points: $x = 0, 1$

f'' + − +

0 1

$\Rightarrow$ concave up at $x < 0$ and $x > 1$

concave down at $0 < x < 1$