COMMITTEE

General Chairs:

Mark Ablowitz	University of Colorado, U.S.A.
Din-Yu Hsieh	Tsinghua Universtiy, China
Jianke Yang	University of Vermont, U.S.A.

Scientific Committee:

Mark Ablowitz	University of Colorado, Boulder, U.S.A
Triantaphyllos R.Akylas	Massachusetts Institute of Technology, U.S.A
David Benney	Massachusetts Institute of Technology, U.S.A
Demetrios Christodoulides	University of Central Florida, U.S.A
Roger Grimshaw	Loughborough University, U.K.
Chaohao Gu	Fudan Universtiy,China
Boling Guo	IAPCM,Beijing,China
Ryogo Hirota	Waseda University,Japan
Shi Jin	University of Wisconsin, U.S.A.
David J.Kaup	University of Central Florida, U.S.A.
Chiang C. Mei	Massachusetts Institute of Technology, U.S.A
Lev Pitaevskii	Universtiy of Trento,Italy
Orlando Ragnisco	Roma Tre University Italy
Walter Strauss	Brown University, U.S.A.
Michael Weinstein	Columbia University, U.S.A.
Ke Wu	Capital Normal Universtiy,China
Yunbo Zeng	Tsinghua Universtiy,China

Local Organizing Committee:

Xingbiao Hu	Chinese Academy of Sciences, China
Qingping Liu	China Universtiy of Mining and Technology, China
Senyue Lou	Shanghai JiaoTong University, Ningbo Universtiy, China
Jianke Yang	University of Vermont, U.S.A

Table of Contents

Conference Program

June 9

8:30～9:30	Opening Ceremony			West Auditorium
9:30～10:30	Fokas (Keynote)			West Auditorium
10:30～11:00	Coffee break			
11:00～12:00	Kivshar (Keynote)			West Auditorium
12:00～13:30	lunch (Jiasuo Restaurant)			
	S5(part 1 of 2)	S6	S10	S15
Organizer	Z.Chen	Z. Feng	Hirota; Takahashi	Tsynkov
Location	Room 104	Room 118	Room 322	Room 213
13:30～14:00	Stegeman	Q. Lu	Takahashi	Abarbanel
14:00～14:30	Firth	W. Yong	Saito	Baruch
14:30～15:00	Fan	S. Ma	Nimmo	Boyd
15:00～15:30	Kartashov	Q. Wang	Ohta	Ditkowski
15:30～16:00	Coffee break			
16:00～16:30	Kip	S.Zheng	Iwao	Z. Huang
16:30～17:00	Morandotti	X. Wang	Willox	J. Yang
17:00～17:30	D. Zhang	S. Fei	Nagai	Steinhoff
17:30～18:00	Skryabin	Z. Feng	Hirota	Lemesurier
18:00～18:30			Matsukidaira	
18:30～20:00	dinner (Jiasuo Restaurant)			

June 10

	S1	S2(part 1 of 2)	S7(part 1 of 2)	C1
Organizer	Biondini;Pelloni	Brand; Lee;Kivshar	Grimshaw; Mei	Mihalache
Location	Room 213	Room 104	Room 322	Room 118
8:30～9:00	Fokas	Drummond	S. Liao	Guyenne Z.Zhang
9:00～9:30	C.Zheng	Machida	Milewski	Mihalache
9:30～10:00	Coffee break			
10:00～10:30	Kaup	Lee	Akylas	W. Duan L. Zhang
10:30～11:00	Matsuno	J. Liu	Nachbin	T.S. Yang
11:00～11:30	Biondini	Ueda	Grimshaw	S. Liu Allen
11:30～12:00	B.Feng		Choi	Dmitriev
12:00～13:30	lunch (Jiasuo Restaurant)			
13:30～14:30	Zakharov (Keynote)			West Auditorium
14:30～15:30	Segev (Keynote)			West Auditorium
15:30～16:00	Coffee break			
	S5(part 2 of 2)	S12(part 1 of 2)	S7(part 2 of 2)	S14(part 1 of 3)
Organizer	Z.Chen	S. Lou; D.Luo	Grimshaw; Mei	Ragnisco;Y. Zeng
Location	Room 104	Room 118	Room 213	Room 322
16:00～16:30	Trillo	Duan	Vlasenko	Ballesteros
16:30～17:00	Vicencio	S. Gao	Stashchuk	Heredero
17:00～17:30	C. Yang	D. Luo	Chow	C. Cao
17:30～18:00	Del Re	H. Lu		Doliwa
18:00～18:40	Poster session			
19:00～20:30	Conference Reception (Quanjude Peking Duck House)			

June 11

8:00～9:00	Kodama(Keynote) Science Building 104			
9:00～9:30	Coffee break			
	S8	S9	S11(part 1 of 2)	S13(part 1 of 2)
Organizer	Haberman	Halburd	Ilan	Patton, M. Wu
Location	Room 213	Room 322	Room 104	Room 118
9:30～10:00	Dmitriev	Filipuk	Ablowitz	Hillebrands
10:00～10:30	B.Feng	Chiang	Ilday	Melkov
10:30～11:00	Marchant	Clarkson	Baruch	Slavin
11:00～11:30	Haberman	Lafortune	Gavish	Guslienko
11:30～12:00	Y. Zhu	Ghassani	Z. Chen	
12:00～13:00	lunch (Jiasuo Restaurant)			
13:00～19:00	tour to the great wall of China			
19:00～20:30	dinner (Jiasuo Restaurant)			

June 12

	C2	S3	S4	S11(part 2 of 2)
Organizer	X. Chen	Calini; Lafortune	M. Chen	Ilan
Location	Room 118	Room 213	Room 322	Room 104
8:30～9:00	X. Chen Leblond	Ivey	Bouard	Z.Shi
9:00～9:30	D.Zhang	Calini	Deconinck	Fibich
9:30～10:00	Coffee break			
10:00～10:30	Hoseini Toda	Deconinck	Carter	C. Sun
10:30～11:00	Mike Hay	Lafortune	M. Chen	Antar
11:00～11:30	C.Shi Phibanchon	Carter	Goubet	Ilan
11:30～12:00	Y. Shi		L. Zhang	
12:00～13:30	lunch (Jiasuo Restaurant)			
	S14(part 2 of 3)	S2(part 2 of 2)	C3	C4
Organizer	Ragnisco;Y. Zeng	Brand; Lee;Kivshar	Firth	Slavin
Location	Room 322	Room 104	Room 213	Room 118
13:30～14:00	J. He	Martin	G. Huang Biondini	M. Wu Kostylev
14:00～14:30	Bogoyavlenskij	W. Liu	J. Yu	Neumann
14:30～15:00	L. Tian	Carr	Stepic L. Dong	Serga Choi
15:00～15:30	Y. Zeng	Konotop	Dohnal	Ezersky
15:30～16:00	Coffee break			
16:00～16:30	Y.Zhang	Berloff	P. Zhang Khomeriki	
16:30～17:00	Conte	Brand	X. Gan	
	S14(part 3 of 3)		S12(part 2 of 2)	S13(part 2 of 2)
Organizer	Ragnisco;Y. Zeng		S. Lou; D.Luo	Patton, M. Wu
Location	Room 322		Room 213	Room 118
17:00～17:30	X. Tang		F. Huang	Khomeriki
17:30～18:00	Villaroel		L. Ran	Leblond
18:00～18:30	Z. Zhou		S. Lou	Y. Cheng
19:00～20:30	dinner (Jiasuo Restaurant)			

List of Talks

KEYNOTE TALKS

1. Thanasis Fokas （University of Cambridge）
"The D-Bar Method, Medical Imaging,Complexification, Harmonicity and Integrability in 3+1"

2. Yuri Kivshar（National University，Australia）
" Energy localization and solitons in nonlinear periodic systems"

3. Vladimir E. Zakharov (Institute of Physics RAN)
"Turbulence in Integrable Systems"

4. Mordechai Segev （Technion - Israel Institute of Technology）
"Nonlinear Waves in Photonic Lattices: from Lattice Solitons to Anderson Localization"

5. Yuji Kodama （Ohio State University，U.S.A.）
"Combinatorics and geometry in shallow water waves:Soliton solutions of the KPII equation "

MINISYMPOSIA

S1: Nonlinear waves and integrability

Organizers:

G.Biondini: The State University of New York at Buffalo, USA
B. Pelloni: University of Reading UK

In this minisymposium, the focus will be the analysis of nonlinear PDEs describing wave phenomena that can be analysed by the inverse scattering and related transforms, in particular the extension due to Fokas. Recent results in the solution of boundary value problems and the extension to multidimension will be presented and discussed alongside recent results for KdV type models obtained by standard PDE techniques.

Speakers List :

1) T. Fokas (University of Cambridge, UK)
"Integrable PDE in multidimensions"

2) Chunxiong Zheng (Tsinghua University, China)
"Numerical solution for some one-dimensional nonlinear wave equations in unbounded domains"

3) David Kaup (University of Central Florida, USA)
"Inverse Scattering on Finite Intervals"

4) Yoshimasa Matsuno (Yamaguchi University, Japan)
"Periodic solutions of the short pulse model equation"

5) G. Biondini (The State University of New York at Buffalo, USA)
"Line-soliton solutions of the Davey-Stewartson equation"

6) Baofeng Feng (University of Texas, Pan American, USA)
"An integrable difference scheme for the Camassa-Holm equation and numerical computation"

S2: Nonlinear matter waves of quantum gases

Organizers :

Joachim Brand (Massey University Auckland, New Zealand)
Chaohong Lee (Australian National University, Australia)
Yuri S. Kivshar (Australian National University, Australia)

Nonlinear wave phenomena in quantum degenerate atomic gases have become a field of increasing interest in last decade as atomic Bose-Einstein condensates are becoming more available. With the possibility to control interactions, impurities, and dissipation at ultra-cold temperatures, these inherently quantum mechanical many-body systems present both new opportunities and challenges for theoretical and experimental studies. At this minisymposium we will bring together researchers from different backgrounds to discuss the current state of the field as well as future directions.

Speakers List :

1) Peter Drummond (University of Queensland, Brisbane, Australia)
"Phase-space methods for quantum gases and interactions"

2) Kazushige Machida (Okayama University, Japan)
"Quantized vortices in various superfluids"

3)Chaohong Lee (The Australian National University, Canberra, Australia)
"Macroscopic and many-body nonlinear quantum behaviors of coupled condensates"

4) Jie Liu (Institute of Applied Physics and Computational Mathematics, Beijing, China)
"Quantum Entanglement Manifestation of Transition to the Self-Trapping of BEC"

5) Masahito Ueda (Tokyo Institute of Technology, Japan)
"Solitons and collapsing dynamics of Bose-Einstein condensates"

6) Andy Martin (University of Melbourne, Australia)
"Dipolar dilute gas Bose-Einstein condensates"

7) Wu-Ming Liu (Institute of Physics, Chinese Academy of Sciences, Beijing, China)
"Soliton and vortex in Bose-Einstein condensates"

8) Lincoln D. Carr (Colorado School of Mines, Golden CO, USA)
"Relativistic Nonlinear Phenomena in Bose-Einstein condensates"

9) Vladimir Konotop (University of Lisbon, Portugal)
"Intra-band tunneling of a Bose-Einstein condensates in 3D lattices"

10) Natalia Berloff (Cambridge University, UK)
"Spontaneous rotating vortex lattices in a pumped decaying condensate"

11) Joachim Brand (Massey University, Auckland, New Zealand)
"Topological solitons in double-ring Bose-Einstein condensates"

S3: Periodic Problems in Soliton Equations

Organizers:

Annalisa Calini and Stéphane Lafortune
Department of Mathematics, College of Charleston, USA

This minisymposium brings together researchers whose work addresses diverse aspects of soliton equations with periodic boundary conditions. The periodic problem of completely integrable nonlinear PDEs is a both rich and challenging area of study, which brings together techniques and results from algebraic geometry and functional analysis, spectral theory and perturbation methods. The proposed minisymposium will discuss both theoretical and applied aspects, including construction and analysis of special solutions, spectral stability analysis of periodic solutions, issues of algebraic complete integrability in the algebro-geometric context, universal aspects of small dispersion limits, and applications of periodic theory to vortex filament dynamics and to rogue waves generation in deep water.

Speakers List :

1) Thomas Ivey (College of Charleston Charleston, USA)
"Finite-Gap Solutions of the Vortex Filament Flow: Genus One Solutions and Symmetric Solutions"

2) Annalisa Calini (College of Charleston Charleston, USA)
"Cable Formation for Finite-Gap Solutions of the Vortex Filament Flow"
Co-author: Tom Ivey (College of Charleston)

3) Bernard Deconinck (University of Washington, USA)
"KdV cnoidal waves are linearly stable. "

4) Stéphane Lafortune (College of Charleston Charleston, USA)
"Stability Analysis of Persisting Periodic Solutions to a Complex Ginzburg-Landau Perturbation of NLS"
Co-author: Tom Ivey (College of Charleston)

5) John Carter **(**Seattle University, USA)
"Stability of plane-wave solutions to a dissipative generalization of the vector NLS equation

S4: Nonlinear Water Waves

Organizer:

Min. Chen (Purdue University, USA)

This session is centered on the propagation of water waves where nonlinearity, dispersion and sometimes dissipation, capillarity and random noises are all acting. Featured in the session will be theoretical work and numerical simulations on issues such as the stability and long-time behavior of solutions.

Speakers List :

1) Anne de Bouard (Centre National de la Recherche Scientifique, France)
"Random perturbations of the Korteweg-de Vries equation"

2) Bernard Deconinck (University of Washington, USA)
"Hill's method: computing spectra of linear operators"

3) John Carter (Seattle University,USA)
"Stability of nontrivial-phase solutions of higher-order generalizations of the cubic nonlinear Schrodinger equation"

4) M. Chen (Purdue Univiversit, USA)
"Numerical simulations of water waves"

5) Olivier Goubet (Université de Picardie, France) (tentative)
"Viscous asymptotical models for water wave propagation"

6) Linghai Zhang (Lehigh University, USA)
"Solutions to some open problems in n-dimensional fluid dynamics

S5: Nonlinear Waves in Optics & Photonics

Organizers:

Zhigang Chen (San Francisco State University, USA and Nankai University, China)

This mini-symposium intends to bring together people who are studying nonlinear waves in optics & photonics, in particular, in the discrete optical systems. The symposium will include topics of theoretical and experimental study on wave phenomena in nonlinear waveguide arrays, photonic microstructures and lattices, as well as photonic crystals.

Speakers List :

1) George Stegeman (University of Central Florida, Florida, USA)
"Discrete Solitons At the Interface Between Periodic Media"
2) William Firth (University of Strathclyde, Glasgow, UK)
"Homoclinic Snaking in Optics: Reconciling Theory and Experiment"
3) Shanhui Fan (Stanford University, CA, USA)
"Dynamic and nonlinear effects in photonic crystals"
4) Yaroslav Kartashov (Universitat Politecnica de Catalunya, Spain)
"Soliton diffusion in random optical lattices"
5) Detlef Kip (Clausthal University of Technology, Clausthal-Zellerfeld, Germany)
"Experimental observation of Rabi oscillations in one-dimensional photonic lattices"
6) Roberto Morandotti (Université du Québec, Montréal, Canada)
"All optical processing in integrated periodic structures"
7) Daozhong Zhang (Institute of Physics, Chinese Academy of Sciences)
"Tunable frequency conversion in a nonlinear photonic crystal"
8) Dmitry Skryabin (University of Bath, England)
"Puzzle of optical fiber supercontinuum or photons under the action of gravity-like force"
9) Stefano Trillo (University of Ferrara, Ferrara, Italy)
"Dynamics and thermodynamics of optical dispersive shocks and soliton gas"
10) Rodrigo Vicencio (Universidad de Chile, Chile)
"Localization and Delocalization phenomena in one-dimensional disordered nonlinear waveguide arrays"
11) Changxi Yang (Tsinghua University, China)
"Incoherent four-wave mixing in highly nonlinear optical fiber"
12) Eugenio Del Re (University of L'Aquila, L'Aquila, Italy)
"Engineering effective soliton-supporting nonlinearities in micro-structured potassium-lithium-tantalate-niobate"

S6: Traveling wave phenomena in biology

Organizer:

Zhaosheng Feng(University of Texas-Pan American, USA)

There is the widespread existence of traveling wave phenomena in biology and chem-biology. This clearly necessitates a study of traveling waves in depth and of the modeling and analysis involved. This Special Session is devoted to traveling wave phenomena of differential equations and dynamical systems arising from biology and ecology. The aim of this session is to get together experts and young researchers in this area to report their recent development and to exchange their ideas in research.

Speakers List (* indicates speaker):

1)Bao-hua Wang, Qi-shao Lu* and Shu-juan Lu (Beijing University of Aeronautics and Astronautics, China)
"Noise induced coherence resonance of Ca2+ in hepatocytes"

2) Wen-An Yong (Tsinghua University, China)
"Effective dynamics for axonal transport"

3) Suqi Ma (China Agricultural University, China)
"Dynamics of a delay hematological cell model"

4) Qingyun, Wang (Peking University, Inner Mongolia Finance and Economics College, China)
"Spatial coherence resonance in Hodgkin-Huxley neuronal networks with the coupling delay"

5) Shenzhou Zheng (Beijing Jiaotong University, China)
"Green function of nonlinear degenerate elliptic operators and its application"

6) Xiaohui Wang (University of Texas-Pan American, USA)
"Survival data analysis with functional predictors"

7) Shuming Fei* (Southeast University, China) and Zhaosheng Feng
"Traveling wave phenomena in Burgers-KdV type equations"

8)Zhaosheng Feng (University of Texas-Pan American, USA)
"Traveling waves in reaction-diffusion systems"

S7: Nonlinear waves in geophysical fluid flows

Organizers:

Roger.Grimshaw (Loughborough University,UK)
Chiang.C.Mei (Massachusetts Institute of Technology,USA)

The talks will describe some recent theoretical and modelling results for nonlinear water waves and internal waves. The topics covered will include freak waves, deterministic and random wave fields.

Speakers List :

1) Shijun Liao (Shanghai Jiaotong University, China):
"On the interaction of non-uniform current and nonlinear waves"

2) Paul Milewski (University of Wisconsin-Madison, USA)
"Dynamics of gravity-capillary waves in deep water"

3) Triantaphyllos Akylas (Massachusetts Institute of Technology, USA)
"Stability of lumps and wave collapse in water waves"

4) Andre Nachbin (IMPA, Brasil)
"An optimal Boussinesq model for solitary wave-microstructure interaction"

5) Roger Grimshaw (Loughborough University,UK)
"Transcritical flow over a step"

6) Wooyoung Choi (New Jersey Institute of Technology, USA)
"Short-wave instability of internal solitary waves and a regularized long wave model"

7) Vasily Vlasenko (University of Plymouth, UK)
"Three-dimensional evolution of large-amplitude internal waves in the Strait of Gibraltar"

8) Natalia Stashchuk (University of Plymouth, UK)
"Nonlinear internal waves forced by supercritical tidal plume"

9) KW Chow (The University of Hong Kong, China)
"Exact solutions for a class of variable coefficient nonlinear Schroedinger equations"

S8: Solitary wave interactions and collisions in non-integrable systems

Organizer:

Richard Haberman (Southern Methodist University, USA)

Weak and strong interactions and collisions of solitary waves are well known to be complicated for non-integrable equations. They have been studied by perturbation methods based on inverse scattering, multiple scale perturbation methods, and collective coordinate variational approximations, as well as direct numerical simulations. Issues addressed by some of our participants include accurately determining excitation of solitons internal modes and radiation. In addition, recent advances will be presented in using the separatrix map of dynamical systems to understand the fractal structure of chaotic scattering associated with collisions and interactions of solitary waves.

Speakers List :

1) Sergey V. Dmitriev (Technical University, Russia)
"Strongly inelastic soliton collisions in weakly perturbed integrable systems"

2) Baofeng Feng (University of Texas –Pan, USA)
"Reduced models for the interactions of discrete breathers in Fermi-Pasta-Ulam lattices"

3) Tim R. Marchant (The University of Wollongong, Australia)
"Solitary wave interaction for a higher-order nonlinear Schrodinger equation"

4) Richard Haberman (Southern Methodist University, USA)
"Chaotic scattering via the separatrix map in solitary wave interactions"

5) Yi Zhu (Tsinghua University, China)
"A universal map for fractal structures in weak solitary wave interactions"

S9: Minisymposium on Painlevé Analysis

Organizer:

Rod Halburd (University College London)

Painleve analysis involves searching for singularity structures of solutions of differential equations that are compatible with integrability. In particular, any ordinary differential possessing the Painleve property (that all movable sinularities of all solutions are poles) is believed to be integrable. Painleve analysis often provides a simple ``test" for integrability. It is especially effective in determining values of parameters for which an equation is integrable. This kind of analysis was first used by Kowalevskaya to find her famous integrable case of the equations of motion of a spinning top.

Speakers List :

1) Galina Filipuk (Loughborough University, UK)
"Movable branch points in solutions of ODEs"

2) Yik Man Chiang (Hong Kong University of Science and Technology, Hong Kong, China)
"Malmquist theorems for second-order ODEs"

3) Peter Clarkson (University of Kent at Canterbury, UK)
"Vortices and polynomials"

4) Stéphane Lafortune (College of Charleston, South Carolina, USA)
"Painlevé analysis for ultra-discrete equations"

5) Asma Al-Ghassani (Loughborough University, UK)
"Analogues of Painlevé analysis"

S10: Discrete and ultradiscrete integrable systems

Organizers:

Ryogo Hirota (Prof. Emeritus, Waseda Univ. Japan)
Daisuke Takahashi (Waseda Univ. Japan)

Main topics:
* Exact solutions to discrete and ultradiscrete systems
* Applications to the traffic flow utilizing ultradiscretization
* Discrete mappings showing recurrence phenomena (cycles)
* Transformations among integrable systems

Speakers List :

1) Daisuke Takahashi (Waseda Univ, Japan)
"Recent progress in discrete and ultradiscrete integrable systems "

2) Satoru Saito (Tokyo Metropolitan Univ, Japan)
"Recurrence equations on invariant varieties of periodic points "

3) Jonathan Nimmo (University of Glasgow, UK)
"Quasideterminant solutions of noncommutative discrete systems " Junta

4) Yasuhiro Ohta (Kobe Univ, Japan)
"Discretization of coupled nonlinear Schrodinger equations "

5) Masataka Iwao (Tokyo Univ, Japan)
"Spectrum decomposition of periodic ultradiscrete maps "

6) Ralph Willox (Tokyo Univ, Japan)
"On the relation between the discrete KP hierarchy, Yang-Baxter maps and Box-and Ball systems "

7) Hidetomo Nagai (Waseda Univ,Japan)
"New expression of ultradiscrete soliton solutions "

8) Ryogo Hirota (Prof. Emeritus, Waseda Univ, Japan)
"New solutions to ultradiscrete soliton equations "

9) Matsukidaira (Ryukoku Univ,Japan)
"Euler and Lagrange equations of traffic flow "

S11: Self-Focusing and Ultrafast Lasers

Organizer:

Boaz Ilan (School of Natural Sciences, University of California at Merced, USA)

This mini-symposium is being organized in order to bring together researchers who are studying self-focusing, ingularity formation, soliton propgation in ultrafast lasers andstability in Nonlinear Schrödinger equations, with emphasis on application to Nonlinear Optics and Bose-Einstein Condensates. These areas of research are seeing intense multidisciplinary research in fundamental science and with a wide scope of applications,e.g., ultra-precise clocks and optical signal processing. Typical mathematical models involve nonlinear Schrodinger-type equations. Because of the novelty and complexity of these physical systems, it is expected that a synthesis of modeling, rigorous and asymptotic analyses, and computational methods will be subjects of significant discussion. The general purpose of this minisymposium is to bring together theoretical and experimental researchers from different disciplines working in these areas, and to provide a forum for the multidisciplinary discussion of current and future research directions.

Speakers List :

1) Mark J. Ablowitz (University of Colorado at Boulder, Department of Applied Mathematics, USA)
"Soltons and Dynamics in Mode Locked Lasers "

2) F. Ömer İlday (Bilkent University, TURKEY)
"Spectrally breathing femtosecond pulses from an Er-doped fiber laser "

3) Guy Baruch (Tel Aviv University, ISRAEL)
"Solution of the nonlinear-Helmholtz equation for self-focusing beams "

4) Nir Gavish (Tel Aviv University, ISRAEL)
"Singular vortex solutions of the nonlinear Schroedinger equation "

5) Zhigang Chen (San Francisco State University, USA)
"Discrete solitons and singularity formation in reconfigurable photonic lattices "

6) Zuoqiang Shi (Tsinghua University, China)
"Instability of Two-dimensional Lattice Solitons Near Edges of Bloch Bands in Periodic Media "

7) Gadi Fibich (Tel Aviv University, ISRAEL)
"Stabilities of lattice solitons "

8) Can Sun (Princeton University, USA)
"Observation of Peakon Profile in Nonlocal Nonlinear Collapse "

9) Nalan Antar (Istanbul Technical University, TURKEY)
" Band Gap Formation, Fundamental Solitons and Vortices in Two Dimensional Lattices "

10) Boaz Ilan (School of Natural Sciences, University of California at Merced, USA)
"NLS stability theory for solitons in inhomogenous media "

S12: Theory and Applications on the Nonlinear Wave Phenomena in the Atmosphere and the Ocean

Organizers:

Sen-yue Lou (Ningbo University, Shanghai Jiao Tong University, China)
De-hai Luo (Ocean University of China)

The goal of this minisymposia is to survey recent advances both in theory and applications on the nonlinear wave phenomena in the atmosphere and the ocean including: Atmosphere and ocean ciuculations, solitary waves, solutions of the Navier-Stokes equations and Euler equations, oceanic oscilations, gravity waves, hurricanes and tsunamis, effects of the atmospheric and oceanic waves on the other natural scientifical fields.

Speakers List :

1) Jinqiao Duan (Illinois Institute of Technology, USA)
"Stochastic Modeling, Analysis and Simulation for Geophysical Flows"

2) Shouting Gao (Institute of Atmospheric Physics, CAS,China)
"A three-dimensional wave-activity relation for pseudomomentum"

3) De-hai Luo, (Ocean Univ Qingdao, China)
"Dynamics of eddy-driven low-frequency dipole modes for North Atlantic Oscillation"

4) Han-Cheng Lu,(PLA University of Science and Technology, Nanjing University of Information Science and Technology, China, University of Maryland,USA)
"The Characteristics Analyses of Nonlinear Mixed Wave in Tropical Cyclone"

5) Fei Huang (Ocean University of China, Qingdao,China)
"Applications of Nonlinear Soliton Theory on Atmospheric and Oceanic Circulation Dynamics"

6) Lingkun Ran, (Institute of Atmospheric Physics, CAS,China)
"Polarization and wave-activity relation of inertial-gravity waves"

7) Sen-yue Lou, (Ningbo University, Shanghai Jiao Tong University,China)
"Hurricanes and Tsunamis"

S13: Nonlinear Waves in Magnetic Systems

Organizers:

Carl E. Patton (Colorado State University, USA)
Mingzhong Wu (Colorado State University, USA)

This symposium will bring together leading experimental and theoretical scientists in the field of nonlinear waves in magnetic systems. The symposium will consist of eight invited talks and three contributed talks. These talks will cover the most recent and important developments in nonlinear waves in magnetic systems. Emphasis will be placed on spin wave bullets, coupled spin wave solitons, spin wave bi-stabilities, spin wave parametric pumping, and chaotic spin waves in ferrite thin films; nonlinear interactions of spin waves in magnetic metallic thin films; topological solitons in magnetic nanometer- sized strips; line solitons in magnetic slabs; parametric instability in ferromagnetic strip elements; and spin-polarized current driven-spin wave bullets in magnetic nanostructures.

Speakers List :

1)Burkard Hillebrands (Kaiserslautern University of Technology, Germany)
"Nonlinear Optics with New Light: Optics with Spin Waves"

2)Gennadiy A. Melkov (Kiev National Taras Shevchenko University, Ukraine)
"Nonlinear Interactions of Spin Waves in Metallic Permalloy Films"

3) Andrei N. Slavin (Oakland University, USA)
"Excitation of a Subcritically Unstable Nonlinear Spin-Wave Mode by Spin-Polarized Current in a Magnetic Nano-Structure"

4) Konstantin Yu. Guslienko (Seoul National University, South Korea)
"Dynamics of Magnetic Topological Solitons in Restricted Geometry"

5) Ramaz Khomeriki (Tbilisi State University, Georgia)
"Bistable Magnetization Profiles in Magnetic Thin Films"

6) Herve Leblond (UniversitdAngers, France)
"Bi-dimensional Soliton Propagation in a Ferromagnetic Slab in the Short Wave Approximation"

7) Yongshan Cheng (Hubei Normal University, China)
"Dynamics of Two Coupled Short Spin Wave Envelope Solitons"

S14: Integrable Systems

Organizers:

Orlando Ragnisco(Università di Roma Tre, Italy)
Yun-Bo Zeng (Tsinghua University, China)

Speakers List :

1) Angel Ballesteros (Universidad de Burgos Plaza Misael Banuelos,Burgos)
"Integrable models from coalgebras"

2) Rafael Hernandez Heredero (Escuela Universitaria de Ingeniería Técnica de Telecomunicación, Madrid)
"Discrete multiscale method and asymptotic symmetries"

3) Cewen Cao (Zhengzhou University, China)
"Finite genus solution to the lattice sine-Gordon equation"

4) Adam Doliwa (University of Warmia and Mazury, Poland)
"Geometry of the noncommutative Darboux equations"

5) J. S. He (University of Science and Technology of China)
"The one-parameter transformation group of soliton equations: gauge transformation"

6) Oleg Bogoyavlenskij (Queen's University, Canada)
"Integrable generalizations of Volterra system"

7) L. X. Tian (Jiangsu University, China)
"A hierarchy of new multi-component evolution equations and its bi-Hamiltonian structures"

8) Y. B. Zeng (Tsinghua University, China)
"Two new multi-component BKP hierarchies"

9) Y.J.Zhang (Tsinghua University, China)
"On quasi-triviality and integrability of a class of evolutionary PDEs"

10) Robert Conte (Service de physique de letat condense, France)
"Doubly periodic, simply periodic and rational solitary waves of partially integrable nonlinear PDEs"

11) X.Y. Tang (Shanghai Jiao Tong University, China)
"Reflection and reconnection interactions of resonant dromions"

12) Javier Villaroel (Universidad de Salamanca Plaza de la Merced, España)
"IST solutions to a 2+1 extension of NLS"

13) Z. X. Zhou (Fudan University, China)
"Finite dimensional Hamiltonian systems related with two dimensional C_n^{(1)} Toda equation"

S15: Numerical Computation of Nonlinear Waves

Organizer:

Semyon V. Tsynkov (North Carolina State University, USA)

The speakers will present recent advances in the development of numerical methods for the simulation of nonlinear wave phenomena. Both time domain and frequency domain methods will be discussed as they apply to a variety of problems in optics, continuum mechanics, and other areas.

Speakers List :

1) Saul Abarbanel (Tel Aviv University, Israel)
"Nonlinear PMLs"

2) Guy Baruch (Tel Aviv University, Israel)
"Numerical Simulations of the nonlinear Helmholtz equation"

3) John Boyd (University of Michigan, USA)
"Radial basis functions methods for large-scale geophysical waves"

4) Adi Ditkowski (Tel Aviv University, Israel)
"Grid generation for singular solutions of PDEs"

5) Zhongyi Huang (Tsinghua University,China)
"An efficient numerical method for the wave equation in an inhomogeneous medium"

6) Jianke Yang (University of Vermont, USA)
"Efficient Iteration Methods for Solitary Waves and Their Stability Spectra"

7) John Steinhoff (University of Tennessee Space Institute, USA)
"Computing sharp wavefronts using nonlinear solitary waves"

8) Brenton Lemesurier (College of Charleston Charleston, USA)
"Discrete Time Hamiltonian Equations and the Periodic Nonlinear Schrodinger Equation"

Contributed Talks

C1: General aspects of nonlinear waves

1. Philippe Guyenne (University of Delaware, USA)
"Numerical simulations of 3D overturning waves "

2. Zhiwen Zhang (Tsinghua University, China)
"Numerical method for one-dimensional linear and nonlinear Klein-Gordon equations on unbounded domain "

3. Dumitru Mihalache (Horia Hulubei National Institute for Physics and Nuclear Engineering, Romania)
"Collisions between co-axial three-dimensional Ginzburg-Landau vortex solitons "

4. Wen-shan Duan (Northwest Normal University, China)
"Interactions between two solitons with arbitrary propagation directions in two-dimensional plasma "

5. Linghai Zhang (Lehigh University, USA)
"Dynamics of Nonlinear Traveling Waves in Synaptically Coupled Neuronal Networks "

6. Tian-Shiang Yang (National Cheng Kung University, Taiwan)
"Asymptotic analysis of an input-shaping scheme for suppressing motion-induced residual vibration of nonlinear mechanical systems "

7. Sheng Liu (Northwestern Polytechnical University, China)
"Anomalous interaction of spatial gap solitons in optically induced photonic lattices "

8. Michael A. Allen (Mahidol University,Thailand)
"Determination of the growth rate curve for transverse instabilities of plane soliton solutions of the extended Zakharov-Kuznetsov equation "

9. S.V.Dmitriev (Institute for Problems of Superplasticity of Metals, RAS, Russia)
"Discrete systems free of the Peierls-Nabarro potential "

C2: Integrable systems

1.Xiang-Jun Chen (Jinan University, China)
"Solitons of the derivative nonlinear Schrodinger equation with nonvanishing boundary conditions "

2.Hervé Leblond (Université d'Angers, France)
"Integrable models for few-cycle optical pulses "

3. Da-jun Zhang (Shanghai University, China)
"Limit Solutions for Soliton Equations "

4. S.M.~Hoseini (Vali-e-Asr University, Iran)
"Evolution of higher-order gray Hirota solitary waves "

5. Kouichi Toda (Toyama Prefectural University, Japan)
"Higher-dimensional integrable hierarchies with non-isospectral parameters "

6.Mike Hay (University of Sydney, Australia)
"Exploring the possible 2×2 partial difference equation Lax pairs "

7. Chang-Guang Shi (Shanghai University of Electric Power, China)
"Exploring vortex solutions of Faddeev model "

8. Sarun Phibanchon (Faculty of Science and Art, Burapha University, Thailand)
"Transverse instability of the modified nonlinear Schrodinger equation "

9. Nalini Joshi and Yang Shi (School of Mathematics and Statistics, University of Sydney, Sydney, NSW 2006, Australia)
"Exact solutions to the 2 by 2 associated linear problem of the second and third q-discrete Painleve equations "

C3: Nonlinear Optics

1.Guoxiang Huang (East China Normal University, China)
"Slow Light and Fast Light Solitons in Resonant Multi-Level Atomic Systems "

2. Gino Biondini (State University of New York at Buffalo, USA)
"The dispersion-managed Ginzburg-Landau equation and its application to femtosecond lasers "

3. Jun Yu (University of Vermont, USA)
"Nonlinear analysis of modulational instability in a layered optical medium "

4. Milutin Stepic (Vinča Institute of Nuclear Sciences, Serbia)
"Modulational instability and spatial solitons in one-dimensional lattices with resonant nonlinearity"

5. Liangwei Dong (Zhejiang Normal University, China)
"Necklace Solitons and Ring Solitons in Bessel Optical Lattices"

6. Tomas Dohnal (University of Karlsruhe, Germany)
"2D Surface Gap Solitons at a Nonlinearity Interface"

7. Peng Zhang (Northwestern Polytechnical University; Nankai University, China)
"Band-gap engineering of two-dimenional photonic lattices with reconfigurable refractive index potential"

8. Ramaz Khomeriki (Tbilisi State University, GEORGIA)
"Linear and Nonlinear Landau-Zener Tunneling in Optical Directional Coupler "

9. Xuetao Gan (Northwestern Polytechnical University, China)
"Evolution of optical vortices in light-induced photonic lattices under nonconventional bias conditions"

C4: Spin waves and water waves

1.Mingzhong Wu (Colorado State University, USA)
"Excitation of chaotic spin waves through four-wave parametric processes"

2. Mikhail Kostylev (University of Western Australia, Australia)
"Nonlinear spin waves in in-plane confined geometries "

3. Timo Neumann (Fachbereich Physik, TU Kaiserslautern, Germany)
"Brillouin light scattering observations of thermal modes in yttrium- iron garnet films"

4. Alexander A. Serga (Fachbereich Physik, Kaiserslautern University of Technology, Germany)
"Negative transitional damping of spin waves at the bottom of magnon spectrum"

5. Wooyoung Choi (New Jersey Institute of Technology, USA)
"Theoretical and experimental studies on the evolution of nonlinear water waves"

6. Alexander Ezersky (Morphodynamique Continentale et hatotigraveere (M2C) Universitacuteede Caen-Basse Normandie 2-4 rue des Tilleuls, France)
"Modulated surface gravity waves in a shallow-water resonator"

POSTER

1."Modulational instability in two-component discrete media with cubic-quintic nonlinearity"
B. A. Umarov1;2 B. B. Baizakov 2, A. Bouketir 1, and A. Messikh 1
1 Department of Computational and Theoretical Sciences, Faculty of Science, International Islamic University Malaysia, Jalan Istana, Bandar Indera Mahkota, 25200 Kuantan, Pahang, Malaysia
2 Physical-Technical Institute of the UzbekAcademy of Sciences, G. Mavlyanov str. 2b, 100084,Tashkent, Uzbekistan

2. "An analytic approach to Faddeev model"
Minoru Hirayama1 and Chang-Guang Shi 2
Department of Mathematics and Physics, Shanghai University of Electric Power, Pinglian Road 2103, Shanghai 200090, China

3. "Nonclassical solitons in DNA dynamics"
Maximo Aguero (a), Ma. de Lourdes Najera(b)
(a) Facultad de Ciencias, (b) Plantel Nezahuaycoyolt, Universidad Autonoma del Estado de Mexico, Instituto Literario 100, Toluca, CP50000 , Mexico

4. "Soliton solutions for two nonlinear partial differential equations using a Darboux transformation of the Lax pairs"
Ji Lin 1, Bo Ren 1, Hua-mei Li 1, Yi-shen Li 2
1Department of Physics, Zhejiang Normal University, Jinhua 321004, China
2Department of Mathematics, University of Science and Technology of China, Hefei, 230026, China

5. "The higher genus generalisation of the Weierstrass theory for elliptic functions"
Matthew England
Mathematics, School of Mathematical and Computer Sciences (MACS),Colin Maclaurin Building, Heriot-Watt University, Riccarton, Edinburgh, EH14 4AS, UK

6. "Tri-stability in a Pendula Chain"
Ramaz Khomeriki,
Department of Physics, Tbilisi State University, Tbilisi 0128, Republic of Georgia.
Jerome Leon, Dominique Chevriaux Laboratoire de Physique Theorique et Astroparticules CNRS-IN2P3-UMR5207, Universite Montpellier 2, 34095 Montpellier, France

7. "Initial-boundary-value problems for linear differential-difference

equations"
Danhua Wang and Gino Biondini
Department of Mathematics, State University of New York at Buffalo, NY 14260,U.S.A

8. "Optical Bloch oscillation stimulated by interaction between discrete solitons"
Fajun Xiao, Peng Zhang, Sheng Liu, Xuetao Gan and Jianlin Zhao
Institute of Optical Information Science and Technology and Shaanxi Key Laboratory of Optical Information Technology, School of Science, Northwestern Polytechnical University,Xi'an 710072, China

9. "A perturbative change of the nature of invariant varities of periodic points"
Noriko Saitoh Department of Applied Mathematics,YokohamaNationalUniversity,Hodogaya-ku Yokohama 240-8501, Japan

10. "Ishimori-I Equation with Self-Consistent Sources"
Juan Hu1; Xing-Biao Hu 2and Hon-Wah Tam3
1 Institute of Computational Mathematics and Scientific Engineering Computing,Academy of Mathematics and System Sciences,ChineseAcademy of Sciences, P.O. Box 2719, Beijing 100080, P.R. CHINA
2 Graduate School of the ChineseAcademy of Sciences, Beijing, P.R. CHINA
3 Department of Computer Science, Hong Kong Baptist university,Kowloon Tong, Hong Kong, P.R. CHINA

11. "A Bilinear Backlund Transformation and Lax Pair for a (1+1) Dimensional Differential-Difference sine-Gordon Equation "
Xian-Min Qian
Department of Physics, Shaoxing University

12. "A connection between HH3 and KdV with one source "
Jun-xiao Zhao 1,2, Robert Conte 1,3
1 Service de physique de létat condensé (URA 2464) CEA--Saclay, F--91191 Gif-sur-Yvette Cedex, France
2 School of Mathematical Sciences, Graduate University of Chinese Academy of Sciences, Bejing, P.R.China
3 Centre de mathématiques et de leurs applications (UMR 8536) École normale supérieure de Cachan, 61, avenue du Président Wilson,F--94235 Cachan Cedex, France.

13. "On a new coupled KP equation and its soliton solution "
Hong-Yan Wang
School of Information, Renmin University of China, Beijing, China

Abstracts

The D-Bar Method, Medical Imaging, Complexification, Harmonicity and Integrability in 3+1

Thanasis Fokas
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, CB3 OWA, UK
email: T.Fokas@damtp.cam.ac.uk

We will first review a method for deriving linear and nonlinear transform pairs. We will then present two applications of this method: (a) The inversion of the so-called attenuated Radon transform, which plays a significant role in medical imaging. (b) The derivation of a nonlinear Fourier transform pair in four dimensions, which can be used for the solution of the Cauchy problem of an integrable generalization of the Kadomtsev-Petviashvilli equation in 4+2, i.e. in four spatial and two temporal dimensions. The question of reducing this equation from 4+2 to 3+1 dimensions and the occurence of a harmonic constraint will also be discussed.

Energy localization and solitons in nonlinear periodic systems

Yuri Kivshar
Nonlinear Physics Center, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia
Tel: +61-26-125-3081, email: ysk124@rsphysse.anu.edu.au

I will overview the recent progress in the study of the energy localization and solitons in a variety of nonlinear systems where the effects of discreteness and periodicity become important. This panoramic presentation will cover (i) generation and control of optical gap solitons in waveguide arrays and photonic lattices, including the most recent observation of polychromatic gap solitons generated by a supercontinuum source, (ii) localized matter waves of Bose-Einstein condensates in two- and three-dimensional optical lattices, (iii) discrete localized magnetic states in composite metamaterials, and (iv) energy localization in carbon nanotubes.

First of all, I will emphasize the most important recent advances in nonlinear optics where many of novel theoretical findings have been verified in experiment. This includes the observation of surface solitons in one- and two-dimensional photonic lattices, the observation of polychromatic "rainbow" gap solitons in photonic lattices generated by a supercontinuum source [1], the generation of topologically stable spatially localized multivortex solitons, etc.

One of the recent concepts in the theory of nonlinear waves is associated with a novel type of broad nonlinear states which appear in the gaps of the bandgap spectra of periodic systems such as light waves in periodic photonic lattices and Bose-Einstein condensates in optical lattices. These localized states cannot be treated by familiar multi-scale asymptotic expansion techniques, and they can be better understood as truncated nonlinear Bloch waves [2]. I will demonstrate that these self-trapped localized nonlinear modes can be found in one-, two-, and three-dimensional periodic potentials, and they have been readily observed in experiments on nonlinear self-trapping of matter waves in one-dimensional optical lattices.

Finally, I will discuss the energy localization in carbon nanotubes and demonstrate the existence of spatially localized nonlinear modes in the form of discrete breathers [3]. In nanotubes with the chirality index $(m, 0)$ there exist three types of discrete breathers associated with longitudinal, radial, and torsion anharmonic vibrations, however only *twisting breathers* survive in a curved geometry remaining long-lived modes even in the presence of thermal fluctuations.

References:

1. D.N. Neshev, A.A. Sukhorukov, A. Dreischuh, R. Fischer, Sangwoo Ha, J. Bolger, Lam Bui, W. Krolikowski, B.J. Eggleton, A. Mitchell, M.W. Austin, and Yu.S. Kivshar, Phys. Rev. Lett. **99**, 123901 (2007).
2. T.J. Alexander, E.A. Ostrovskaya, and Yu.S. Kivshar, Phys. Rev. Lett. **96**, 040401 (2006).
2. A.V. Savin and Yu.S. Kivshar, arXiv:0705.4482 (2007).

Turbulence in Integrable Systems

Vladimir E. Zakharov
Department of Mathematical Physics, Institute of Physics RAN, Leninskii Pr. 53, Moscow 119951, Russia
Tel: 499-135-4264, email: zakharov@math.arizona.edu

Dynamical systems integrable by Inverse Scattering method could demonstrate such complex behavior that require statistical description. Development of statistical description leads to creation of rich and complicated theory of Integrable Turbulence. There are two approaches for this development.

The first approach is kinetic description of rarified ensemble of solitons. First kinetic equation for solitons in KdV equation was found by author in 1971 [1]; now this equation is essentially modernized [2]. Much more complicated kinetic equations can be written for bright and dark solitons in the focusing and defocusing Nonlinear Schrodinger equations. These kinetic equations describe "second sound" on background of spatially Kolmogorov solitonic spectra as well as instability of such spectra. This instability leads to formation of "secondary turbulence", formation and interaction of solitonic clusters. Statistical description of solitons in 3-wave interacting system is fascinatingly interesting problem. In this case solitons interact nonelastically, they can glue and decay, and as a result, behavior of soliton ensemble can be very complicated. Another fundamental problem is statistical description of 2-D system of solitons in framework of KP-2 equation. In this case, solitons can glue and form complicated clusters. The theory should be developed as expansion in powers of density of solitons.

Second approach is traditional to nonlinear wave dynamics study of kinetic equations for waves. For the system integrable by Inverse Scattering method these equations are very peculiar. For instance, in KP-2 case all collision terms of kinetic equation are cancelled [3,4,5] and only nontrivial theory appears due to dependence of wave frequency of wave spectra. This theory can be compared with collisionless kinetic equation of plasma in framework of Vlasov-Maxwell equation. In the case of KP-1 even the first three-wave collision term is non-zero. However, in this case the kinetic equation admits infinite amount of motion constants, and the kinetics of wave spectra is pretty complicated. More sophisticated problem of kinetic behavior of turbulent spectra in integrable system, such as structure of correlation functions, is not explored yet. This problem even is not formulated in a proper way so far, and in my talk I will try to do this.

References:

1. V.E. Zakharov, Sov. Phys. JETP, 33, 538–541(1971).
2. G.A. El and A.M. Kamchatnov, Phys. Rev. Lett. 95, 204101 (2005).
3. V.E. Zakharov and E.I Shulman, Physica D, 29, 283-320, (1988).
4. V.E. Zakharov, Amer. Math. Soc. Transl. 182 (2), 167-197, (1998).

Nonlinear Waves in Photonic Lattices: from Lattice Solitons to Anderson Localization

Mordechai Segev
Physics Department, Technion - Israel Institute of Technology
Email: msegev@techunix.technion.ac.il

Waves in nonlinear periodic potentials are ubiquitous in nature, manifesting themselves in a variety of systems: from optical and matter-waves to sound waves, spin waves, and charge-density waves (see [1] and references therein). The past five years have witnessed major progress in photonic lattices. The key concepts in this area will be reviewed, with an emphasis on the universal ideas that apply to all periodic physical systems in which nonlinear waves propagate. Topics covered will be discrete / lattice solitons [2,3], two-dimensional lattice solitons [4] and vortex lattice solitons [5], random-phase (incoherent) lattice solitons [6], nonlinear waves in quasi-crystals [7], and Anderson localization of light [8].

References:

1 D.N. Christodoulides et al., Nature 424, 817 (2003).
2 D. N. Christodoulides and R. I. Joseph, Opt. Lett. 13, 794 (1988)
3 H.S. Eisenberg et al., Phys. Rev. Lett. 81, 3383 (1998).
4 J.W. Fleischer et al., Nature 422, 147 (2003).
5 J.W. Fleischer et al., Phys. Rev. Lett. 92, 123904 (2004); Neshev et al., ibid 92, 123903 (2004).
6 O. Cohen et al., Nature 433, 500 (2005).
7 B. Freedman et al., Nature 440, 1166 (2006)
8 T. Schwartz et al., Nature 446, 52 (2007).

Combinatorics and geometry in shallow water waves: Soliton solutions of the KPII equation

Yuji Kodama
Department of Mathematics, Ohio State University, Columbus OH, 43210 U.S.A.
Tel: 614-292-0692, email: kodama@math.ohio-state.edu

The KPII equation describes two-dimensional shallow water waves, which is an two-dimenisonal extension of the KdV equation. In the previous papers, we found a large variety of line-soliton solutions of the KPII equation. A line-soliton solution consists of several infinite, semi-infinite and finite lines connected with the nonlinear interactions, and it forms a web-like pattern in the x-y plane (the surface of the water). In this talk, I will explain a classification of the line-soliton solutions of the KP equation based on the Schubert decomposition of the Grassmannian $\mathrm{Gr}(N, M)$, the set of N-dimensional subspaces in $\mathbf{R}^M$. Each soliton solution can be identified as a point on a positive Grassmann cell which is obtained by a further decomposition of the Schubert cells. In particular, I will show that the classification of N-soliton solutions can be obtained by the chord diagrams with $2N$ vertices and N chords having overlapping and crossings, where each chord diagram corresponds to a permutation of the symmetric group S_{2N}. Then a combinatorial problem is to count the number of those chords and to find a generating function for the numbers. The set of those chord diagrams in each Schubert cell forms a partial polytope given by the symmetric group S_N. I will also present a real shallow water experiment for those soliton solutions. This work is under a collaboration with S. Chakravarty (UCCS).

References:

1. G. Biondini and Y. Kodama, On a family of solutions of the Kadomtsev-Peviashvili equation which also satisfy the Toda lattice hierarchy, *J. Phys. A: Math. Gen.* **36** 10519-10536 (2003).
2. Y. Kodama, Young diagrams and N-soliton solutions of the KP equation, *J. Phys. A: Math. Gen.* **37**, 11169-11190 (2004).
3. G. Biondini and S. Chakravarty, Elastic and inelastic line-solitons solutions of the Kadomtsev-Petviashvili II equation, *J. Math. Phys.* **47**, 033514 (2007).
4. S. Chakravarty and Y. Kodama, Classification of the line-soliton solutions of KPII, (arXiv:0710.1456 [nlin.SI]).
5. S. Chakravarty and Y. Kodama, A generating function for the N-soliton solutions of the Kadomtsev-Petviashvili II equation, *submitted to Comtem. Math.* (2007).
6. S. Chakravarty and Y. Kodama, Combinatorics and geometry of the N-soliton solutions of the KP equation, *in preparation.*

Initial-boundary value problems in three dimensions

Athanassios S. Fokas

Department of Applied Mathematics and Theoretical Physics, Cambridge University Cambridge, CB3 0WA U.K.; t.fokas@damtp.cam.ac.uk

Abstract:

A method for analysing initial-boundary value problems for linear and integrable nonlinear PDEs in two dimensions was introduced in 1997 and further developed in the works of about fifty researchers. The extension of this method to three dimensions will be discussed.

Numerical solution for some one-dimensional nonlinear wave equations in unbounded domains

Chunxiong Zheng

Department of Mathematical Sciences, Tsinghua University Beijing 100084 - People's Republic of China; czheng@math.tsinghua.edu.cn

Abstract:

We report the numerical technique for some one-dimensional nonlinear wave equations in unbounded domains. Exact absorbing boundary conditions will be presented based on the work of A. Boutet de Monvel, A.S. Fokas and D. Shepelsky [Lett. Math. Phys. 65 (3) 199-212, 2003], which considered the generalized Dirichlet-to-Neumann mapping for some integrable systems by the inverse scattering theory. A discretization technique will be proposed for the boundary conditions, and some numerical tests will be given to demonstrate its performance.

Inverse Scattering on Finite Intervals

D.J. Kaup & Heinz Steudel

Kaup: Dept. of Mathematics, PO Box 161364, University of Central Florida
Orlando, FL, 32816-1364 Tel: 407-423-2795, email: kaup@mail.ucf.edu
Steudel: Institut für Physik der Humboldt–Universität, Newtonstraße 15,
12489 Berlin, Germany email: steudel@physik.hu-berlin.de

Abstract:

The initial value problem for hyperbolic integrable systems are frequently given on a finite or semi-infinite interval. In these cases, simplifications to the solution of the Marchenko equations can be given [1]. The common features of all these problems is that the initial value problem is a Goursat problem and the asymptotic form of the effective reflection coefficients is stationary[2]. One feature of the latter is that all contributions of the continuous spectra (radiation) becomes equivalent to countable set of pure solitons and/or virtual solitons [3]. This procedure is of interest for solving initial–boundary value problems by the inverse scattering transform applied to a semi-infinite or finite interval. In this case one typically obtains an *effective* $S-$matrix for which the reflection coefficients have an infinite number of poles, whereby the careful selection of a finite number of poles can lead to an approximate solution. In this case, the Marchenko equations can be solved by pure N-soliton solutions, upon taking the limit of $N \to \infty$. This can be done by numerically constructing the N-soliton solution for some finite value of N, and then repeating the calculations for N taken to be successively larger and larger values [2]. We will demonstrate that in these situations, the reflection coefficients become equivalent to rational coefficients. When one restricts the problem to a certain symmetry reduction it can be demonstrated that rational reflection coefficients always lead to truncated $N-$soliton potentials. We will also discuss some of the subtle features of these scattering problems.

References:

1. H. Steudel and D. J. Kaup, *The inverse scattering transform on a finite interval*, J. Phys. A **32** 6219–6231 (1999)
2. D. J. Kaup and H. Steudel, *Recent Results on Second Harmonic Generation*, Contemporary Mathematics **326**, 33-48 (2003).
3. D.J. Kaup and H. Steudel, *Virtual Solitons and the Asymptotics of the Second Harmonic Generation*, Inverse Problems **17**, 959-970 (2001).

Periodic solutions of the short pulse model equation

Yoshimasa Matsuno
Division of Applied Mathematical Science, Graduate School of Science and Engineering
Yamaguchi University, Ube, Yamaguchi 755-8611, Japan
email: matsuno@yamaguchi-u.ac.jp

Abstract:

The short pulse (SP) equation was proposed as a model nonlinear equation describing the propagation of ultra-short optical pulses in nonlinear media [1]. Originally, it has stemmed from an attempt to construct integrable differential equations associated with pseudospherical surfaces [2]. It is an alternative of the cubic nonlinear Schrödinger (NLS) equation. Although the integrability of the SP equation has been established from various points of view [2-6], only a few results are available about solutions of the equation [7-10]. Here we develop a systematic procedure for constructing special solutions of the SP equation which include both periodic and soliton solutions. The main concern is a new class of nonsingular two-phase periodic solutions which reduce to breather solutions in the long-wave limit. We first introduce a novel hodograph transformation to convert the SP equation into the integrable sine-Gordon (sG) equation. With the known solutions of the sG equation, the system of linear partial differential equations governing the inverse mapping can be integrated analytically to obtain various solutions of the SP equation in the form of the parametric representation. The properties of the solutions are investigated in detail. In particular, we obtain a criterion under which the solutions become single-valued functions. The nonsingular breather solutions presented here may play an important role in studying the propagation of ultra-short pulses in an optical fibre.

References:

1. T. Schäfer and C. E. Wayne, Physica D **196** (2004) 90.
2. M. L. Robelo, Stud. Appl. Math. **81** (1989) 221.
3. R. Beals, M. Rabelo and K. Tenenblat, Stud. Appl. Math. **81** (1989) 125.
4. A. Sakovich and S. Sakovich, J. Phys. Soc. Jpn. **74** (2005) 239.
5. J. C. Brunelli, J. Math. Phys. **46** (2005) 123507.
6. J.C. Brunelli, Phys. Lett. A **353** (2006) 475.
7. A. Sakovich and S. Sakovich, J. Phys. A **39** (2006) L361.
8. V. K. Kuetche, T. B. Bouetou and T. C. Kofane, J. Phys. Soc. Jpn. **76** (2007) 024004.
9. Y. Matsuno, J. Phys. Soc. Jpn. **76** (2007) 084003.
10. E.J. Parkes, Chaos, Solitons and Fractals, to appear.

Line-soliton solutions of the Davey-Stewartson equation

Gino Biondini[1], Ken-ichi Maruno[2] and Sarbarish Chakravarty[3]

[1] Department of Mathematics, State University of New York at Buffalo, Buffalo, NY 14260 U.S.A. - Email: biondini@buffalo.edu

[2] Department of Mathematics, University of Texas-Pan American, Edinburg, TX, 78539 U.S.A. - Email: kmaruno@utpa.edu [3] Department of Mathematics, University of Colorado at Colorado Springs, Colorado Springs, CO 80933 U.S.A. - Email: chuck@math.uccs.edu

Abstract:

Soliton solutions of (2+1)-dimensional soliton equations have received renewed interest in recent years, and several works have showed that these solutions describe a rich phenomenology of soliton interactions [1–11]. The phenomenon of soliton resonance was first discussed in the context of the Kadomtsev-Petviashvili (KP) equation [10] (see also [11]). Recently [1–4,6,9], more general resonant solutions of KP, which possess a web-like structure, have been studied.

It was also conjectured in [4] that soliton resonance and web-like structure are a generic feature of (2+1)-dimensional integrable systems whose solutions can be expressed in determinant form. Indeed, resonant solutions with web structure have recently been found in a coupled KP (cKP) system [5], in the so-called DKP equation [7], and in the two-dimensional Toda lattice and its fully discrete and ultra-discrete analogues [8].

In this talk we characterize a large class of soliton solutions of the Davey-Stewartson (DS) equation, and we study their asymptotics and interaction patterns in the xy-plane. Many of these solutions describe phenomena of soliton resonance and web structure. In particular, a subclass of solutions of DS is the analog of the soliton solutions of the Kadomtsev-Petviashvili II equation. In addition to these, however, we show that more general solutions exist, which describe resonant phenomena that have no counterpart in the KP equation.

References:

1. Biondini G, , *Phys. Rev. Lett.* **99**, 064103:1-4 (2007)
2. Biondini G and Chakravarty S, , *J. Math. Phys.* **47**, 033514:1–26 (2006)
3. Biondini G and Chakravarty S, , *Math. Comp. Simul.* **74**, 237-250 (2007)
4. Biondini G and Kodama Y, , *J. Phys. A* **36**, 10519–10536 (2003)
5. Isojima S, Willox R and Satsuma J, , *J. Phys. A, Math. Gen.* **35**, 6893–6909 (2002); , *J. Phys. A, Math. Gen.* **36**, 9533–9552 (2003)
6. Kodama Y, , *J. Phys. A* **37**, 11169-11190 (2004)
7. Kodama Y and Maruno K-i, , *J. Phys. A* **39**, 4063–4086 (2006)
8. Maruno K-i and Biondini G, , *J. Phys. A* **36**, 11819–11839 (2004)
9. Medina E, , *Lett. Math. Phys.* **62**, 91–99 (2002)
10. Miles J W, , *J. Fluid Mech.* **79**, 171–179 (1977)
11. Ohkuma K and Wadati M, , *J. Phys. Soc. Japan* **52**, 749–760 (1983)

An integrable difference scheme for the Camassa-Holm equation and numerical computation

Baofeng Feng[1], Yasuhiro Ohta[2] and Kenichi Maruno[3]
[1]Department of Mathematics, University of Texas-Pan American, Edinburg, TX, 78539-2999 U.S.A.
Tel: (956) 381-2269, email: feng@utpa.edu
[2]Department of Mathematics, Kobe University, Rokko, Kobe 657-8501 Japan
[3]Department of Mathematics, University of Texas-Pan American, Edinburg, TX, 78539-2999 U.S.A.
Tel: (956) 381-3536, email: kmaruno@utpa.edu

Abstract:

The Camassa-Holm (CH) equation

$$w_T + 2\kappa^2 w_X - w_{TXX} + 3ww_X = 2w_X w_{XX} + ww_{XXX}$$

has attracted considerable interest since it has been derived as a model equation for shallow-water waves [1]. Originally, the CH equation has been found in a mathematical search of recursion operators connected with the integrable partial differential equations [2]. The CH equation is shown to be completely integrable, admitting peakon solutions represented by piecewise analytic functions when $\kappa = 0$ [3]. When $\kappa \neq 0$, solutions recover analytic nature, but expressed in a parametric form. The existence of cusped soliton solutions were also shown by Kraenkel and Zenchuk [4].

It is extremely difficult to perform the numerical computations of the CH equation due to the singularities of the cuspon and peakon solutions. So far, none of the numerical methods for the CH equations gives a satisfactory result. In this talk, we present an integrable semi-discretization of the CH equation. Determinant formulas for N-soliton solutions of the continuous and semi-discrete CH equations are presented. Based on determinant formula, we can generate multi-soliton and cusped-soliton solutions for both continuous and semi-discrete CH equations. The numerical computations using the integrable semi-discrete CH equation are performed. We show that the integrable scheme gives very accurate numerical results even for the soliton-solition and cuspon-cuspon interactions.

References:

1. R. Camassa and D. Holm, Phys. Rev. Lett. **71** (1993) 1661.
2. B. Fuchssteiner and A. Fokas, Physica **D4** (1981) 47.
3. R. Camassa, D.D. Holm and J.M. Hyman, Adv. Appl. Mech. **31** (1994) 1.
4. R.A. Kraenkel and A. Zenchuk, J. Phys. A: Math. Gen. **32** (1999) 4733.

Phase-space methods for quantum gases and interactions

Peter D. Drummond, S. Hoffmann, S. Wales, J. Hedditch
Australian Centre for Quantum-Atom Optics, University of Queensland, Brisbane, Australia.
Tel: +61733653404, email: drummond@physics.uq.edu.au

Abstract:

In this talk, recent developments of phase-space techniques useful for quantum gases will be analyzed, including large-scale first-principles simulations of quantum many-body dynamics with over 10^5 atoms, and a comparative evaluation of the scaling properties of different methods. The talk will focus on techniques suitable for the treatment of quantum dynamics in Brownian motion, where one quantum particle is immersed in a sea of particles of a different species, but without making a Markov or similar reservoir approximations. We focus particularly on the use of hybrid techniques, where different operator orderings are used for the quantum particle in Brownian motion, and the quantum gas which acts as the host.

Historically, phase-space methods originated with Wigner and Moyal's ideas of obtaining classical-like equations for quantum mechanics. However, there is no generally equivalent stochastic process in a purely classical phase-space, except for systems with a quadratic Hamiltonian and linear equations of motion. Instead of these methods, we will examine the use of non-classical phase-space, in which a phase-space of higher dimension than the classical one is utilized.

In particular, we consider the application of hybrid phase-space techniques, which are useful when a highly degenerate quantum gas interacts dynamically with a nano-oscillator or simply another quantum gas of much lower density. We show that this technique is capable of solving quantum dynamical behaviour in a non-perturbative way, even with very large numbers of host particles and very low temperatures.

References:

1. P. Deuar and Peter D. Drummond, Phys. Rev. Lett. 98, 120402 (2007).
2. P. D. Drummond, P. Deuar, T. G. Vaughan and J. F. Corney, J. Mod Optics 54, 2499 (2007).
3. Ruifang Dong, Joel Heersink, Joel F. Corney, Peter D. Drummond, Ulrik L. Andersen, and Gerd Leuchs, Optics Letters 33, 116 (2008).

Vortices in various superfluids

Kazushiga Machida
Department of Physics, Okayama University, Okayama 720-8530, Japan
email: machida@mp.okayama-u.ac.jp

Abstract:

Quntized vortices are a hallmark of superfuidity. Those are realized in various Fermionic and Bosonic systems, ranging from superconductors with charged electrons, to superfluid ^{3}He and ^{4}He, to recent neutral atomic gases. In this talk, we will focus on low-lying excitations with Bosonic and Fermionic charactors, trying to extract common features independent of the underlying each superfluid.

We will cover here
(1) Fermionic excitation spectrum associated with chiral superfluid characterized by two component order parameter where time reversal symmetry is spontaneously broken. We examine possible existence of the so-called Majorana particles in superfluid ^{3}He A Phase confined in two-dimensional parallel plates.
(2) The Kelvin wave excitations of vortices in harmonically trapped BEC atomic gases. We simulate the Kelvin wave motions, namely vortex oscillation motions along the vortex line direction for a three dimensional BEC system. We numerically solve the Gross-Pitaevskii equation for this geometry.

The present work is done in collaboration with M. Ichioka, T. Mizushima, T. Simula, M. Takahashi, Y. Tsutsumi, K.M. Suzuki and P. Miranovic.

Macroscopic and many-body nonlinear quantum behaviors of coupled condensates

Chaohong Lee
Nonlinear Physics Center and ARC Center of Excellence for Quantum-Atom Optics, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia
Tel: 61-2-61259074, email: chl124@rsphysse.anu.edu.au

Abstract:

To detect and control the quantum coherence of condensates, it is natural to link different condensates with an internal or external coupling. The study of coupled condensates should not only shine some light on the fundamental problems of quantum interference, quantum decoherence and classical-quantum correspondence, but also provide possible applications in technologies of atomic interferometry and high-precision measurement.

Using the mean-field theory, the coupled system is treated as multiple coupled modes of macroscopic matter waves which obey coupled Gross-Pitaevskii equations. Due to the nonlinear atom-atom interaction, many novel nonlinear macroscopic quantum effects have been explored, such as, macroscopic quantum self-trapping [1], dynamical bifurcation & chaos [2], and nonlinearity-assisted quantum tunnelling [3].

Beyond the mean-field theory, the many-body quantum effects become important due to energy level degeneracy. To explore the many-body quantum effects, one can employ a second quantization theory for coupled multiple modes which obey Hubbard-like Hamiltonians. Several many-body quantum phenomena such as quantum phase transition [4] and conditional tunnelling [5] in these models have been observed. For a quantized Bose-Josephson junctions, we find that it is possible to realize Heisenberg-limited interferometers via controlling the inter-particle interaction [6] and create single-atom devices via utilizing the interplay between asymmetry and interaction blockade [7]. For three coupled condensates on a ring, we reveal that the breakdown of coherent structures through quantum fluctuations accompanies the superfluid-insulator crossover [8].

References:

1. A. Smerzi *et al.*, Phys. Rev. Lett. 79, 4950 (1997).
2. C. Lee, W. Hai, L. Shi, and K. Gao, Phys. Rev. A 69, 033611 (2004); C. Lee, W. Hai, X. Luo, L. Shi, and K. Gao, Phys. Rev. A 68, 053614 (2003); W. Hai, C. Lee, G. Chong, and L. Shi, Phys. Rev. E 66, 026202 (2002); C. Lee, W. Hai, L. Shi, X. Zhu, and K. Gao, Phys. Rev. A 64, 053604 (2001).
3. C. Lee, E. A. Ostrovskaya, and Yu. S. Kivshar, J. Phys. B 40, 4235 (2007).
4. M. Greiner *et al.*, Nature 415, 39 (2002).
5. S. Fölling *et al.*, Nature 448, 1029 (2007).
6. C. Lee, Phys. Rev. Lett. 97, 150402 (2006).
7. C. Lee, L. -B. Fu, and Yu. S. Kivshar, arXiv: 0708.0253v2.
8. C. Lee, T. J. Alexander, and Yu. S. Kivshar, Phys. Rev. Lett. 97, 180408 (2006).

Complex Dynamics and Adiabatic Manipulation of Bose-Einstein condensates (BECs) in a Double-Well trap

Jie Liu
Institute of Applied Physics and Computational Mathematics, P.O.Box 8009, Beijing 100088, China
email: liu_jie@iapcm.ac.cn

Abstract:

In this talk, I will review our group's work on double-well BECs systems. This includes the following four aspects:
i) Quantum entanglement manifestation of transition to nonlinear self-trapping;
ii) Nonlinear quantum Zeno effect;
iii) Nonlinear Rosen-Zener transition;
iv) Nonlinear adiabatic theorem and coherent manipulation.

References:
1 Jie Liu, Biao Wu, and Qian Niu, Phys. Rev. Lett. 90, 170404 (2003)
2 Di-Fa Ye, Li-Bin Fu, and Jie Liu,Phys. Rev. A 77, 013402 (2008)
3 L. Fu and J. Liu ,Phys. Rev. A 74, 063614 (2006)
4 WeiDong Li and Jie Liu, Phys. Rev. A 74, 063613 (2006)

Issues on Spinor-Dipolar Bose-Einstein Condensates

Masahito Ueda
Department of Physics, Tokyo Institute of Technology, 2-12-1 Ookayama, Meguro-ku, Tokyo 152-8551, Japan, and ERATO Macroscopic Quantum Project, JST
Tel: +81-3-5734-2570, ueda@ap.titech.ac.jp

Abstract:

The fundamental properties of a spinor Bose-Einstein condensate (BEC) are determined by both spin-exchange and dipole interactions. The spin-dependent part of the interaction (spin-exchange interaction) is three orders of magnitude smaller than the spin-independent one and usually much smaller than the temperature of the system. Yet, due to bosonic stimulation, it profoundly affects the properties of the system. The magnitude of the local spin is primarily determined by the sign of the spin-exchange interaction. The magnetic dipole-dipole interaction is even smaller in magnitude than the spin-exchange interaction; however, it plays a crucial role in forming the spin texture, which is the spatial distribution of the spin orientation, and controls the long-term dynamics of magnetization. Spinor condensates can exhibit various topological excitations such as fractional vortices and non-abelian excitations. However, the structure of the singular core has remained unclear, because the internal degrees of freedom allow various possible states to fill in the core. Dynamical instabilities, which will ensue when we start from the "wrong" ground state, can yield a rich variety of excitations and will be an ideal testing ground for the Kibble mechanism. By suddenly altering the strength of the interaction, we can induce a second-order quantum phase transition from one phase to another. The ensuing dynamics follow two stages, which have different time scales. The first stage involves formation of spin domains, each domain having an independent phase coherence. This domain formation is driven by dynamical instabilities, and the final pattern is very sensitive to the initial seeds, of both thermal and quantum origins. The second stage involves generating topological defects that reconcile phase relations independently established among different domains. In this second stage, we might also envisage two-body spin correlations being caused by the dipolar interaction. The many-body nature of the spinor condensate remains a big challenge. In the single-mode approximation, where we ignore the spatial dependence of the spin, we have a complete understanding of the spin-1 BEC, and a good knowledge of the spin-2 BEC. However, the single-mode approximation is likely to be violated by the dipole interaction, and fragmented BECs are highly vulnerable to symmetry-breaking perturbations. It is conceivable that an exotic pair, trio, quartet, or even sextet state could emerge in high-spin systems, but the real question is which state is viable and under what conditions. I will attempt to provide an overview of what we know and what we have yet to learn about spinor BECs.

Dipolar dilute gas Bose-Einstein condensates

A.M. Martin[1], C. Ticknor[2], N.G. Parker[3], R.M.W. van Bijnen[4] and D.H.J. O'Dell[3]
[1]School of Physics, University of Melbourne, Parkville, Victoria 3010, Australia
Tel: +61 3 8344 0550, email: martinam@unimelb.edu.au
[2] ARC Centre of Excellence for Quantum-Atom, Swinburne University of Technology, Hawthorn, Victoria 3122, Australia
[3] Department of Physics and Astronomy, McMaster University, Hamilton, Ontario, L8S 4M1, Canada
[4] Eindhoven University of Technology, PO Box 513, 5600 MB Eindhoven, The Netherlands

We review the current experimental and theoretical achievements in the field of dilute gas Bose-Einstein condensation in the presence of dipolar interactions, focusing on vortex lattice formation in such systems. We initially consider the problem of a dilute gas Bose-Einstein condensate (BEC) in a rotating anisotropic trap, in the absence of dipolar interactions, where short-range interactions dominate. In this regime a vortex lattice forms when the rotational frequency (Ω) of the system is $\approx 0.7\omega_r$ (ω_r is the trapping frequency perpendicular to the axis of rotation). Insight into the mechanism of vortex formation can be gained by noting that $0.7\omega_r$ coincides with the frequency at which hydrodynamic surface excitations become unstable [1,2]. Through comparison with experimental results [3,4] and numerical solutions of the Gross-Pitaevskii equation (GPE) [5,2] such instability has been related to the formation of a vortex lattice.

The above results apply to conventional BECs with short-range s-wave interactions, parameterized via $g = 4\pi\hbar^2 a/(m)$, where a is the s-wave scattering length. However, recent experimental advances in the condensation of chromium, with dipolar interactions [6-8], opens the door to experimentally study the effect of dipolar interactions in BECs. Parallel theoretical work, using a modified GPE, has studied the effect of such long-range interactions on the ground state vortex lattice solutions [9,10]. However, the route to generating such states has not been explored. For this purpose we solve the hydrodynamic equations of motion for a dipolar BEC in rotating anisotropic harmonic traps [11]. We show that the solutions depend on both the strength of the dipolar interactions, ε_{dd}, and the aspect ratio of the trap, $\gamma = \omega_z/\omega_r$, in stark contrast to conventional BECs where the instability is independent of both the strength of the interactions, g, and γ [12,1]. In addition we evaluate the dynamical stability of our solutions, showing that the region of Ω for which the solutions are unstable can be controlled via ε_{dd} and γ.

References:

1. S. Sinha and Y. Castin, Phys. Rev. Lett. **87**, 190402 (2001).
2. N.G. Parker *et al.*, Phys. Rev. A **73**, 061603(R) (2006).
3. K.W. Madison *et al.*, Phys. Rev. Lett. **86**, 4443 (2001).
4. E. Hodby *et al.*, Phys. Rev. Lett. **88**, 010405 (2002).
5. C. Lobo *et al.*, Phys. Rev. Lett. **92**, 020403 (2004).
6. A. Griesmaier *et al.*, Phys. Rev. Lett. **94**, 160401 (2005).
7. T. Koch *et al.*, arXiv:cond-mat/0710.36.43.
8. Q. Beaufils *et al.*, arXiv:cond-mat/0712.3521.
9. N.R. Cooper *et al.*, Phys. Rev. Lett. **95**, 200402 (2005).
10. J. Zhang and H. Zhai, Phys. Rev. Lett. **95**, 200403 (2005).
11. R.M.W. van Bijnen *et al.*, Phys. Rev. Lett. **98**, 150401 (2007).
12. A. Recati *et al.*, Phys. Rev. Lett. **86**, 377 (2001).

Interference, Solitons and Dynamics of Bose-Einstein Condensates

Wu-Ming Liu
Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China
Tel: 86-10-82649531, email: wmliu@aphy.iphy.ac.cn

Abstract:

Nonlinear effects in the interference of Bose-Einstein condensates are studied using exact solutions of the one-dimensional nonlinear Schrodinger equation, which is applicable when the lateral motion is confined or negligible. With the inverse scattering method, the interference pattern is studied as a scattering problem with the linear Schrodinger equation, whose potential is profiled by the initial density distribution of the condensates. Our theory not only provides an analytical framework for quantitative predictions for the one-dimensional case, it also gives an intuitive understanding of some mysterious features of the interference patterns observed in experiments and numerical simulations [1].

We present a family of exact solutions of the one-dimensional nonlinear Schrodinger equation which describes the dynamics of a bright soliton in Bose-Einstein condensates with the time-dependent interatomic interaction in an expulsive parabolic potential. Our results show that, under a safe range of parameters, the bright soliton can be compressed into very high local matter densities by increasing the absolute value of the atomic scattering length, which can provide an experimental tool for investigating the range of validity of the one-dimensional Gross-Pitaevskii equation. We also find that the number of atoms in the bright soliton keeps dynamic stability: a time-periodic atomic exchange is formed between the bright soliton and the background [2].

We present how to control interactions between solitons, either bright or dark, in Bose-Einstein condensates by synchronizing Feshbach resonance and harmonic trap. Our results show that as long as the scattering length is to be modulated in time via a changing magnetic field near the Feshbach resonance, and the harmonic trapping frequencies are also modulated in time, exact solutions of one dimensional nonlinear Schrödinger equation can be found in a general closed form, and interactions between two solitons are modulated in detail in currently experimental conditions. We also propose experimental protocols to observe the new phenomena such as fusion, fission, warp, oscillation, elastic collision in future experiments [3].

By means of the Darboux transformation, we obtain analytical solutions for a soliton set on top of a plane-wave background in coupled Gross-Pitaevskii equations describing a binary Bose-Einstein condensate. We consider basic properties of the solutions with and without the cross interaction (cross phase modulation) between the two components of the background. In the absence of the cross phase modulation, this solutions maintain properties of one-component condensates, such as the modulation instability; in the presence of the cross interaction, the solutions exhibit different properties, such as restriction of the modulation instability and soliton splitting [4].

Magnetic solitons in spinor Bose-Einstein condensates confined in a one-dimensional optical lattice are studied by the Holstein-Primakoff transformation method. It is shown that due to the long-range light-induced and static magnetic dipole-dipole interactions, there exist different types of magnetic solitary excitations in different parameter regions. Compared to conventional solid-state materials, the parameters of this type of magnetic solitons in an optical lattice can be easily tuned by the above dipole-dipole interactions,

which are highly controllable in experiments [5].

We study the magnetic soliton dynamics of spinor Bose-Einstein condensates in an optical lattice which results in an effective Hamiltonian of anisotropic pseudospin chain. A modified Landau-Lifshitz equation is derived and exact magnetic soliton solutions are obtained analytically. Our results show that the time oscillation of the soliton size can be controlled in practical experiment by adjusting of the light-induced dipole-dipole interaction. Moreover, the elastic collision of two solitons is investigated [6].

We find one-, two-, and three-component solitons of the polar and ferromagnetic types in the general nonintegrable model of a spinor three-component model of the Bose-Einstein condensate, based on a system of three nonlinearly coupled Gross-Pitaevskii equations. The stability of the solitons is studied by means of direct simulations and, in a part, analytically, using linearized equations for small perturbations. Global stability of the solitons is considered by means of an energy comparison. As a result, ground-state and metastable soliton states of the ferromagnetic and polar types are identified. For the special integrable version of the model, we develop the Darboux transformation. As an application of the DT, analytical solutions are obtained that display full nonlinear evolution of the modulational instability of a continuous-wave state seeded by a small spatially periodic perturbation. Additionally, by dint of direct simulations, we demonstrate that solitons of both the polar and ferromagnetic types, found in the integrable system, are structurally stable; i.e., they are robust under random changes of the relevant nonlinear coefficient in time [7].

References:

1. W. M. Liu, B. Wu, Q. Niu, Nonlinear effects in interference of Bose-Einstein condesates, Phys. Rev. Lett. 84, 2294 (2000).
2. Z. X. Liang, Z. D. Zhang, W. M. Liu, Dynamics of a bright soliton in Bose-Einstein condensates with time-dependent atomic scattering length in an expulsive parabolic potential, Phys. Rev. Lett. 94, 050402 (2005).
3. X. F. Zhang, Q. Yang, J. F. Zhang, X. Z. Chen, W. M. Liu, Controlling Soliton Interactions in Bose-Einstein Condensates by Synchronizing Feshbach Resonance and Harmonic Trap, Phys. Rev. A 77, in press (2008).
4. L. Li, B. A. Malomed, D. Mihalache, W. M. Liu, Exact soliton-on-plane-wave solutions for two-component Bose-Einstein condensates, Phys. Rev. E 73, 066610 (2006).
5. Z. W. Xie, W. P. Zhang, S. T. Chui, W. M. Liu, Magnetic solitons of spinor Bose-Einstein condensates in optical lattice, Phys. Rev. A 69, 053609 (2004).
6. Z. D. Li, P. B. He, L. Li, J. Q. Liang, W. M. Liu, Magnetic soliton and soliton collisions of spinor Bose-Einstein condensates in an optical lattice, Phys. Rev. A 71, 053611 (2005).
7. L. Li, Z. D. Li, B. A. Malomed, D. Mihalache, W. M.Liu, Exact soliton solutions and nonlinear modulation instability in spinor Bose-Einstein condensates, Phys. Rev. A72. 033611 (2005).

Nonlinear Dirac Equation

Lincoln D. Carr and Laith Haddad
Department of Physics, Colorado School of Mines, Golden, Colorado 80401, U.S.A.
Tel: +1 303-273-3759, email: lcarr@mines.edu

Abstract:

We present a relativistic generalization of the nonlinear Schrodinger equation, the nonlinear Dirac equation (NLDE). Although different versions of a nonlinear Dirac equation have appeared in numerous fields in the past (for a recent summary, see [1]), we present a novel version of the NLDE which is of immediate experimental relevance in Bose-Einstein condensates and has a "speed of light" ten orders of magnitude slower than c. We discuss the symmetry properties of this new equation.

References:

1. Wei-Khim Ng and Rajesh R. Parwani, e-print arXiv:0707.1553 (2007).

Intra-band tunneling of Bose-Einstein condensates in three-dimensional lattices

Vladimir V. Konotop[1] and Valery S. Shchesnovich[2]
[1]Departamento de Física, Faculdade de Ciências, Universidade de Lisboa, Campo Grande, Ed. C8, Piso 6, Lisboa 1749-016, Portugal
email: konotop@cii.fc.ul.pt
[2]Instituto de Física - Universidade Federal de Alagoas, Maceió AL 57072-970, Brazil
email: vs_shchesnovich@yahoo.co.uk

Abstract:

The intra-band tunneling of a Bose-Einstein condensate between three degenerate high-symmetry X-points of the Brillouin zone of a cubic optical lattice is studied in the quantum regime by reduction to a three-mode model. The mean-field approximation of the deduced model is described. Compared to the previously reported two-dimensional (2D) case [1], which is reducible to the two-mode model, in the case under consideration there exist a number of new stable stationary atomic distributions between the X-points and a new critical lattice parameter. The quantum collapses and revivals of the atomic population dynamics are absent for the experimentally realizable time span. The 2D stationary configurations, embedded into the 3D lattice, turn out to be always unstable, while existence of a stable 1D distribution, where all atoms populate only one X-state, may serve as a starting point in the experimental study of the nonlinear tunneling in the 3D lattice.

References:

1. V. S. Shchesnovich and V. V. Konotop, Phys. Rev. A **75,** 063628 (2007)

Spontaneous rotating vortex lattices in a pumped decaying condensate

Natalia Berloff
Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge, CB3 0WA, United Kingdom
Tel: 44-(0)1223-337860, email: N.G.Berloff@damtp.cam.ac.uk

Abstract:

Injection and decay of particles in an inhomogeneous quantum condensate can significantly change its behaviour. We model trapped, pumped, decaying condensates by a complex Gross-Pitaevskii equation and analyse the density and currents in the steady state. With homogeneous pumping, rotationally symmetric solutions are unstable. Stability may be restored by a finite pumping spot. However if the pumping spot is larger than the Thomas-Fermi cloud radius, then rotationally symmetric solutions are replaced by solutions with spontaneous arrays of vortices. These vortex arrays arise without any rotation of the trap, spontaneously breaking rotational symmetry.

Topological solitons in double-ring Bose-Einstein condensates

Joachim Brand[1], Tania Haigh[2], and Ulrich Zülicke[2]

1. Centre of Theoretical Chemistry and Physics and Institute of Fundamental Sciences, Massey University Auckland, Private Bag 102904, North Shore City 0745, Auckland, New Zealand
email: J.Brand@massey.ac.nz
2. Centre of Theoretical Chemistry and Physics and Institute of Fundamental Sciences, Massey University, Palmerston North, New Zealand

Abstract:

We consider a Bose-Einstein condensate in a double-ring trap under rotation. We find that all ground-state configurations are stable in contrary to the findings of Ref. [1]. However, the phase diagram of the system shows a surprisingly complex structure with localized defects appearing in ground state configurations. The defects have the properties of topological solitons and can be seen as BEC analogs of Josephson Vortices. We study the dynamics of the defects under the influence of external fields.

References:

1. I. Lesanovsky and W. v. Klitzing, Phys. Rev. Lett. **98**, 050401 (2007).

Finite-Gap Solutions of the Vortex Filament Flow: Genus One Solutions and Symmetric Solutions

Thomas Ivey

Department of Mathematics, College of Charleston, Charleston, SC 29424, U.S.A.
Tel. +1-843-953-5730, email: iveyt@cofc.edu

Abstract:

A simplified model of vortex filament motion in an ideal fluid leads to an integrable nonlinear evolution equation, known as the localized induction approximation or vortex filament flow (VFF), closely related to the cubic focussing nonlinear Schroedinger (NLS) equation. In particular, finite-gap VFF solutions provide examples of evolving curves whose geometric and topological features can be related to the spectrum of the AKNS scattering system for the corresponding NLS solution.

In the first of two talks, we discuss the construction of quasi-periodic VFF solutions, a complete characterization of two-phase solutions (the genus one case), including geometrically interesting special cases (Euler elastica, constant torsion curves, self-intersecting filaments, and symmetric solutions), and generalizations of these features to higher genus.

Co-authors: Annalisa Calini (College of Charleston)

Finite-Gap Solutions of the Vortex Filament Flow: Isoperiodic Deformations

Annalisa Calini
Department of Mathematics, College of Charleston, Charleston, SC 29424, U.S.A.
Tel. +1-843-953-5730, Email: calinia@cofc.edu

Abstract:

The simplest model of vortex filament motion in an ideal fluid leads to an integrable nonlinear evolution equation closely related to the cubic focussing nonlinear Schrödinger equation. Closed finite-gap solutions of the vortex filament flow provide examples of evolving curves whose topological features can be related to their algebro-geometric description.

In the second of two talks, we describe how the theory of isoperiodic deformations (developed by Grinevich and Schmidt, after Krichever) can be used to generate a family of closed finite-gap solutions of increasingly higher genus close to a multiply covered circle. Each step of the deformation process corresponds to constructing a cable on the previous filament, whose knot type is determined from the deformation scheme, and is invariant under the time evolution.

Co-authors: Tom Ivey (College of Charleston)

KdV cnoidal waves are linearly stable

Bernard Deconinck
University of Washington Department of Applied Mathematics Seattle, WA 98195-2420, USA
Tel. +1-206-543-6069, Email: bernard@amath.washington.edu

Abstract:

Going back to considerations of Benjamin (1974), there has been significant interest in the question of stability for the stationary periodic solutions of the Korteweg-deVries equation, the so-called cnoidal waves. In this talk, we exploit the squared-eigenfunction connection between the linear stability problem and the Lax pair for the Korteweg-deVries equation to completely determine the spectrum of the linear stability problem for eigenfunctions that are bounded on the real line. We find that this spectrum is confined to the imaginary axis, leading to the conclusion of spectral stability. An additional completeness argument allows for a statement of linear stability.

Stability Analysis of Persisting Periodic Solutions to a Complex Ginzburg-Landau Perturbation of NLS

Stéphane Lafortune
Department of Mathematics, College of Charleston Charleston, SC 29424, USA
Tel: +1-843-953-5869, email: lafortunes@cofc.edu

Abstract:

It was shown in [Cruz-Pacheco, Levermore, and Luce (2004)] that a certain class of periodic solutions to the nonlinear Schrodinger equation (NLS) persist when the NLS is subject to a perturbation leading to the Complex Ginzburg Landau equation (CGL). In this presentation, I will show how one can use methods coming from the theory of integrability together with the Evans function to study the spectral stability of these persisting solutions. In particular we show that the solutions of NLS are spectrally stable with respect to periodic perturbations. However, the solutions can become unstable when NLS is perturbed to CGL.
Co-Author: Tom Ivey (College of Charleston)

Stability of plane-wave solutions to a dissipative generalization of the vector nonlinear Schrödinger equation

John D. Carter
Mathematics Department, Seattle University, Seattle, WA 98122, U.S.A.
Tel: 206-296-5956, email: carterj1@seattleu.edu

Abstract:

The vector nonlinear Schrödinger (VNLS) equation can be used as a model for the evolution of a uniform train of surface waves with a two dimensional, bi-periodic surface pattern propagating on deep water. Such wave trains are linearly unstable. The VNLS equation does not include any dissipative terms. When the VNLS equation is modified to include a specific form of dissipation, then the corresponding wave train is linearly stable [1]. We examine the VNLS equation with multiple forms of dissipation [2]. We study the stability of the corresponding plane-wave solutions. This is joint work with Wilhelmina Chik.

References:

1. W. Craig, D.M. Henderson, M. Oscamou, and H. Segur, Math. Comp. Sim. 74 (2007).
2. T.J. Bridges and F. Dias, Phys. Fluids 19, 104104 (2007).

Random perturbations of the Korteweg-de Vries equation

Anne de Bouard
Centre de Mathématiques Appliquées, CNRS and Ecole Polytechnique,
91128 Palaiseau Cedex, France
tel : 331-69334587, email: debouard@cmapx.polytechnique.fr

Abstract:

We consider the KdV equation perturbed by additive or multiplicative noise, white in time, and analyze the dynamical behavior of the solutions which are close to solitons, in the limit where the amplitude of the noise goes to zero. We estimate the exit time of neighborhoods of the soliton, with or without random modulations of the soliton parameters. In the additive case, the estimates we obtain are sharp, while in the multiplicative case, we obtain additional properties of the solution as e.g. "superdiffusion" of its order one part. We also estimate the exit time in the multi-soliton case. The results have been obtained in collaborations with A. Debussche, E. Gautier and K. El Dika.

Hill's method: computing spectra of linear operators

Chris Curtis and Bernard Deconinck
Department of Applied Mathematics, University of Washington
Tel: 206-543-6069, email: bernard@amath.washington.edu

Abstract:

Hill's method is a numerical method to compute spectra of linear operators with periodic coefficients. Such problems often arise when examining the linear stability of periodic stationary solutions of nonlinear wave equations. I will discuss new results proving the convergence of Hill's method for large classes of problems. Time permitting theorems for the computation of the eigenfunctions and for the rate of convergence of the eigenvalue computations will be given as well.

Stability of nontrivial-phase solutions to the modified NLS equation

John D. Carter
Mathematics Department, Seattle University, Seattle, WA 98122, U.S.A.
Tel: 206-296-5956, email: carterj1@seattleu.edu

Abstract:

The two-dimensional cubic nonlinear Schrödinger (NLS) equation is a model for the evolution of two-dimensional nearly monochromatic gravity-capillary waves that are slowly modulated in deep water. All nontrivial-phase solutions of the NLS equation are known to be unstable. The NLS equation is derived from the Euler equations via a perturbation analysis. If this perturbation analysis is taken one order further, the modified nonlinear Schrödinger (MNLS) equation is obtained. We establish that nontrivial-phase solutions of the MNLS equation are unstable. We compare the NLS and MNLS models by examining the growth rates of the instabilities of solutions to these equations. This is joint work with Eddie Feeley.

Results on Boussinesq systems for water waves

Min Chen
Department of Mathematics, Purdue University 47907, U.S.A.
Tel: 7654941964, email: chen@math.purdue.edu

Abstract:

This talk is a report on some of the results we have achieved on the study of Boussinesq systems. The results consist of existence proofs, explicit form and numerical construction of one-dimensional special solutions, namely solitary, cnoidal and standing waves, and two-dimensional symmetric and asymmetric patterns; and of numerical simulations on waves generated from a rectangular source, on two-dimensional wave pattern formations and on blow-up solutions of large data for some of the systems.

Viscous asymptotical models for water wave propagation

Olivier Goubet
LAMFA UMR 6140 CNRS Université de Picardie Jules Verne 33 rue saint-Leu 80039 Amiens cedex email: olivier.goubet@u-picardie.fr

Abstract:

In this talk, we will discuss the effect of the viscosity on asymptotical models for the propagation of water waves. We focus on the decay rate towards equilibrium of solutions. Part of this work is joint work with Min Chen (Purdue University).

References:

1. M. Chen and O. Goubet, Long-Time Asymptotics of 2D Dissipative Boussinesq Systems, to appear

Solutions to some open problems in n-dimensional fluid dynamics

Linghai Zhang
Department of Mathematics, Lehigh University
14 East Packer Avenue, Bethlehem, Pennsylvania 18015 USA
Telephone: +1-610-758-4116. Fax: +1-610-758-3767. Email: liz5@lehigh.edu

Abstract:

The focus of this work is on the solutions to some open problems of the global weak solutions of the Cauchy problems for a general nonlinear dissipative partial differential equation

$$\mathbf{u}_t - \varepsilon\triangle\mathbf{u}_t - \alpha\triangle\mathbf{u} + \mathcal{D}\mathbf{u} + \mathcal{N}(\mathbf{u}, \nabla\mathbf{u}) = \mathbf{0}, \qquad \mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x}),$$

in n-dimensional space, where $n \geq 1$ is an integer, $\alpha > 0$ and $0 \leq \varepsilon \leq 1$ are real constants, and $\triangle = \sum_{k=1}^{n} \frac{\partial^2}{\partial x_k^2}$ denotes the classical Laplace operator. More precisely, suppose that the initial function $\mathbf{u}_0 \in L^1(\mathbf{R}^n) \cap H^2(\mathbf{R}^n)$, let $\mathbf{u} = \mathbf{u}(\mathbf{x}, t, \mathbf{u}_0)$ represent the global solutions of the Cauchy problem, we will study the limit

$$\lim_{t\to\infty} \left\{ (1+t)^{2m+\lambda+n/2} \int_{\mathbf{R}^n} \left[|\triangle^m \mathbf{u}(\mathbf{x}, t)|^2 + \varepsilon |\nabla\triangle^m \mathbf{u}(\mathbf{x}, t)|^2 \right] \mathrm{d}\mathbf{x} \right\}$$

in terms of the initial data $\mathbf{u}_0$ and the model parameters, such as the dissipation coefficient, where $m \geq 0$ is any integer, $\lambda = 0$ if $\int_{\mathbf{R}^n} \mathbf{u}_0(\mathbf{x})\mathrm{d}\mathbf{x} \neq \mathbf{0}$, and $\lambda = 1$ if $\int_{\mathbf{R}^n} \mathbf{u}_0(\mathbf{x})\mathrm{d}\mathbf{x} = \mathbf{0}$. The limit problem has been open for a long time. The model includes the n-dimensional Burgers equation, the n-dimensional Benjamin-Bona-Mahony-Burgers equation, the one-dimensional nonlinear cubic Korteweg-de Vries-Burgers equation, the one-dimensional nonlinear Benjamin-Ono-Burgers equation, the two-dimensional nonlinear nonlocal quasi-geostrophic equation, the n-dimensional incompressible Navier-Stokes equations and the n-dimensional incompressible Magnetohydrodynamics equations as particular examples.

The main ideas in the analysis are Fourier transform, Plancherel's identity, new decomposition of frequency space, lower limit estimate and upper limit estimate.

References:

1. Charles J. Amick, Jerry L. Bona and Maria E. Schonbek, Decay of solutions of some nonlinear wave equations. *Journal of Differential Equations*, **81**(1989), 1-49.
2. Linghai Zhang, Decay estimates for the solutions of some nonlinear evolution equations. *Journal of Differential Equations*, **116**(1995), 31-58.
3. Linghai Zhang, Sharp rate of decay of solutions to 2-dimensional Navier-Stokes equations. *Communications in Partial Differential Equations*, **20**(1995), 119-127.
4. Linghai Zhang, Decay estimates for solutions to initial value problems for the generalized nonlinear Korteweg-de Vries-Burgers equation. *Chinese Annals of Mathematics*, **16A**(1995), 22-32.
5. Linghai Zhang, Solutions to some open problems in one-dimensional fluid dynamics. Chinese Annals of Mathematics, Series B. (2008). In press.

Surface Soliton Families at Boundaries Between Periodic Media

G. I. Stegeman, S. Suntsov, K. G. Makris and D. N. Christodoulides
College of Optics and Photonics, CREOL and FPCE, University of Central Florida, 4000 Central Florida Blvd., Orlando Florida 32816, USA
email: george@creol.ucf.edu

R. Morandotti
Institut national de la recherch scientifique, Universit du Qubec, Varennes, Qubec, Canada J3X 1S2

Mate Volatier, Vincent Aimez, and Richard Ars
Centre de Recherche en Nanofabrication et en Nanocaractrisation, CRN, Universit de Sherbrooke, Sherbrooke, Qubec, Canada J1K2R1

H. Yang, G. Salamo
Physics Department, University of Arkansas, Fayetteville, Arkansas 72701, USA

Abstract:

We have investigated families of nonlinear waves (surface solitons) at the boundaries between dissimilar periodic media. It is shown both theoretically and experimentally that an interface between two periodic Kerr media consisting of arrays of 1D, parallel channel waveguides can support interface solitons peaked on different sides of the boundary.

The band structure was calculated by solving for the linear Floquet-Bloch modes of the composite structure and a soliton centered on each boundary channel was obtained, i.e. a pair of solitons. Both solitons were stable on propagation. The waveguides had different widths and therefore different propagation constants on the two sides of the boundary. As a result the threshold powers for the formation of the solitons centered on opposite sides of the boundary is different. For solitons peaked in channels near and at the boundary, the variation in threshold power for soliton formation varied with channel number differently in the two coupled arrays, indicating both an attractive and a repulsive soliton potential on opposite sides of the boundary.

Supporting experiments were performed in AlGaAs arrays. Two different families of solitons on opposite sides of the boundary were identified and the difference in soliton potential near the interface in the two arrays was verified.

Dissipative optical solitons and cavity soliton lasers

William Firth
Department of Physics, University of Strathclyde, 107 Rottenrow, Glasgow G4 ONG, Scotland, UK
Tel: +44 141 548 3269, email: w.j.firth@strath.ac.uk

Abstract:

Dissipative solitons [1] are self-localized structures in lossy systems, which require a nonlinear balance between loss and gain in addition to the balance between nonlinearity and dispersion which characterizes ordinary solitons. In optics, they are often termed *cavity solitons*, being most intensively studied in optical cavities (see Chapters 3-6 of [1], and *op cit*).

This talk will review some of the main features of transverse (spatial) cavity solitons in theory and experiment, including their links to spontaneous pattern formation. These links imply the existence of clusters of cavity solitons. Experiment and theory do not agree on the existence ranges and stability of such clusters, and a possible resolution involving an additional nonlocal nonlinearity [2] will be discussed.

Some potential applications will be mentioned, in particular taking advantage of their transverse mobility [3]. For many applications, the cost-effectiveness of generating cavity solitons directly within a laser is attractive. A recent demonstration of such a *cavity soliton laser* based on a VCSEL with frequency-selective feedback [4] will be described, and its theory discussed.

Acknowledgements: This work was supported by the EU STREP FunFACS (www.funfacs.org). The author is grateful to FunFACS partners for their many contributions to this work.

References:

1. *Dissipative solitons*, Vol. 661 of *Lecture Notes in Physics*, edited by N. Akhmediev and A. Ankiewicz (Springer, New York, 2005).
2. W. J. Firth, L. Columbo and A. J. Scroggie, Phys. Rev. Lett. 99, 104503 (2007).
3. F. Pedaci, et al, Appl. Phys. Lett. 92 011101 (2008).
4. Y. Tanguy, T. Ackemann, W. J. Firth and R. Jäger, Phys. Rev. Lett. 100, 013907 (2008).

Dynamic and nonlinear effects in photonic crystals

Shanhui Fan
Department of Electrical Engineering, Stanford University, Ginzton Laboratory, AP 273, Stanford, CA 94305, USA
Tel: 1-650-724-4759; Email: shanhui@stanford.edu

Abstract:

In photonic crystal, the introduction of dynamic modulation creates new opportunities. In this paper, we discuss the possibility of complete capturing and storing of an optical pulse, with only the use of a few resonators, whose dimensions are far smaller than the spatial length of the pulse. We also argue for the implication of such pulse capturing scheme for nonlinear optics in photonic crystals.

Soliton diffusion in random optical lattices

Yaroslav V. Kartashov[1], Victor A. Vysloukh[2], and Lluis Torner[1] [1] ICFO-Institut de Ciencies Fotoniques, and Universitat Politecnica de Catalunya, Mediterranean Technology Park, 08860 Castelldefels (Barcelona), Spain
[2] Departamento de Fisica y Matematicas, Universidad de las Americas - Puebla, Santa Catarina Martir, 72820, Puebla, Mexico
email: Yaroslav.Kartashov@icfo.es

Evolution of excitations in disordered nonlinear medium give rise to a number of unique physical phenomena. A universal feature of wave packet and particle dynamics in disordered me-dia in different areas of physics is percolation [1]. Percolation occurs in all types of physical set-tings, including high-mobility electron systems, Josephson-junction arrays, two-dimensional GaAs structures near the metal-insulator transition, or charge transfer between super-conductor and hoping insulator, to mention a few. In this presentation we will discuss the nonlinear optical analogue of biased percolation [2] that is related to disorder-induced soliton transport in ran-domly modulated optical lattices with Kerr-type focusing nonlinearity in the presence of linear variation of the refractive index in the transverse plane, thus generating a constant deflecting force for light beams entering the medium. When such a force is too small, solitons in perfectly periodic lattice are trapped in the vicinity of the launching point due to Peierls-Nabarro potential barriers, provided that the launching angle is smaller than a critical value. Under such conditions soliton transport is suppressed, and thus the lattice acts as a soliton insulator. However, random modulations of the lattice parameters turns soliton transport possible again, with the key parame-ter determining the soliton current being the standard deviation of phase/amplitude fluctuations.

We will show that the soliton current in lattices with amplitude and phase fluctuations reaches its maximal value at intermediate disorder levels and that it drastically reduces in both, almost regu-lar and strongly disordered lattices. We will also discuss the phenomenon of Brownian-type soliton motion in a random poten-tial made by an optical lattice. Analogies between nonlinear excitations and particles are well known. Thus, soliton motion in materials with inhomogeneous transverse properties mimics the random motion of mechanical particles. In this context, optical induction techniques opened up a wealth of opportunities to the generation of reconfigurable refractive index landscapes. Optical induction allowed the recent observation of Anderson localization with linear light beams propa-gating in disordered optical lattices [3]. In this presentation we will discuss new analogies be-tween the dynamics of strongly nonlinear excitations in optically induced random lattices and Brownian motion. We will consider lattices featuring nondiffracting speckle-like patterns, since such lattices allow flexible control of disorder characteristics, as well as the possibility to experi-mentally vary the random realization. It will be illustrated that the small-scale randomly located potential spots act as a random potential that produces random forces acting on soliton and thus causing its Brownian- type motion. As a result, we find that the statistically averaged squared dis-placement of the soliton center grows linearly with distance, in analogy with Brownian motion.

References:

1. G. R. Grimmett, Percolation (Springer, Berlin, 1999).
2. Y. V. Kartashov, V. A. Vysloukh, L. Torner, Opt. Express 15, 12409 (2007).
3. T. Schwartz et al., Nature 446, 05623 (2007).

Experimental observation of Rabi oscillations in one-dimensional photonic lattices

Detlef Kip, Ksenia Shandarova, Christian E. Rüter, Rong Dong
Institute of Physics and Physical Technologies, Clausthal University of Technology, 38678 Clausthal-Zellerfeld, Germany, E-Mail: d.kip@pe.tu-clausthal.de

Abstract:

Since the famous work of Isidor Rabi in 1936, it is well known that in quantum systems, different energy levels may resonantly interact with an appropriate periodic modulation [1]. A typical example is a two-level atomic system, where an electromagnetic wave whose frequency is tuned to the energy gap between the two states, causes periodic population exchanges accompanied by emission and re-absorption of a photon. Here, we realize the optical analogue of this process, by a transversely-periodic photonic lattice that is additionally modulated along the propagation direction [2,3]. We demonstrate all-optical Rabi-like transitions of extended Floquet-Bloch (FB) modes occurring when the corresponding selection rules of momentum and parity conservation are fulfilled. We observe direct transitions of FB modes between bands, as well as indirect transitions assisted by coherent phonons. For the latter, such Rabi oscillations corresponding to indirect transitions are especially intriguing, because their presence necessitates phonons that remain coherent throughout the oscillation, which makes this phenomenon extremely rare in atomic lattices.

References:

1. I. I. Rabi, Phys. Rev. 49, 324 (1936).
2. V. S. Shshesnovich and S. Chvez-Cerda, Opt. Lett. 32, 1920 (2007).
3. F. Chen, M. Stepic, C. E. Rüter, D. Runde, D. Kip, V. Shandarov, O. Manela, and M. Segev, Opt. Express 13, 4314 (2005).

All optical processing in integrated periodic structures

Roberto Morandotti
INRS-EMT, 1650 Boulevard Lionel Boulet, Varennes, Qubec, Canada, J3X 1S2
Tel: 450-9298124, email: morandot@emt.inrs.ca

Abstract:

Integrated periodical structures often lead to an intriguing spatio-temporal dynamics and to various configurations for all-optical pulse shaping. In this talk, I will describe a series of novel schemes and structures for signal processing in the temporal domain, with applications in both the linear and the nonlinear regimes.

Tunable frequency conversion in a nonlinear photonic crystal

Yan Sheng, Junhong Dou, Jingjuan Li, Boqin Ma, Bingying Cheng, Zhiyuan Li and Daozhong Zhang

Optical Physics Lab., Beijing National Laboratory for Condensed Matter Physics, Institute of Physics, Chinese Academy of Sciences, Beijing 100080, China

Abstract:

A tunable efficient quasi-phase matched second harmonic generation (SHG) in nonlinear $LiNbO_3$ crystal has been demonstrate, where the distribution of ferroelectric domains possess only the short-range order. In such domain structures the reciprocal vectors form a series of concentric rings. In each ring the reciprocal vectors are continuously distributed, which can be used to compensate continuous wave vector differences and achieve the phase-matching for different frequencies and multiple parametric interactions. Therefore, the tunable output can be realized with higher conversion efficiency. This process is similar to the broadband SHG in random media, but the existence of short range ordered domain results in much higher frequency conversion efficiency.

Experiments show that in our sample the tunable range of SHG output can be 685 nm to 584 nm and 538 nm to 475 nm, where the phase matching conditions are satisfied by using the reciprocal vectors in the two lowest concentric rings respectively. As the input wavelength is tuned continuously from 1.317 to 1.185μm, corresponding SHG efficiencies keep almost unchanged and it is around 12%. Meanwhile, The dependence of SHG output on the crystal temperature and incident angle were also measured at a fixed output wavelength 634nm. The results indicate that the variation of temperature does not lead to a drop of SHG conversion efficiency and the SHG output changes just a little within an incident angle of 18°.

The achieved broadband SHG with high conversion efficient might be beneficial to the development of optics integration and multiple-channel devices.

Puzzle of optical fiber supercontinuum or photons under the action of gravity-like force

Dmitry V. Skryabin
Department of Physics, University of Bath, Bath, BA2 7AY, UK.
email: d.v.skryabin@bath.ac.uk

Abstract:

Femtosecond pulses of light propagating along photonic-crystal bres can generate a broad optical supercontinuum. This striking discovery has applications ranging from spectroscopy and metrology to telecommunication and medicine. Among the physi- cal principles underlying supercontinuum generation are soliton emission, a variety of four-wave mixing processes, Raman-induced soliton self-frequency shift, and disper- sive wave generation mediated by solitons. Although all of the above eects contribute to supercontinuum generation, none of them can explain the generation of blue and violet light from infrared femtosecond pump pulses. In this talk I will argue that the most profound role in the shaping of the short-wavelength edge of the continuum is played by the eect of radiation trapping in a gravity-like potential created by accel- erating solitons. The underlying physics of this eect has a straightforward analogy with the inertial forces acting on an observer moving with a constant acceleration. This observation allows to use ber solitons as ecient tools for targeted frequency conversion and for emulation of light bending and trapping by gravity in the ber optics experiments.

References:

1. A. Gorbach, D.V. Skryabin, Nature Photonics 1, 653 - 657 (2007).

Dynamics and thermodynamics of optical dispersive shocks and soliton gas

S. Trillo, C. Conti, A. Fratalocchi, M. Peccianti, G. Ruocco
Dipartimento di Ingegneria, Università di Ferrara, Via Saragat 1, 44100 Ferrara, Italy
and
SOFT INFM-CNR Università di Roma "La Sapienza", P. A. Moro 2, 00185, Roma, Italy
Tel: +39-0532-974838, email:stefano.trillo@unife.it

Abstract:

We discuss the formation of dispersive shock waves of the spatial type occurring in defocusing media [1]. When developing from a dark input waveform, the oscillations characteristic of the shock waves behave as multiple pairs of dark solitons that emanate from a focus (shock) point. Stimulated by our experimental results obtained in a nonlocal (thermal) medium, we have developed a full theory of the phenomenon. In the local limit, we establish a connection between the structure of the dispersive shock wave and the integrability of the underlying wave equation, which allows us to develop a thermodynamical description of the soliton gas and to interpret the shock wave in terms of a phase-transition [2]. Robustness of the phenomenon against nonlocality will be also discussed.

References:

1. W. Wan, S. Jia, And J. W. Fleischer, Nature Phys. **3**, 46 (2007); N. Ghofraniha, C. Conti, G. Ruocco, S. Trillo, Phys. Rev. Lett. **99**, 043903 (2007).
2. A. Fratalocchi, C. Conti, G. Ruocco, S. Trillo, in preparation.

Localization and Delocalization phenomena in one-dimensional disordered nonlinear waveguide arrays

Rodrigo A. Vicencio
Departmento de Física, Facultad de Ciencias, Universidad de Chile, Chile.
Tel: 56-2-9787446, email: rodrigov@uchile.cl

Abstract:

In this talk, I will show you our recent results about nonlinear spatial delocalization in disordered lattices [1]. 50 years ago, Anderson showed that a discrete system with a random distribution of on-site energies possesses a set of linear localized modes, denominated as Anderson States (AS's), resulting in the absence of diffusion across the lattice [2]. If we consider nonlinearity and discreteness we could also expect localization, because the interplay of these two effects is responsible for the appearance of nonlinear localized states called discrete solitons/breathers [3]. Therefore, an important question would be: What happen if discreteness, disorder and nonlinearity are present simultaneously? Many different approaches can be considered to answer that question. In particular, we study a nonlinear-induced delocalization of an initially very localized state in a disordered lattice. When a cubic nonlinearity is switched on, increasing from zero with a constant velocity, the effective frequency of the state crosses the linear band and the profile starts to resonate with linear modes inside this band. We demonstrate that a delocalization transition, due to resonances with AS's, is possible if the nonlinearity increases with a velocity below some threshold value, for a given strength of disorder. This means that, if we increase the nonlinearity too fast (non-adiabatically), the profile is not able to resonate with distant AS's and does not delocalize. On the contrary, if the nonlinearity increases slowly (adiabatically), the profile resonates with more states - close or not in space - increasing its size and consequently being delocalized.

References:

1. Rodrigo A. Vicencio and Sergej Flach, in preparation (2008).
2. P.W. Anderson, Phys. Rev. **109**, 1492 (1958).
3. D.K. Campbell, S. Flach, and Yu.S. Kivshar, Phys. Today **57** (1), 43 (2004).

Incoherent four-wave-mixing in highly nonlinear optical fiber

Yu Tian and Changxi Yang
Department of Precision Instruments, Tsinghua University, Beijing 100084 China
email: cxyang@tsinghua.edu.cn

Abstract:

We report on polarization independent wavelength conversion, ultrafast depolarization, and wavelength conversion of multi-channel signals by use of incoherent four-wave mixing in highly nonlinear optical fiber. Polarization independent wavelength conversion is realized with a non-polarized pump from a spectrum-sliced amplified spontaneous emission (ASE) light source. The signal is coherent and polarized from a tunable diode laser. When adjusting the signal polarization randomly using a polarization controller, the variation of wavelength conversion efficiency is less than 0.2 dB. The conversion efficiency is almost the same as that using a coherent laser pump under the same power. Simultaneous wavelength conversion of 22 incoherent signals over 16 nm with 0.8 nm interval is presented using the incoherent four-wave mixing configuration. Ultrafast depolarization of a polarized signal is achieved through incoherent FWM by using an ASE pump. Theoretical investigation shows that for a signal with any fixed polarization state, the generated idler wave is randomly polarized and the degree-of-polarization (DOP) of the idler measured is 0.33. The speed of the depolarizer is determined by the intrinsic nonlinear response of silica, which could be only a few femtoseconds. Vector theory of four-wave mixing is implemented to analyze the polarization independent nature of wavelength conversion, ultrafast depolarization, and wavelength conversion of multi-channel incoherent signals by using randomly polarized pumps in highly nonlinear optical fiber.

Engineering eective soliton-supporting nonlinearities in micro-structured potassium-lithium-tantalate-niobate

Eugenio DelRe, Alessandro Ciattoni, Carlo Rizza, and Andrea Marini
Department of Electrical and Information Engineering, University of L'Aquila, 67100 L'Aquila, Italy
Tel: +39 0862 434463, email: edelre@ing.univaq.it

Abstract:

In this talk we will discuss a method to achieve and design optical non- linear responses through a light-mediated spatial hybridization of dierent standard nonlinearities. The mechanism is based on the fact that optical propagation through a spatial composite of dierent nonlinear media is governed by an eective nonlinear response if the spatial scale of the sequence is much smaller than light diraction length. We apply our general approach to the signicant case of centro-symmetric photorefractive crystal biased by a periodically modulated external voltage to predict strictly bending-free miniaturized soliton propagation.

References:

1. A. Ciattoni, E. DelRe, C. Rizza, A. Marini, Tailoring wave nonlinearity through spatial composites, submitted to Physical Review Letters.

Noise induced coherence resonance of $[Ca^{2+}]$ in hepatocytes

Bao-hua Wang Qi-shao Lu and Shu-juan Lü
School of Science, Beijing University of Aeronautics-Astronautics,
Beijing 100083, China
E-mail: qishaolu@hotmail.com, wangbh06@126.com

Abstract:

Calcium ion (Ca^{2+}) almost exists in all kinds of human's cells, which participates in a great deal of cell functions such as secretion, reproduction and movement [1]. When a cell is stimulated, under the action of IP_3 and $[Ca^{2+}]$ itself together, $[Ca^{2+}]$ begins oscillation and hands on to the next one by the gap junction between cells. As a result, a high spatiotemporal organization of the concentration coming in an intact organization [2,3].

In this paper, it is found that the Ca^{2+} current in excitable media coupled by a number of hepatocytes can exhibit remarkable waves in the present of Gaussian white noise without any external signal. The phenomenon can be considered as coherence resonance(CR), which indicates that the excitable system shows an maximal regularity only by noise disposing of signals [4].

The model constructed by T.Höfer that describes the synchronization of calcium oscillation among hepatocytes [5]. When hepatocytes are excitable, noise cannot wake any cell vibrating in potent time so there is not any wave in the network. As the noise intensity increasing and exceeding a subthreshold value, the cells begin oscillating and the frequency varies with the fluctuation. At the same time the regular spatial $[Ca^{2+}]$ wave pattern appears, but it becomes abnormity when noise mounts beyond some value. It is evident that there exist some noise values making the spatiotemporal CR optimal. Changing the intercellular coupling strength, it is found that the pattern varies from small size spiral waves to circle waves under the same fluctuation. In the case of the large couple strength, the pattern turns into some solid high Ca^{2+} concentration region in the network. The emergence of this phenomenon is resulted from that small coupling blocks the calcium ion current transferring between cells and then the waves only develop in bounded regions such that no large size waves can not form.

In order to quantifying CR, the rate of $[Ca^{2+}]$ excitation is tracked with time varying. By calculating the autocorrelation function of the excitation rate, we obtain that CR rises with the fluctuation increasing and then descends with the noise further augmenting after attaining the critical value. In Addition, it is observed that smaller coupling between cells can make them more sensitive to puniness noise than the larger one can do.

References:

1. M.J. Berridge, M.D. Bootman and P.Lipp, Nature. 395, 645 (1998).
2. A.P. Thomas, G.S.J.Bird, G.Hajnoczky, L.D. Robb-Gaspers and J.W. Putney, FASEB J. 10, 1505 (1996).
3. M.H. Nathanson, A.D. Burgstahler, A. Mennone, M.B. Fallon, C.B.Gonzalez and J.C. Sáez, AM.J.Physiol. 450, 273 (1992)
4. A. Pikovsky and J. Kurths, Phys. Rev.Lett. 78, 775 (1997).
5. T. Höfer, Biophys. J. 77, 1244 (1999).

Effective Dynamics for Axonal Transport

Wen-An Yong
Zhou Pei-Yuan Center for Applied Mathematics
Tsinghua University, Beijing, 100084, China
Tel: 10-62792813, email: wayong@tsinghua.edu.cn

Abstract:

In nerve cells, protein synthesis occurs solely in the soma and many of these proteins are transported via the axons, which are typically extremely long and narrow, to the synaptic region. According to Blum and Reed [1], the transport processes can be described mathematically by initial-boundary value problems of partial differential equations of the form

$$(\partial_t + M\partial_x)u = C(u)/\epsilon,$$

where $u = (u_1, u_2, \cdots, \cdots)^T$ is a vector of chemical concentrations, M is a constant and diagonal matrix, $C(u)$ is a nonlinear mapping representing the interactions of the various chemical constituents, and ϵ is a positive parameter. When the chemistry is fast, the parameter ϵ is small.

In this talk, we consider the regime with fast chemistry and show that, in this regime, the dynamics of the transport processes can be reduced to a scalar advection-diffusion equation. This result indicates that the transport processes are essentially characterized with 2 quantities: advection and diffusion coefficients. Moreover, these two macroscopic and measurable quantities are related to the detailed reaction rate constants which are microscopic and difficult to be measured. This work can also be regarded as a sensitivity analysis of the dynamics with respect to the reaction rate constants.

References:

1. Michael C. Reed and Jacob J. Blum, Lectures on Mathematics in the Life Sciences, Vol. 24, 1–22 (1994).

Dynamics of A Delay Hematological Cell Model

Suqi Ma
Department of Mathematics, China Agricultural University, Beijing 100083, China
Tel: 010-81737303, email: cau-masuqi@163.com

Abstract:

The complex dynamics of a simple two-compartment model of the production and regulation of circulating blood neutrophil number are investigated. It is addressed that the possibility of the proliferative disorders may be due to both the factors of the apoptosis rate r_s of the haematopoietic stem cell and the cell cycle duration τ_s. Using the developed geometrical criterion for the Hopf bifurcation and transient behavior of the delay systems to this model, we separate the stable regime from unstable regime in the $r_s - \tau_s$ plane. The patterned periodic oscillation regimes with low periodicity in the number of circulating blood cells are illustrated in the $r_s - \tau_s$ plane. As the value of τ_s above its critical level is increased, the transitions from period-1 to period-5 by adding period bifurcation and eventually to complex strange attractors are found. As the value of r_s above its critical level is increased, however, the simple period-adding bifurcation phenomena occur which eventually lead to periodic solutions with large periodicity.

References:

1. M. C. Mackey, A. A. G. Aprikyan and D. C. Dale, The rate of apoptosis in post mitotic neutrophil precursors of normal and neutropenic humans, Cell Prolif., 36, 2003, 27-34.
2. Elnashaie S. S. E. H., Harraz H. M., Abashar M. E., Homoclinical chaos and the period-adding route to complex non-chaotic attractors in fluidized bed catalytic reactors, Chaos, Solitons and Fractals, 12, 2001, 1761-1792.
3. Suqi Ma, Zhaosheng Feng, Qishao Lu, A two-parameter geometrical criteria for delay differential equations, Discrete and continuous dynamical systems series B, 9(2), 2008, 397-413.

Spatial coherence resonance in Hodgkin-Huxley neuronal networks with the coupling delay

Qingyun Wang
State Key Laboratory for Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering, College of Engineering, Peking University, Beijing 100871, China
School of Statistics and Mathematics, Inner Mongolia Finance and Economics College, Huhhot 010051, China
E-mail: nmqingyun@163.com

Abstract:

Spatial coherence resonance in a spatially extended Hodgkin-Huxley (HH) neuronal networks with transmission delay is studied in the present paper. We focus on the ability of additive temporally and spatially uncorrelated Gaussian noise to extract a particular spatial frequency of excitatory waves in the neuronal networks with transmission delay. It is shown that there exists an intermediate noise level, spatial coherence resonance in neuronal networks is maximum irrespectively of the delay. Furthermore, it is found that the delay has a nontrivial effect on spatial coherence resonance. The peak values of signal to noise ratio (SNR) fluctuate substantially in dependence on delay and that no clear trend could be inferred with the computer resources available to us. The relative magnitudes of these peaks are found to be a sensitive function of time delay. Moreover, the impact of diffusive and small-world network topology on spatial coherence resonance has also been investigated. It is shown that as the diffusive topology of the medium is relaxed via the introduction of shortcut links introducing small-world properties amongst coupled HH neurons, decoherence of the spatial dynamics is increased, which implies the absence of spatial coherence resonance. Results obtained in this paper may provide a guidance for the understanding of the impact of noise and delay on a realistic ensemble of neurons.

Green Function of Nonlinear Degenerate Elliptic Operators and Its Application

Shenzhou Zheng
Department of Mathematics, Beijing Jiaotong University, Beijing 100044, China
Tel: 86-10-51685244, email: zhengsz@vip.sina.com

Abstract:

We are concerned with traveling waves and the Green function associated with the nonlinear degenerate elliptic operator of divergence form. We present several theorems on local estimates and comparisons with the p-Laplacian for the Green function under certain conditions in the sense of distribution. As an application of these estimates of the Green function, the regularity in Morrey spaces of the so-called inhomogeneous A-harmonic equation is derived.

Survival Data Analysis With Functional Predictors

Xiaohui Wang
Department of Mathematics, University of Texas-Pan American, Edinburg, 78539, USA
Tel: 956-381-3454, email: xhwang@utpa.edu

Abstract:

In this talk, we study time to event type of responses with sparse functional predictors. Our semi-parametric methodology utilizes spline bases function such that the function is represented by a spline coefficient vector with small dimension. Applications and various examples are also illustrated.

Traveling Wave Phenomena in Burgers-KdV Type Equations

Shumin Fei[a] and Zhaosheng Feng[b]
[a]School of Automation, Southeast University, Nanjing 210096, China
Tel: 025-83792720, email: smfei@seu.edu.cn
[b]Department of Mathematics, University of Texas-Pan American, Edinburg, 78539, USA

Abstract:

In this talk, we survey some recent advances in the study of travelling wave solutions to the BurgersKortewegde Vries type equation. Some comments are given on the existing results. A class of travelling solitary wave solutions in terms of elliptic functions with arbitrary velocity is obtained by means of the first-integral method as well as the method of compatible vector fields.

References:

1. R.S. Johnson, A Modern Introduction to the Mathematical Theory of Water Waves, Cambridge University Press, Cambridge, 1997.
2. Z. Feng, Nonlinearity, 20, 343-356 (2007).

Traveling Waves in Reaction-diffusion Systems

Zhaosheng Feng
Department of Mathematics, University of Texas-Pan American, Edinburg, 78539, USA
Tel: 956-292-7483, email: zsfeng@utpa.edu

Abstract:

There is the widespread existence of wave phenomena in physics, chemistry and biology [1, 2]. This clearly necessitates a study of traveling waves in depth and of the modeling and analysis involved. In this talk, we present a nonlinear reaction-diffusion system, which can be considered as a generalization of the Fisher equation. Applying the qualitative theory of planar dynamical systems, we show that under certain conditions, the nontrivial bounded traveling wave solution for the system is monotone or oscillatory. We then present a class of traveling wave solutions to the nonlinear reaction-diffusion system by using the Cole-Hopf transformation and the Lie symmetry method.

References:

1. J.D. Murray, Mathematical Biology, Springer–Verlag, New York, 2004.
2. N.F. Britton, Reaction–Diffusion Equations and Their Applications to Biology, Academic Press, New York, 1996.

On the interaction of non-uniform current and nonlinear waves

Shijun LIAO, Jun CHENG and Jie CANG
State Key Lab of Ocean Engineering School of Naval Architecture, Ocean and Civil Engineering
Shanghai Jiao Tong University, Shanghai 200030, China.
email: sjliao@sjtu.edu.cn

Abstract:

A train of periodic waves propagating on a non-uniform current which possesses an exponential distribution of vorticity is investigated by a relatively new analytic method for general nonlinear problems, namely the homotopy analysis method (HAM). Convergent series solutions are derived for not only small amplitude waves on a weak current but also large amplitude waves on a relatively strong current. The dispersion relationships in cases of different currents are compared each other. It is found that, for a co-flowing current, the vorticity tends to increase the wave phase speed and decrease the wave height. For an opposite current, the vorticity tends to decrease the wave phase speed but increase the wave height. This work also shows the potential of the homotopy analysis method for complicated problems with strong nonlinearity.

Dynamics of gravity-capillary waves in deep water

Paul Milewski, Benjamin Akers
Department of Mathematics, University of Wisconsin–Madison, Madison, WI 53706, U.S.A.
Tel: 608-262-3220, email: milewski@math.wisc.edu

Abstract:

When the effects of capillarity and gravity are balanced, solitary surface waves have been found numerically in several model equations and also in the full Euler equations [1,2,3]. This is true in two- and three-dimensions and for all depth regimes, from shallow water to infinite depth. In shallow water there are rigorous proofs of the existence of these waves and there have been several studies, based on model KdV and KP-type equations, of their dynamics [4,5]. In the more relevant infinite depth regime there is little known about the stability and dynamics of these waves. In two dimensions the stability was studied in [6] and the dynamics of the waves within a model equation in [7]. We present here some results from [7] and new results on the stability and dynamics of these waves in three-dimensions.

References:

1. Vanden-Broeck, J.-M., Dias, F., (1992) "Gravity-capillary solitary waves in water of infinite depth and related free-surface flows," *J. Fluid Mech.* **240**, 549–557.
2. Parau, E. I., Jean Mark Vanden-Broeck, and M. J. Cooker, (2005) "Nonlinear three-dimensional gravity-capillary solitary waves", *J. Fluid Mech.* **536**, 99–105
3. Milewski, P.A., (2005) "Three-Dimensional Localized Solitary Gravity-Capillary Waves", *Comm. Math. Sci.* **3**, 89–99.
4. Milewski, P. A., Vanden-Broeck, J.-M., (1999) "Time-dependent gravity-capillary flows past an obstacle", *Wave Motion* **29**, 63–79.
5. Berger, K., Milewski, P. A. (2000) "The generation and evolution of lump solitary waves in surface-tension-dominated flows", *SIAM J. Appl. Math.*, **61**, 731–750.
6. Calvo, D. C., Yang, T.-S., Akylas T. R. (2002), "Stability of steep gravity-capillary waves in deep water". *J. Fluid Mech.*,**452**, 123–143.
7. Akers, B. F., Milewski, P. A., (2007) "Model Equations for Gravity-Capillary Waves in Deep Water", to appear in *Studies in Appl. Math.*

Stability of lumps and wave collapse in water waves

T. R. Akylas and Yeunwoo Cho
Department of Mechanical Engineering, Massachusetts Institute of Technology,
Cambridge, MA 02139, U.S.A.
Tel: 617-2535356, email: trakylas@mit.edu

Abstract:

This study is concerned with the class of gravity-capillary lumps recently found on water of finite or infinite depth for Bond number, $B < \frac{1}{3}$. In the near-linear limit, these lumps resemble locally confined wavepackets and can be approximated in terms of a particular steady solution (ground state) of an elliptic system of the Benney-Roskes-Davey-Stewartson (BRDS) type. According to the BRDS equations, however, initial conditions above a certain threshold develop a singularity in finite time owing to nonlinear focusing; the ground state, in fact, being on the borderline between existence and wave collapse suggests that the newly discovered lumps are unstable. Based on the fifth-order KP equation, a model for gravity-capillary waves when B is close to $\frac{1}{3}$, it is pointed out that an exchange of stability occurs at a certain finite wave steepness, lumps being unstable (stable) below (above) this critical value. As a result, a small-amplitude lump, that is linearly unstable and would be prone to wave collapse according to the BRDS equations, depending on the perturbation, either decays into dispersive waves or evolves to an oscillatory state near a finite-amplitude stable lump. The effect of external forcing on the dynamics of lumps will also be discussed.

An optimal Boussinesq model for solitary wave-microstructure interaction

André Nachbin
IMPA, Est. D. Castorina, 110, Rio de Janeiro, RJ, 22460-320, Brazil.
Tel: 55-21-25295214, email: nachbin@impa.br

Abstract:

This is joint work with J. Garnier (Paris 7, France), R. Kraenkel (IFT, Brazil) and J.C. Muñoz (Universidad del Valle, Colombia). In previous works we have considered the propagation of long water waves when the bottom topography is highly disordered, including the case where it is modeled as a random microstructure. Apparent diffusion and waveform inversion, through a time-reversal procedure, were topics of interest [1-8]. A very brief overview will be presented. Through numerical multiple scattering experiments it was observed that, even in the presence of a highly disordered microstructure, Nwogu's [9] numerical strategy to optimize the depth parameter $Z_\alpha = -0.53$ still plays a crucial role [7]. Having in mind an effective KdV equation (i.e. solitary waves), in this talk we present a multiscale, weakly dispersive, weakly nonlinear asymptotic analysis in the presence of a rapidly varying topography. Remarkably the same depth value arises and its exact value is revealed by the theory [10].

References:

1. J.C. Muñoz Grajales and A. Nachbin, SIAM J. Appl. Math. 64, 977-1001 (2004).
2. J.P. Fouque, J. Garnier and A. Nachbin, SIAM J. Appl. Math. 64, 1810-1838 (2004).
3. J.P. Fouque, J. Garnier and A. Nachbin, Physica D 195, 324-346 (2004).
4. J.P. Fouque, J. Garnier, J.C. Muñoz Grajales and A. Nachbin, Phys. Rev. Lett. 92, 094502-1 (2004).
5. W. Artiles Roqueta and A. Nachbin, Phys. Rev. Lett. 93, 234501, (2004).
6. J. Garnier and A. Nachbin, Phys. Rev. Lett. 93, 154501 (2004)
7. J.C. Muñoz Grajales and A. Nachbin, IMA J. Appl. Math. 71, 600-633 (2006).
8. J. Garnier, J.C. Muñoz Grajales and A. Nachbin, SIAM. Multiscale Model. Simul. 6, 995 (2007).
9. O. Nwogu, J. Waterway, Port, Coastal, Ocean Eng., 119 (1993).
10. J. Garnier, R. Kraenkel and A. Nachbin, Phys. Rev E 76, 046311 (2007).

Transcritical flow over a step

Roger Grimshaw
Department of Mathematical Sciences, Loughborough University,
Loughborough, LE11 3TU, UK
Tel: 44-1509-223480, email: R.H.J.Grimshaw@lboro.ac.uk

Abstract:

The interaction of a flow with topography can generate large-amplitude, horizontally propagating solitary waves. Often these waves appear as a wave-train, or undular bore. In this talk we focus on the situation when the flow is critical, that is, the flow speed is close to that of a linear long wave mode, and the obstacle consists of a single step, either up or down. In the weakly nonlinear regime, this is modeled by the forced Korteweg de Vries equation [1,2], or the extended forced Korteweg-de Vries equation when a cubic nonlinear term is added.

References:

[1] R.H.J. Grimshaw and N.F. Smyth, Resonant flow of a stratified fluid over topography. *J. Fluid Mech.*, **169**, 429-464 (1986).

[2] R.H.J. Grimshaw, D.-H Zhang and K.W. Chow, Generation of solitary waves by transcritical flow over a step. *J. Fluid Mech.*, **587**, 235-354 (2007).

Short-wave instability of internal solitary waves and a regularized long wave model

Wooyoung Choi[(1)], Ricardo Barros[(1)] and Tae-Chnag Jo[(2)]

[(1)] Dept. of Mathematical Sciences, New Jersey Institute of Technology (NJIT)
Newark, NJ 07102, U.S.A.
wychoi@njit.edu, rbarros@njit.edu

[(2)] Dept. of Mathematics, Inha University, Korea
taechang@inha.ac.kr

Abstract:

The propagation of large amplitude internal solitary waves in a system of two constant density layers are studied using a strongly nonlinear long wave model. While steady solitary wave solutions of the model show excellent agreement with numerical solutions of the Euler equations and laboratory experiments, a local stability analysis reveals that the time-dependent inviscid model suffers from the Kelvin- Helmholtz instability due to a tangential velocity discontinuity across the interface. To suppress this undesirable short wave instability that is often absent in real experiments, an attempt is made to regularize the model by modifying the short wave behavior of the dispersion relation and introducing the effect of viscosity.

Three-dimensional evolution of large-amplitude internal waves in the Strait of Gibraltar

Vasiliy Vlasenko and Nataliya Stashchuk
School of Earth, Ocean and Environmental Sciences, University of Plymouth, Drake Circus, Plymouth PL4 8AA, UK
Tel: +44-1752-232419, email: vvlasenko@plymouth.ac.uk
Tel: +44-1752-233044, email: nstashchuk@plymouth.ac.uk

Abstract:

The modelling of large amplitude internal waves (LAIW) propagating in the Strait of Gibraltar is carried out using a fully-nonlinear nonhydrostatic numerical model. The vertical fluid stratification, parameters of the propagating waves and bottom topography were taken close to those observed in the strait. Amplitudes of the first mode LAIWs radiating from Camarinal Sill and propagating along the strait towards the Alboran Sea were taken in a wide range, from 10 to 90m (as observed in the studied area). The focus of the modelling efforts was on three-dimensional peculiarities of LAIWs evolution, viz. cross-strait variability, interaction with lateral boundaries (including wave breaking and water mixing), radiation of secondary waves from orographic features and interaction of secondary scattered internal waves.

It was found that propagating along the channel packets of LAIWs reveal remarkable three-dimensional behaviour. Due to the Coriolis effect and multiple reflections from the lateral boundaries, the largest leading LAIW loses its energy much faster than that in the attached wave tail. The latter, in turn, effectively adsorbs the scattered energy from the leading wave in the course of propagation and growths in amplitude. As a result of the energy transition, the initially rank-ordered wave packet loses its regular structure transforming into a non-rank-ordered wave structure. One-month observations collected in the eastern part of the Strait of Gibraltar confirm that the non-rank-ordered structure is a common feature in the wave packets emerging from the strait into the Alboran Sea.

Nonlinear internal waves forced by supercritical tidal plume

Nataliya Stashchuk and Vasiliy Vlasenko
SEOES, University of Plymouth, Drake Circus, Plymouth, PL4 8AA, UK
Tel: +44-1752-233044, email: nstashchuk@plymouth.ac.uk
Tel: +44-1752-232419, email: vvlasenko@plymouth.ac.uk

Abstract:

There are some important sources of oceanic baroclinic waves such as barotropic tides, shear instability, atmospheric impact, generation by local initial disturbances, exciting by oscillations of buoyancy fluxes, production of internal waves by stationary stratified currents and some others. A new mechanism of generation of internal waves (IWs) has been formulated recently by Nash and Moum (2005) who reported observation of packets of IWs in the area of the Columbia River plume and assumed that they are probably generated in the coarse of transition from supercritical to subcritical regime of evolution. The groups of these waves are regularly observed in the area by Synthetic Aperture Radars. We use a fully non-linear and non-hydrostatic numerical model to investigate the generation of IWs by the Columbia River plume. The vertical fluid stratification, parameters of the tides and river discharge and bottom topography were taken close to those observed near the Columbia River mouth. The model investigation shows that river water being less saline intrudes into the ocean like a surface layer supercritical jet, which then starts to decelerate due to radial spreading. The fast moving frontal part of the plume collides with "stagnant" oceanic water and sinks beneath the lens of the plume resulting in formation the head of gravity current. Such a structure contains arrested IWs which phase velocities are smaller than the propagation speed of the plume. When the velocity of the plume falls below the speed of the first baroclinic mode, the head of the gravity current separates from the plume and disintegrates into packet of IWs. The structure and dynamics of propagating river plume are studied in a wide range of parameters of three basic driving forces: buoyancy, river discharge and ebb tide.

References:

1. J.D. Nash and J.N. Moum, Nature, 437 (2005).

Exact solutions for a class of variable coefficient nonlinear Schrödinger equations

H.S. Chiu, C.K. Lam, K.W. Chow, and D.H. Zhang
Department of Mechanical Engineering, The University of Hong Kong, Pokfulam Road Hong Kong
Tel: 852-28592641, email: kwchow@hkusua.hku.hk

Abstract:

The nonlinear Schröodinger equation (NLS) is a widely applicable model for wave packets dynamics [1]. For wave propagation in an inhomogeneous medium, e.g. a fluid with variable depth or an optical fiber with spatially dependent dispersion, variable coefficient NLS equation (VCNLS) or variable coefficient Korteweg-de Vries models [2], are relevant. Here VCNLS with real dispersion in the presence of linear and/or nonlinear gain/loss is solved exactly. A modified Hirota bilinear method which has been used earlier in the literature to treat the complex Ginzburg Landau equation is employed [3]. An additional ingredient is the usage of time- or space-dependent wave numbers [4]. One-soliton solution of such VCNLS is obtained in an analytical form. One restriction of the present algorithm is that the coefficient of the second-order dispersion must be real. A simple example of an exponentially modulated dispersion profile is worked out in detail to illustrate the principle. The competition between the linear gain and nonlinear loss, and vice versa, is investigated. The stability of the solitary waves is tested in direct simulations. They appear to be very robust objects.

References:

1. C.C. Mei, The Applied Dynamics of Ocean Waves, Wiley, New York, (1983).
2. S.R. Clarke, and R.H.J. Grimshaw, Journal of Fluid Mechanics, 415, 323-345, (2000).
3. K. Nozaki and N. Bekki, Journal of the Physical Society of Japan, 53, 1581-1582, (1984).
4. C.C. Mak, K.W. Chow and K. Nakkeeran, Journal of the Physical Society of Japan, 72, 3070-3074, (2005).

Strongly inelastic soliton collisions in weakly perturbed integrable systems

Sergey V. Dmitriev
Institute for Metals Superplasticity Problems RAS, Khalturina St. 39, Ufa 450001, Russia
Email: dmitriev.sergey.v@gmail.com

Abstract:

In fully integrable systems solitons interact as quasi-particles, completely preserving their physical properties such as momentum and energy. Although realistic systems supporting solitary waves are typically non-integrable, and the result of interaction between coherent structures in such systems can be highly nontrivial.

Firstly, we overview the existing literature on soliton collisions in non-integrable systems. Various manifestations of inelasticity of soliton collisions such as radiation of small-amplitude phonons, excitation of soliton's internal modes, and radiationless energy exchange between colliding solitons are discussed in a systematic way. Such basic manifestations of inelasticity of soliton collisions can explain formation of short-lived multi-soliton quasiparticles and fractal soliton scattering.

Secondly, we report on several recent results related to soliton collisions. Particularly, we continue our studies on the weakly perturbed sine-Gordon equation. Here we discuss the peculiarities of the three-soliton collisions and present a simple three-particle model where the fractal soliton scattering and many other interesting effects observed in numerical simulations are explained without involving into consideration the soliton's internal modes. It is demonstrated that, in this case, the radiationless energy exchange between colliding solitons [1] is responsible for the observed effects.

References:

1. Kevrekidis P.G. and Dmitriev S.V., Soliton Collisions, in Encyclopedia of Nonlinear Science, Ed. Alwyn Scott (Routledge, New York, 2005), P. 148-150.

Interaction of discrete breathers in Fermi-Pasta-Ulam lattice

Bao-Feng Feng
Department of Mathematics, University of Texas - Pan American,
Edinburg, TX 78539-2999, USA
Tel: 956-381-2269, email: feng@utpa.edu

Abstract:

The inelastic nature and the resulting energy exchange in collisions of intrinsic localized modes, or discrete breathers, has been an open problem for quite a while. A detailed numerical investigation was conducted in Fermi-Pasta-Ulam lattice by Doi (see Phys. Rev. E 68 066608(2003)), which called for a theoretical explanation for the mechanism of energy exchange. In this talk, we first propose a regularized PDE model for discrete breathers in FPU lattice. Breather solutions are obtained via a variational approach, and their interactions are explored numerically. Then, based on the modified conservation law, a simplified ODE model is derived from the PDE model. The direct simulations for the ODE model achieve a good agreement with the numerical results in FPU lattice. Further, like the cases of coupled nonlinear Schrdinger (NLS) equation and phi-four equation, a possibility of fractal structures in breather-breather collisions is investigated.

Solitary wave interaction for a higher-order nonlinear Schrödinger equation

Tim Marchant and Sayed Hoseini[†]
School of Mathematics and Applied Statistics
s The University of Wollongong,
Wollongong, N.S.W., Australia.
email: tim@uow.edu.au
and
[†] Mathematics Department,
Vali-e-Asr University,
Rafsanjan, Iran.

Abstract:

Solitary wave interaction for a higher-order version of the nonlinear Schrödinger (NLS) equation is examined. An asymptotic transformation is used to transform a higher-order NLS equation to a higher-order member of the NLS integrable hierarchy, if an algebraic relationship between the higher-order coefficients is satisfied. The transformation is used to derive the higher-order two-soliton solutions. It is shown that the higher-order collision is asymptotically elastic and analytical expressions are found for the higher-order phase and coordinate shifts.

Numerical simulations of the interaction of two higher-order solitary waves are also performed. For an example which satisfies the algebraic relationship, the numerical results confirm that the collision is elastic. The numerical and theoretical predictions for the higher-order phase and coordinate shifts are also in strong agreement. For an example which does not satisfy the algebraic relationship, the numerical results show that the collision is inelastic; radiation is shed by the solitary wave collision. As the bed of radiation shed by the waves decays very slowly (like $t^{-\frac{1}{2}}$), it is computationally infeasible to calculate the final phase and coordinate shifts for the inelastic example.

Chaotic Scattering Via the Separatrix Map in Solitary Wave Interactions

Richard Haberman
Department of Mathematics, Southern Methodist University,
Dallas, TX 75275, USA
Tel: 214-768-2519, email: rhaberma@smu.edu

Abstract:

We present a new and complete analysis of the n-bounce resonance and chaotic scattering in solitary wave collisions. In these phenomena, the speed at which a wave exits a collision depends in a complicated fractal way on its input speed. We present a new asymptotic analysis of collective-coordinate ordinary differential equations (ODEs), reduced models that reproduce the dynamics of these systems. We reduce the ODEs to discrete-time iterated separatrix maps and obtain new quantitative results unraveling the fractal structure of the scattering behavior. These phenomena have been observed repeatedly in many solitary wave systems over 25 years. Joint work with Roy Goodman.

A Universal Map for Fractal Structures in Weak Solitary Wave Interactions

Yi Zhu
Zhou Pei-Yuan Center for Applied Mathematics, Tsinghua University,
Beijing 100084, China
Tel: +86-10-62796750, email: zhuyi03@mails.tsinghua.edu.cn

Abstract:

It is known that solitary wave interactions exhibit fractal scatterings in non-integrable systems. This phenomenon has been reported and studied in various physical systems. Most of these studies focused on strong interactions, i.e. collisions. In this talk, fractal scatterings in weak solitary wave interactions is analyzed for the generalized nonlinear Schrödinger equations with arbitrary nonlinearities. Using the Karpman-solov'ev perturbation method, the dynamics of these interactions is first reduced to a universal two-degree-of-freedom Hamiltonian ODE system. Then using asymptotic methods along the separatrix orbits of the reduced ODEs, a simple but universal second-order map is derived. Comparison with direct PDE and ODE computations confirms that the map completely captures all major aspects of fractal scatterings in the PDEs and their reduced ODEs, and the map's prediction is asymptotically accurate. The criterion for the existence of fractal scatterings and the scaling laws of these fractals are also derived analytically from the analysis of the map.

Joint work with Richard Haberman and Jianke Yang

References:
1. Y.Zhu and J. Yang, Phys. Rev. E 75, 036605 (2007).
2. Y.Zhu, R. Haberman and J. Yang, Phys. Rev. Lett. 100, 143901
3. Y.Zhu, R. Haberman and J. Yang, to appear in Physica D

Movable branch points in solutions of ODEs

Galina Filipuk
Department of Mathematical Sciences, Loughborough University,
Loughborough LE11 3TU, UK.
Tel: +44 1509 22 3483, email: G.Filipuk@lboro.ac.uk

Abstract:

Any nonlinear equation of the form $y'' = \sum_{n=0}^{N} a_n(z) y^n$ has a (generally branched) solution with leading order behaviour proportional to $(z - z_0)^{-2/(N-1)}$ about a point z_0, where the a_n's are analytic at z_0 and $a_N(z_0) \neq 0$. We consider the subclass of equations for which each possible leading order term of this form corresponds to a one-parameter family of solutions represented near z_0 by a Laurent series in fractional powers of $z - z_0$. For this class of equations we show that the only movable singularities that can be reached by analytic continuation along finite-length curves are of the algebraic type just described. This work generalises previous results of S. Shimomura. The only other possible kind of movable singularity that might occur would be an accumulation point of algebraic singularities that can only be reached by analytic continuation along infinitely long paths ending at a point in the finite plane. This behaviour cannot occur for constant coefficient equations in the class considered. However, an example of R. A. Smith shows that such singularities do occur in solutions of a simple autonomous second-order differential equation outside the subclass we consider here.

Malmquist-type theorems for second-order ODEs

by Edmund Y. M. Chiang*, R. G. Halburd and E. Lingham

*Hong Kong University of Science and Technology**

At the beginning of the twentieth century, Painlevé and Gambier analyzed the possible forms of f when the equation $y''(z) = f(z, y(z), y'(z))$, with $f(z, \zeta, \xi)$ rational in ζ and ξ and with z dependent coefficients, has *all* of its solutions to be single-valued around their movable singularities. They found fifty possible classes for f so that the equations possess the required property. Forty four of the f are known to be solvable in terms of known functions, and the remaining six forms give raise to what we now know to be the six Painlevé equations. On the other hand, Malmquist showed in 1931 that if the equation $y'(z) = R(z, y(z))$, where R is rational in y and with polynomial coefficients, admits a meromorphic solution then $R(z, y) = a_0(z) + a_1(z)y + a_2(z)y^2$, that is, the equation must be reduced to a Riccati equation. We extend Malmquist's and others' results that when f in $y''(z) = f(z, y, y')$ is suitably restricted then we can recover some of the six Painlevé equations by *only assuming* the equation to admit a meromorphic solution. The method of approach is based on a combination of Painlevé analysis and Nevanlinna's theory of value distribution of meromorphic functions. In fact, the Nevanlinna theory even allows us to consider coefficients of f to be transcendental meromorphic functions which have small Nevanlinna order when compared to that of the solution y. This is a joint project with R. G. Halburd and E. Lingham.

Vortices and Polynomials

Peter Clarkson
Institute of Mathematics, Statistics & Actuarial Science
University of Kent, Canterbury, CT2 7NF, UK
Tel: +44(0)1227 827781, email: P.A.Clarkson@kent.ac.uk

Abstract:

In this talk I will discus the relationship between vortex dynamics and properties of polynomials with roots at the vortex positions. Classical polynomials such as the Hermite and Laguerre polynomials have roots which describe vortex equilibria. Stationary vortex configurations with vortices of the same strength and positive or negative configurations are located at the roots of the *Adler-Moser polynomials*, which are associated with rational solutions of the Kortweg-de Vries equation. Aref [1] remarks that the relationship between vortex dynamics and the KdV equation is "quite unexpected and very beautiful".

I will discuss the polynomials associated with rational solutions of other soliton equations such as the Boussinesq and nonlinear equations, and the motion of the poles of rational solutions of these equations.

References:

1. H Aref, Fluid Dyn. Res.3920075–23; J. Math. Phys.482007065401.
2. P A Clarkson, Europ. J. Appl. Math.172006293–322.
3. P A Clarkson, "*Special polynomials associated with rational solutions of the Boussinesq equation*", preprint (2008).

Painlevé analysis for Ultra-discrete equations

Stéphane Lafortune
Department of Mathematics
College of Charleston
Robert Scott Small Building, Room 339
Charleston, SC, 29424, USA
email: lafortunes@cofc.edu

Abstract:

Ultra-discrete equations are generalized cellular automata in the sense that the dependent (and independent) variables take only integer values. We present a new method to identify integrable ultra-discrete equations which is the equivalent of the singularity confinement property for difference equations and the Painlevé property for differential equations. Using this criterion, we find new integrable ultra-discrete equations which include the ultra-discrete Painleve equations.
Co-Author: Nalini Joshi, Sydney University

References:

1 N. Joshi and S. Lafortune, *Integrable ultra-discrete equations and singularity analysis*, Nonlinearity **19**, 1295–1312 (2006).

2 N. Joshi and S. Lafortune, *How to detect integrability in Cellular Automata*, J. Phys. A **38**, L499-L504 (2005).

Analogues of Painlevé analysis

Asma Al-Ghassani
Department of Mathematical Sciences, Loughborough University,
Loughborough, Leicestershire, LE11 3TU, UK.
Tel: +44 1509 228203 email: a.al-ghassani@lboro.ac.uk

Abstract:

Several analogues of Painlevé analysis for discrete equations will be described in this talk. Each approach involves detecting equations for which some measure of complexity of solutions grows much slower than in the generic case. In particular, the degree growth on a function field and the growth of heights on number fields will be explored. The relationship to algebraic entropy and singularity confinement will be described. Several new results concerning dP_{IV} will be presented.

Recent progress in discrete and ultradiscrete integrable systems

Daisuke Takahashi
Department of Applied Mathematics, Waseda University, 3-4-1, Okubo, Shinjuku-ku, Tokyo 169-8555, Japan
Tel: +81-3-5286-3353, email: daisuket@waseda.jp

Abstract:

Discrete and ultradiscrete integrable systems are important objects in the integrable system theory. In this talk, an overview and some special topics on recent progress of the sytems are introduced as a preface of the minisymposium.

Recurrence equations on invariant varieties of periodic points

Satoru Saito[1] and Noriko Saitoh[2]

[1] Hakusan 4-19-10, Midori-ku, Yokohama 226-0006 Japan
Tel: 81-45-934-6191, email: saito@phys.metro-u.ac.jp

[2] Department of Applied Mathematics, Yokohama National University, Hodogaya-ku, Yokohama 240-8501, Japan
Tel: 81-45-339-4206, email: nsaitoh@ynu.ac.jp

Abstract:

A recurrence equation is a discrete integrable equation whose solutions are all periodic with a fixed period for arbitrary initial points. Some of them had been known for some years, while some others have been found recently. In this talk we would like to show the existence of infinitly many recurrence equations associated with invariant varieties of periodic points for some higher dimensional integrable maps [1].

References:

1. S. Saito and N. Saitoh, J. Phys. A: Math. Theor. 40, 12775-12787 (2007).

Quasideterminant solutions of noncommutative discrete systems

Jonathan J C Nimmo
Department of Mathematics, University of Glasgow, Glasgow, G12 8QW, UK
Tel: +44 141 330 4385, email: j.nimmo@maths.gla.ac.uk

Abstract:

Solutions of noncommutative integrable equations can be constructed using Darboux transformations and expressed in terms of quasideterminants which are close analogues of wronskian/casoratian or grammian determinants. This talk will review recent work on this topic particular in connection with discrete integrable systems.

References:

1. Gelfand, I. M. and Retakh, V. S. *Funktsional. Anal. i Prilozhen.*, **25**, 13 (1991).
2. Gelfand, I., Gelfand, S., Retakh, V. and Wilson, R. L. *Adv. Math.*, **193**, 56 (2005).
3. Gilson, C. R., Nimmo, J. J. C. and Ohta, Y. *J. Phys. A*, **40**, 3839 (2007).

Discretization of coupled nonlinear Schrödinger equations

Yasuhiro Ohta
Department of Mathematics, Kobe University, Rokko, Kobe 657-8501, Japan
Email: ohta@math.kobe-u.ac.jp

Abstract:

In this talk, several types of discrete coupled nonlinear Schrödinger equations will be discussed. The equations are transformed into the bilinear form and the soliton solutions of dark and bright types are given in terms of Casorati determinant.

References:

1. M. J. Ablowitz, B. Prinari and A. D. Trubatch, Discrete and Continuous Nonlinear Schrödinger Systems (Cambridge University Press, 2004).
2. R. Hirota, The Direct Method in Soliton Theory (Cambridge University Press, 2004).

Spectrum Resolution of Ultradiscrete Periodic Mappings

Masataka Iwao
Department of Mathematics, School of Fundamental Science and Engineering, Waseda University, 3-4-1 Okubo, Shinjuku-ku, Tokyo 169-8555, Japan
Tel: 011-81-3-5386-2484, email: m.iwao@aoni.waseda.jp

Abstract:

In this talk, some nonlinear mappings having periodic behavior, which is in the sense as generating sequences of identically periodic for all choices of initial data, will be analyzed in the way of spectrum resolution on matrix representation. Especially, we will investigate some concrete examples of piecewise-linear periodic mappings together with the rational mappings related through the ultradiscrete limit. We will see that the matrix representation of such mapping is still preserved under the ultradiscretization and the spectrum resolution gives us more detailed information than the period.

References:

1. M. Csörnyei and M.Laczkovich, Monatshefte für Mathematik 132, 215 (2001).
2. R.L. Graham, D.E. Knuth, and O. Patashnik, *Concrete Mathematics*, Addison-Wesley (1989).
3. R. Hirota and H. Yahagi, J. Phys. Soc. Jpn. 71, 2867 (2002).
4. M. Iwao, J. Phys. Soc. Jpn. 74, 3167 (2005).
5. M. Iwao and D. Takahashi, Glasgow Math. J. A 47, 87 (2005).
6. T. Tokihiro, D. Takahashi, J. Matsukidaira and J. Satsuma, Phys. Rev. Lett. 76, 3247 (1996).
7. T. Tsuda, J. Phys. A 37, 2721 (2004).
8. G.R.W. Quispel, J.A.G. Roberts and C.J. Thompson, Physica D 34, 183 (1989).

Discrete KP, "Box and Ball" systems and Yang-Baxter maps.

R. Willox[1], S. Kakei[2] and J.J.C. Nimmo[3]

[1] Graduate School of Mathematical Sciences, The University of Tokyo, 3-8-1 Komaba, Meguro-ku, 153-8914 Tokyo, Japan willox@ms.u-tokyo.ac.jp
[2] Department of Mathematics, Rikkyo University, Nishi-ikebukuro, Toshima-ku, 171-8501 Tokyo, Japan kakei@rkmath.rikkyo.ac.jp
[3] Department of Mathematics, University of Glasgow, Glasgow G128QQ, UK jjcn@maths.gla.ac.uk

Abstract:

In this talk we shall discuss certain reductions of the discrete KP hierarchy to 1+1 dimensional integrable lattice systems. The connection between these reduced systems and integrable cellular automata (so-called Box and Ball systems) will be commented upon and a general construction will be presented. The systems resulting from this construction provide novel examples of so-called Yang-Baxter maps (i.e., of set-theoretical solutions of the Yang-Baxter equation). If time permits, we shall also discuss the fundamental symmetries of these systems.

New expression of ultradiscrete solition solutions

Hidetomo Nagai
Graduate School of Fundamental Science and Engineering, Waseda University, Japan
Tel: 5286-3353, email: n1a9g8a1i@toki.waseda.jp

Abstract:

New type of multi-soliton solutions to ultradiscrete equations are proposed. The solutions are expressed by a form of ultradiscretized permanent which is defined by a signature-free determinant. The relation between the solutions and the known perturbed solutions is also given.

References:

1. D. Takahashi, R. Hirota, J. Phys. Soc. Japan 76 (2007) 104007–104012
2. J. Matsukidaira, J. Satsuma, D. Takahashi, T. Tokihiro, and M. Torii, Phys. Lett. A 225 (1997) 287–295.
3. M. Murata, S. Isojima, A. Nobe, and J. Satsuma, J. Phys. A: Math. Gen. 39 (2006) L27–L34.

New solutions to ultradiscrete soliton equations

Ryogo Hirota
Professor Emeritus,Waseda University,Tokyo Japan

Abstract:

We discuss new solutions to the ultradiscrete soliton equations such as the Box and Ball system, the Toda equation,etc. The solutions which we call ”negative-solitons” are unstable solutions to the discrete system but are stable in the ultradiscrete limit. Exact solutions describing collisions of solitons with negative-solitons are obtained.

Ultradiscretization and traffic models

Junta Matsukidaira
Department of Applied Mathematics and Informatics, Ryukoku University, Seta, Ohtsu, Shiga 520-2194, Japan
email: junta@rins.ryukoku.ac.jp

Abstract:

In the first part of this talk, we briefly review the ultradiscretization (UD) method and show how it works well in the analysis of traffic models including the ultradiscrete Burgers equation. After that, we propose the stochastic Fukui-Ishibashi model, which is the higher velocity extension of the stochastic ultradiscrete Burgers equation, and analyze the relation between density and flow of traffics using the two cluster approximation.

References:

1. T.Tokihiro, D.Takahashi, J.Matsukidaira, J.Satsuma, Phys. Rev. Lett. Vol.76, No.18, 3247-3250 (1996)
2. J.Matsukidaira, J.Satsuma, D.Takahashi, T.Tokihiro and M.Torii, Phys. Lett. A Vol.225, No.4-6, 287-295 (1997)
3. K. Nishinari and D. Takahashi, J. Phys. A. Vol 31, 5439-5430(1998)
4. J. Matsukidaira and K. Nishinari, Phys. Rev. Lett. Vol.90, No.8, 088701 (2003).
5. J. Matsukidaira and K. Nishinari, Int. J. Mod. Phys. C, Vol.14, No.4, 507-515 (2004).

Pulses and dynamics in mode-locked lasers

Mark J. Ablowitz
Department of Applied Mathematics, University of Colorado, Boulder, CO 80303 U.S.A.
Tel: 303-492-5502, email: mark.ablowitz@colorado.edu

Abstract:

Mode-locked lasers can generate ultra-short optical pulses, with durations ranging from hundreds of picoseconds down to a few femtoseconds. An important mathematical model includes the so-called "master" laser equation [cf. 1,2] which contain specific gain-loss terms. Recently a distributed modification of this system, with power saturation effects included, has been introduced and studied. Such power saturation equations are associated with Ti:sapphire and fiber lasers. The power stauration model admits localized pulse solutions and mode-locking under wide ranges of the parameters for both constant dispersion as well as dispersion managed systems [3,4]. Localized modes are found, which approximately satisfy the system without gain-loss terms, but for unique values of the propagation constant. Modes are found by mode-finding methods and direct numerical simulation indicate that they evolve from general initial data.

References:

1. H. A. Haus, IEEE J. Sel. Topics Quantum Electron. 6, 1173 (2000)
2. R. Paschotta , Haus Master Equation, Encyclopedia of Laser Physics and Technology, updated 2007-04-20
3. M.J. Ablowitz, T.P. Horikis amd B. Ilan, Preprint, U. CO., (2007)
4. T.P. Horikis and M.J. Ablowitz, Preprint, U. CO., (2007)

Spectrally breathing pulses from an Er-doped fiber laser

F Ömer Ilday, Bülent Öktem, Coskun Ülgüdür

Department of Physics and Institute of Material Science and Nanotechnology, Bilkent University, Ankara 06800, Turkey
Tel: 90-312-290-1076, email: ilday@bilkent.edu.tr

Abstract: We report order-of-magnitude spectral breathing in a similariton Er-fiber laser with an intracavity bandpass filter. This is to our knowledge the highest of any laser reported.

Although a wide range of pulse shaping schemes have been reported for ultrafast lasers, including soliton, stretched-pulse, similariton and all-normal dispersion regimes, in all of these cases spectral modification of the pulse remain minor. Here, we report a laser containing a narrow-band optical filter, with the spectral width of the pulse changing by as much as 7-8 times within a single roundtrip. This is the highest spectral variation reported to our knowledge for any laser cavity; this laser could be regarded as the "most nonlinear" laser constructed. Remarkably, the laser operation is very stable. Experimentally, the cavity consists of 3.5 m of single-mode fiber (SMF) with negative dispersion and 1 m of highly doped Er-doped fiber with positive dispersion. The net dispersion of the laser cavity is $0.01ps^2$. We measured bandwidths of 12 nm and 85 nm for the optical spectra at different points within the cavity. The laser generates chirped pulses, which are compressed externally to 110 fs.

We seek maximal understanding of this mode of operation using numerical simulations. The model is based on a nonlinearch Schrodinger equation, generalized to include higher-order dispersion, Raman scattering, gain with saturation and bandwidth filtering, saturable absorption and the bandpass filter. Experimentally observed behavior is reproduced well with the simulations. A simple picture of how the pulse evolves emerges: upon filtering, the pulses enter the SMF, where they are of too low power to regenerate the lost spectral width. The broadening takes place predominantly within the normal-GVD Er-fiber, exhibiting an extreme case of similariton propagation [1] maintained without pulse break-up owing to beginning the evolution with a particularly narrow spectrum. The pulse shape evolves into the parabolic shape, which is characteristic of similaritons. Note that, all previous observations of similaritons in a laser cavity exhibited mild changes in the spectral width.

In conclusion, we report a novel mode of operation of an ultrafast fiber laser, corresponding to extremely strong nonlinear shaping of the pulse, with spectral width breathing by a factor of 7. In analogy to the stretched-pulse laser, this laser could be regarded as a "stretched-spectrum" laser.

References:

1. M. E. Fermann, V. I. Kruglov, B. C. Thomsen, J. M. Dudley, and J. D. Harvey, Phys. Rev. Lett. 84, 6010 (2000).

Simulations of the Nonlinear Helmholtz equation: Arrest of Beam Collapse, Nonparaxial Solitons and Counter-Propagating Beams

Guy Baruch
School of Mathematical Sciences, Tel Aviv University Tel Aviv 69978, Israel
Tel: 972-36406210, email: guybar@tau.ac.il

Abstract:

The scalar nonlinear Helmholtz equation (NLH) models the propagation of intense laser beams in Kerr media such as water, silica and air. This semilinear elliptic equation requires boundary-conditions, which makes it hard to solve. Indeed, it remains unsolved in many configurations.

The standard parabolic approximation for the NLH is the nonlinear Schr ödinger equation (NLS). It is known to possess singular solutions in three dimensions, in which beams self-focus to zero radius. the wavelength, a condition which is violated near the singularity. The parabolic approximation, however, is valid only for beams whose radii is much larger than the wavelength, a condition which is violated near the singularity.

We therefore consider the question: do $3D$ nonlinear Helmholtz solutions exist, under conditions for which the NLS solutions become singular ? In other words, is the singularity removed in the elliptic model, or only in a more comprehensive model ?
This question has been open for forty years. In this work we develop a numerical method which produces such solutions in some cases, thereby showing that collapse is arrested in the scalar elliptic model, and not in a more comprehensive model.

In two dimensions, the NLS possesses soliton solutions. We consider this prediction for narrow beams, for which the NLS model is invalid. For this, we solve the $2D$ NLH for an input beam with a radius of half a wavelength, and observe stable propagation over forty diffraction lengths of solitons whose radius is half a wavelength.

As an additional benefit of solving the Helmholtz equation, we are able to calculate the waves scattered back from the domain, which may be important for applications.

Finally, we consider two counter-propagating solitons in $2D$ and confirm the validity of the coupled-NLS model.

Joint work with Gadi Fibich and Semyon Tsynkov.

Theory of singular vortex solutions of the nonlinear Schrödinger equation

Nir Gavish
School of Mathematical Sciences, Tel Aviv University, Tel Aviv 69978, Israel
Tel: 972-3-6406210, email: nirgvsh@tau.ac.il

Abstract:

In this talk, we consider *vortex solutions* of the NLS of the form

$$\psi(t, r, \theta) = A(t, r)e^{im\theta}, \quad m \in Z.$$

Vortices have been intensively studied, both theoretically and experimentally, in nonlinear optics and in Bose-Einstein condensates (BEC), see [1] for a review. However, almost all of this research effort has been on non-collapsing vortices. In fact, only three papers have considered collapsing vortices [2-4]. Therefore, there is a huge gap between the available theory on non-vortex and vortex singular NLS solutions.

In this study, we present a systematic study of singular vortex solutions of the critical and supercritical NLS. In particular, we extend the available theory for singular NLS solutions and show that, in some cases (e.g., critical power for collapse), stronger rigorous results can be obtained for collapsing vortices. In addition, we find that some of the non-rigorous results for singular non-vortex solutions that were derived using asymptotic analysis and numerical simulations (e.g., stability of blowup profiles) surprisingly change for vortex solutions. Finally, we extend the results of [5] to find new families of supercritical vortex blowup solutions.

Joint work with Gadi Fibich.

References:

1. A. S. Desyatnikov, Yu. S. Kivhsar, and L. Torner, *Progress In Optics, E. Wolf*, 47,219–319, 2005.
2. V.I. Kruglov, YU.A. Logvin, and V.M. Volkov, *J. Modern Optics*, 39,2277–2291, 1992.
3. V.I. Kruglov, V.M. Volkov, R.A. Vlasov, and V.V. Drits., *J.Phys. A: Math. Gen.*, 21,4381–4395, 1988.
4. L.T. Vuong, T.D. Grow, A. Ishaaya, A.L. Gaeta, G.W. Hooft, E.R. Eliel, and G. Fibich,*Phys. Rev. Lett.*, 96,133901, 2006.
5. G. Fibich and N. Gavish and X.P. Wang, Physica D,231,55–86,2007.

Discrete solitons and singularity formation in reconfigurable photonic lattices

Zhigang Chen
San Francisco State University, USA and Nankai University, China
Email: zchen@stars.sfsu.edu

Abstract:

In this talk, I will present our recent work on the formation of discrete solitons and vortex gap solitons in optically induced periodic structures. I will discuss the experimental demonstrations of localized soliton states in the semi-infinite gap, in the photonic bandgap, as well as embedded inside a photonic band of the reconfigurable optical photonic structures. Our work may be relevant for studies of similar phenomena in other periodic systems.

Instability of Two-dimensional Lattice Solitons Near Edges of Bloch Bands in Periodic Media

Zuoqiang Shi
Zhou Pei-Yuan Center for Applied Mathematics, Tsinghua University, Beijing, China
Email: shizq03@mails.tsinghua.edu.cn

Abstract:

Low-amplitude two-dimensional lattice solitons near edges of Bloch bands in periodic media were determined analytically in a recent study [1]. In this talk, linear-stability properties of such lattice solitons near band edges are analyzed by asymptotic methods. It is shown that under focusing nonlinearity, 2D lattice solitons centered at a potential minimum are unstable if the slope of the power curve at the band edge is negative. However, under defocusing nonlinearity, such lattice solitons are unstable if the slope of the power curve at the band edge is positive, which is the opposite of the focusing case. The asymptotic formula for the unstable eigenvalue is also derived analytically. This analytical formula is checked against numerically computed eigenvalues, and excellent agreement is obtained.

Joint work with J. Wang and J. Yang.

References:

1. Z. Shi and J. Yang, Phys. Rev. E 75, 056602 (2007).

Stability and Instability of lattice solitons

Gadi Fibich
School of Mathematical Sciences, Tel Aviv University, Tel Aviv 69978, Israel
Tel: +972-3-640-7164, email: fibich@tau.ac.il

Abstract:

Typically, studies of stability of lattice solitons consider this problem as a yes/no (i.e. stable/unstable) question. In this talk, I will first present a qualitative approach that characterizes the dynamics of unstable solitons. Then, I will present a quantitative approach that quantifies the magnitude of stability or instability. In particular, under this approach the transition from stability to instability is continuous.

Joint work with Yonatan Sivan and Boaz Ilan

Observation of Peakon Profile in Nonlocal Nonlinear Collapse

Can Sun, Christopher Barsi, and Jason W. Fleischer
Department of Electrical Engineering, Princeton University, Princeton, NJ 08544 USA
Email: jasonf@princeton.edu

Abstract:

Wave collapse in a nonlinear medium occurs when the self-focusing effect overcomes diffraction. For example, in the 2D Kerr case, a beam of sufficient power will collapse in a radially-symmetric, self-similar fashion, maintaining a Townes profile [1] as it focuses, regardless of its initial shape [2]. More complex responses, such as nonlocality, are known to prevent catastrophic collapse [3,4]. Here, we report the surprising result that for a critical input power, nonlocal collapse can lead to a transverse profile with a peaked origin and exponentially-decaying tails. Unlike previous peakon/cuspon solutions, this profile is stable but not steady-state. For longer propagation distances, the beam undergoes focusing/defocusing cycles. These observations are confirmed experimentally by demonstrating peakon profiles and beam oscillations in self-focusing lead glass.

Theoretically, we consider spatial beam systems which are governed by the nonlinear Schrodinger (NLS) equation. For nonlocal media, such as the thermal materials considered here, this index change is distributed and depends on heat diffusion, boundary conditions, and the thermo-optic effect [5]. Beam profiles can take on a variety of forms, including shock waves [6] and breathing solutions that result from the multimode nature of the extended, induced potential [7].

It is well-known that the NLS equation can be mapped into Euler-like fluid equations using a polar (Madelung) transformation. For defocusing nonlinearity, the resulting system is a pair of shallow-water equations with an additional dispersive term, known as "quantum" pressure, due to diffraction. In this case, profiles with discontinuous first derivatives have been predicted theoretically, e.g. for surface waves [8] or 1D behavior in plasma waves [9], For focusing nonlinearity, the nonlinear pressure term is negative (attractive). For moderate nonlocality, we show that collapse continues until the quantum pressure becomes dominant. (This is only one of the diffraction terms, but it is the most significant due to the extra derivatives). Beyond this point, the beam defocuses, eventually reducing the intensity gradient and allowing self-focusing to recover. Details and issues of the collapse/bounce cycles will be discussed.

References:

1 R. Y. Chiao, E. Garmire and C. H. Townes, PRL 13, 479 (1964).
2 K.D. Moll, A. L. Gaeta and G. Fibich, PRL 90, 203902 (2003).
3 V, Perez-Garcia, V. Konotop and J. Garcia-Ripoll, PRE 62, 4300 (2000).
4 O. Bang, W. Krolikowski, J. Wyller and J. J. Rasussen, PRE 66, 046619 (2002).
5 C. Rotschild, O. Cohen, O. Manela and M. Segev, PRL 95, 213904 (2005)
6 C. Barsi, W. Wan, C. Sun, and J.W. Fleischer, Opt. Lett. 32, 2930 (2007).
7 I. Kaminer, C. Rotschild, O. Manela and M. Segev, Opt. Lett. 32, 3209 (2007).
8 R. Camassa, D. Holm, PRL 71 1661-1664 (1993)
9 A.G. Litvak and A.M. Sergeev, Pis'ma Zh. Eksp. Teor. Fiz. 27, 549 (1978) [JETP Lett. 27, 517 (1978)].

Band Gap Formation, Fundamental Solitons and Vortices in 2D Nonlinear Lattices

M. J. Ablowitz[a], N. Antar[b], İ. Bakırtaş[b] and B. Ilan[c]

[a]Department of Applied Mathematics, University of Colorado, Colorado 80309-0526,

[b]Department of Mathematics, Istanbul Technical University, Maslak 34469, Turkey,

[c]School of Natural Sciences, University of Colorado at Merced, Merced, CA 95344

Tel: +90 0212 285 33 25, email: antarn@itu.edu.tr

Abstract:

In this talk, we present the band gap formation and the existence of localized nonlinear modes of the underlying nonlinear Schrödinger equation (NLS) with two dimensional irregular lattices, possessing dislocations, defects and quasicrystal structures.We next investigate vortex solitons on quasicrytal lattices. We use a spectral fixed-point numerical scheme to obtain the nonlinear localized modes and vortex solitons. By using direct computational simulations, stability properties of fundamental and vortex solitons are investigated.

References:

1. N.K. Efremidis et al.,Phys. Rev. Lett. 91, 213906 (2003).
2. D. Neshev, Y.S.Kivshar, H. Martin, and Z. Chen, Opt. Lett., 29,486 (2004).
3. E.Schonbrun and R.Piestun, Opt. Eng., 45,028001 (2006)
4. M. J. Ablowitz and Z. H. Musslimani, Opt. Lett., 30,2140 (2005)
5. M. J. Ablowitz, B. Ilan, E. Schorbrun, and R. Piestun, Theo. and Math. Phys., 151 (3) (2007)
6. H. Sakaguchi and B. Malomed, Phys. Review E, 74, 026601 (2006)

NLS stability theory for solitons in inhomogenous media

Boaz Ilan
School of Natural Sciences, University of California, Merced, CA 95344, USA
Tel: +1-209-228-4178, Email: bilan@ucmerced.edu

Abstract:

Orbital stability theory for the localized bound states (solitons) of the Nonlinear Schrödinger (NLS) equation is generalized to arbitrary potentials in any dimension and with a general-type nonlinear term. The key assumptions on the spectrum of the linearized NLS equation are identified and their physical meaning elucidated. This is corroborated by direct numerical simulations of multidimensional solitons with periodic and other lattice-type potentials.

Joint work with Michael Weinstein

Stochastic Modeling, Analysis and Simulation for Geophysical Flows

Jinqiao Duan
Department of Applied Mathematics, Illinois Institute of Technology, Chicago, IL 60616

Abstract: Geophysical flows largely define the environment in which we live. A challenge for better understanding geophysical phenomena is due to multiscale variability, nonlinearity, and uncertainty in geophysical flows.

The need to take stochastic effects into account for modeling geophysical flows has become widely recognized. Stochastic partial differential equations arise naturally as geophysical flow models. The speaker will discuss dynamical issues such as stochastic parameterization, homogenization, synchronization, random boundary conditions, and quantifying the impact of noise, in the context of modeling and simulating geophysical flows.

A three-dimensional wave-activity relation for pseudomomentum

Shouting Gao
Laboratory of Cloud-Precipitation Physics and Severe Storms (LACS),
Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China

Abstract: A three-dimensional, non-hydrostatic local wave-activity relation for pseudomomentum is derived from the non-hydrostatic primitive equations in Cartesian Coordinates by using the extension of Momentum-Casimir method. The stationary and zonally symmetric basic states are chosen and a Casimir function, which is the single-valued function of potential vorticity and potential temperature, is introduced in the derivation.

The wave-activity density and wave-activity flux of the local wave-activity relation for pseudomomentum are expressed entirely in terms of Eulerian quantities so that they are easily calculated with atmospheric data and do not require the knowledge of particle placements. Constructed in the ageostrophic and non-hydrostatic dynamical framework, the local wave-activity relation for pseudomomentum is applicable to diagnosing the evolution and propagation of meso-scale weather systems.

Dynamics of eddy-driven low-frequency dipole modes for NAO

De-hai Luo

Ocean Univ Qingdao, Coll Phys & Environm Oceanog, Phys Oceanog Lab, Qingdao 266003, PR China

Abstract:A simple theoretical model is proposed to clarify how synoptic-scale waves drive the life cycle of the North Atlantic Oscillation (NAO) with a period of nearly two weeks. This model is able to elucidate what determines the phase of the NAO and an analytical solution is presented to indicate a high similarity between the dynamical processes of the NAO and zonal index, which is not derived analytically in previous theoretical studies. It is suggested theoretically that the NAO is indeed a nonlinear initial-value problem, which is forced by both preexisting planetary-scale and synoptic-scale waves. The eddy forcing arising from the preexisting synoptic-scale waves is shown to be crucial for the growth and decay of the NAO, but the preexisting low-over-high (high-over-low) dipole planetary-scale wave must be required to match the preexisting positive-over-negative (negative-over-positive) dipole eddy forcing so as to excite a positive (negative) phase NAO event. The positive and negative feedbacks of the preexisting dipole eddy forcing depending upon the background westerly wind seem to dominate the life cycle of the NAO and its life period. An important finding in the theoretical model is that negative-phase NAO events could be excited repeatedly after the first event has decayed, but for the positive phase downstream isolated dipole blocks could be produced after the first event has decayed. This is supported by observed cases of the NAO events presented in this paper. In addition, a statistical study of the relationship between the phase of the NAO and blocking activity over Europe in terms of the seasonal mean NAO index shows that blocking events over Europe are more frequent and long-lived for strong positive-phase NAO years, indicating that the positive-phase NAO favors the occurrence of European blocking events. Through calculating the scatter diagrams of the strearnfunction (psi(P) or psi(T)) versus potential vorticity (PV) (q(P) or q(T)), where psi(P) and psi(T) are the planetary-scale strearnfunction and total streamfunetion, respectively, and using a weakly nonlinear NAO model proposed in Part I of this paper, it is suggested that negative- and positive-phase NAO events may approximately correspond to free modes even though driven by synoptic-scale eddies.

The Characteristics Analyses of Nonlinear Mixed Wave in Tropical Cyclone

LU Han-Cheng[1], ZHONG Wei[1] , Da-Lin ZHANG[2,3]

[1] Institute of Meteorology, PLA University of Science and Technology, Nanjing 211101

[2] Department of Atmospheric and Oceanic Science, University of Maryland, College Park, Maryland 20742, U.S.A.

[3] Institute of Atmospheric Science, Nanjing University of Information Science and Technology, Nanjing 210044

Abstract: Based on the discussion of the balanced, quasi-balanced and non-balanced dynamic characteristics in hurricanes life cycle, a potential vorticity (PV)-ω inversion system is applied to analyze the TC Bonnie(1998), which exhibited the characteristics of asymmetry and steady intensity state. The results show that the basic vortex can be well described by the balanced flows derived through the PV and nonlinear balanced (NLB) equations, whereas by adding the quasi-balanced ω equation, the quasi-balanced flows can reflect the structure of convergent inflow in planetary boundary layer (PBL), divergent outflow in upper troposphere, strong slantwise updraft in eyewall and subsidence motion in hurricane eye. Because the quasi-balanced flows can describe the organized deep moist convective systems with long life span and the correlative convergence motion have the same magnitude with vortex motion, the quasi-balanced flows have the co-existence feature of divergence and vorticity motions. It is found, however, that the inflow area in boundary layer and outflow area in upper-level contain significant unbalanced supergradient flows. The nonlinear mixed-wave dynamics are investigated using the potential vorticity conservation principle. Furthermore, the nonlinear mixed vortex Rossby inertial-gravity waves in tropical storms are explored in the context of quasi-balanced dynamicsshowing the co-existence of rotational and divergent flows associated with these waves.

Acknowledgments. This work was supported by the National Nature Science Foundation of China (Grant No. 40575022), the National Key Basic Research and Development Project of China (Grant No. 2004CB418301) and the NSF of the U.S.A Grant ATM-0342363.

Soliton study on atmospheric and oceanic circulation dynamics

Fei Huang

Dept. of Marine Meteorology, Ocean University of China, Qingdao, China, 266100

Abstract: The analytical solution obtained from a variable coefficient Korteweg de Vries equation can be successfully used to explain the evolution of atmospheric dipole-type blocking (DB) life cycle. Analytical diagnoses show that basic westerly has great influence on evolution of DB during its life cycle. Weak westerly is necessary for blocking development and the high/low centers of DB decrease and move southward and westward, the horizontal scale enlarges and meridional span reduces, as well as the blocking life period shortens accompanying with the enhanced westerly. Shear of the basic westerly also plays important role on evolution of blocking. Weak cyclonic shear is preferable for the development of blocking and when the cyclonic shear increases, the intensity of blocking decreases and the life period of DB becomes shorter. Very weak anticyclonic shear is also favorable for DB formation, but a critical threshold shear exists, beyond which polar anticyclone develops dramatically instead of envelope Rossby soliton forming the high anticyclone center of DB. Inside the critical threshold of anticyclonic shear, the intensity of DB increases and the life period of DB lengthens when the anticyclonic shear increases. Time-dependent basic westerly (TBW) in the life cycle of DB has some modulates on blocking life period and intensity due to the behavior of basic westerly. The effect of TBW is mostly owing to altering the DB life period and leading to the asymmetry of the DB life cycle evolution.

The (2+1)-dimensional nonlinear 1.5-layer and 2.5-layer ocean circulation model without external wind stress forcing is analyzed by using the classical Lie group approach. Some Lie point symmetries and their corresponding two-dimensional reduction equations are obtained. The theoretical solution for describing wind-driven subtropical gyre circulation was obtained. The solutions considered stratification effect with different layers reduced gravity and layer thickness in the two and a half layer model, which is more general and a modified solution of that by Young and Rhains (1982). The Influences of stratification and thermocline structure on subtropical gyre and western boundary currents are analyzed based on the theoretical solutions.

Polarization and wave-activity relation of inertial-gravity waves

Lingkun Ran

Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029, China

Abstract: The dynamical properties of three-dimensional non-hydrostatic and ageostrophic inertial-gravity waves, such as polarization and wave-activity relation, are examined. It is shown that inertial-gravity wave processes elliptical polarization between any two velocity components. The phase difference between disturbed pressure and vertical velocity is . The phase difference between disturbed potential temperature and disturbed vertical velocity, the same as that between disturbed horizontal divergence and disturbed vertical vorticity, is $\frac{(2n+1)\pi}{2}$.

The multi-scale method is employed to constructed wave-activity relation for inertial-gravity waves. The result shows that the y-direction variation of Coriolis parameter results in nonconservation of wave activity. The development of inertial-gravity wave energy depends on the combination of y-direction variation of Coriolis parameter and vertical variation of basic-stated stratification parameter.

Hurricanes and Tsunamis

S. Y. Lou[1,2], Man Jia[2] Xiao-yan Tang[2] and Fei Huang[3]
[1]Department of Physics, Shanghai Jiao Tong University, Shanghai, 200030, China
[2]Department of Physics, Ningbo University, Ningbo, 315211, China
[3] Dept. of Marine Meteorology, Ocean University of China, Qingdao, China, 266100

Abstract: The Euler equation (EE) is one of the basic equations in many physical fields such as fluids, plasmas, condensed matter, astrophysics, oceanic and atmospheric dynamics. A new symmetry group theorem of the (2+1)-dimensional EE is obtained via a simple direct method which is thus utilized to find exact analytical vortex and circumfluence solutions. A weak Darboux transformation theorem of the (2+1)-dimensional EE can be obtained for arbitrary spectral parameter from the general symmetry group theorem. The possible application of the vortex and circumfluence solutions to the tropical cyclones, especially the Hurricane Katrina 2005, is demonstrated. Tsunami waves usually are described by the exact solutions of the (1+1)-dimensional dispersionless water wave equations (Science 309, 2045 (2005); Phys. Rev. Lett. 97 (2006) 148501), the soliton solutions of the integrable models like KdV and KP equations (arXiv:nlin.PS/ 0702052 (2007)), In this report, we discuss also the possibility to describe the Tsunami waves by folded soliton or folded solitary waves (X. Y. Tang and S. Y. Lou, J. Math. Phys. 44 (2003) 4000).

Nonlinear optics with new light: optics with spin waves

Burkard Hillebrands
Fachbereich Physik, Kaiserslautern University of Technology, Kaiserslautern 67653, Germany
Tel: 49-631-2054228, email: hilleb@physik.uni-kl.de

Abstract:

Spin waves are the excitations of a magnetically ordered solid state material. In magnetic films they have unusual dispersion properties compared to electromagnetic waves and allow the realization of novel experiments in the field of linear and nonlinear propagation and manipulation. The experimental technique of space-, time- and phase resolved Brillouin light scattering spectroscopy is a powerful, versatile tool to investigate spin-wave properties. I will give an introduction into the basic phenomena of spin wave propagation in magnetic films and into fundamental experiments. In the field of nonlinear spin-wave optics I will discuss new experiments on solitons and so-called bullets, the two-dimensional counterparts of solitons, which both are eigen modes of the respective systems. In analogy to well-known experiments in the field of photon optics I will present experiments addressing caustic effects appearing upon collapse of spin-wave modes, spin waves travelling in magnetic stripes on the sub-micrometer scale, radiation of spin waves and damping mechanisms at nanocontacts, as well as experiments addressing parametric adiabatic and non-adiabatic amplification and phase conjugation.

Nonlinear interactions of spin waves in metallic Permalloy films

G. A Melkov[1], Yu.V. Koblyanskiy[1], A. N. Slavin[2], and V. S. Tiberkevich[2]

[1]Faculty of Radiophysics, Kiev National Taras Shevchenko University, Kiev, Ukraine
Tel: 380-44-2660600, email: melkov@mail.univ.kiev.ua

[2]Department of Physics, Oakland University, Rochester, Michigan, U.S.A.

Abstract:

Nonlinear interactions dipolar, dipole-exchange, and purely exchange spin waves between themselves and with high-frequency electromagnetic pumping were studied experimentally in Permalloy (NiFe) metallic films. The main characteristic feature of spin waves in metallic films (in contrast with similar waves in dielectric ferrite films) is their large dissipation, which results in rather small values of mean free path not exceeding several tens of microns. The method of parallel pumping of spin wave instability by external electromagnetic signal was used to compensate dissipation and increase mean free path of excited spin waves. It was found that at large (supercritical) values of the pumping power electromagnetic radiation at the half pumping frequency can be registered in a Permalloy film, and two independent mechanisms of such radiation were suggested and experimentally confirmed. It was, also, discovered that at a relatively small (subcritical) values of the pumping power nonlinear interaction of two contra-propagating pulses takes place in a Permalloy film. As a result of this nonlinear interaction the dipolar spin waves excited initially at half pumping frequency in the film are converted into exchange-dominated short spin waves, which were amplified by pumping and were stored in the film for several microseconds (i.e. for the times three orders of magnitude larger than the natural spin wave lifetimes in Permalloy). A wide spectrum of other interesting nonlinear phenomena including spin wave front reversal, delayed electromagnetic radiation from the film after the pumping was switched off, and spin wave auto-modulation processes were found and investigated.

Excitation of a subcritically unstable nonlinear spin-wave mode by spin-polarized current in a magnetic nano-structure

Andrei N. Slavin and Vasil S. Tiberkevich
Department of Physics, Oakland University, Rochester, Michigan 48309, U.S.A.
Tel: 001-248-3703401, email: slavin@oakland.edu

Abstract:

It was shown analytically and by micromagnetic simulation that a current-driven in-plane magnetized magnetic nano-contact, besides a quasi-linear propagating ("Slonczewski") spin wave mode, can also support a subcritically unstable nonlinear self-localized spin wave "bullet" mode that exists in a much wider range of bias currents. The frequency of the "bullet" mode lies below the spectrum of linear propagating spin waves, which makes this mode evanescent and determines its spatial localization. The threshold current for the excitation of the self-localized "bullet" is substantially lower than for the linear propagating mode, but finite-amplitude initial perturbations of magnetization are necessary to generate a "bullet" in our numerical simulations, where thermal fluctuations are neglected. Consequently, in these simulations the hysteretic switching between the propagating and localized spin wave modes is found when the bias current is varied.

Dynamics of magnetic topological solitons in restricted geometry

K. Yu. Guslienko and S.-K. Kim
Research Center for Spin Dynamics and Spin-Wave Devices and Nanospintronics Laboratory,
Seoul National University, Seoul 151-744, South Korea
Tel: 82-2-8804059, email: kguslienko@snu.ac.kr

Abstract:

FFundamental understanding of magnetization dynamics in a system with a reduced dimensionality is essential in the future advancements of nanomagnetism. Thus, it becomes important to explore the magnetization oscillations in sub-micron magnetic particles such as dots. For mesoscopic and nanoscale sizes of dots, non-uniform magnetization distributions with zero remanence ("vortex" states) are often observed at equilibrium. The vortex states can be stable within a wide range of dot sizes from a few tens of nm up to a few tens of microns. Vortex related phenomena offer insight into magnetization dynamics on a fundamental level, and also govern magnetization reversal.
In this talk, we will present a review of calculations and measurements of the low-frequency (sub-GHz range) and high-frequency (above 1 GHz) vortex dynamic excitations in soft magnetic dots [1]. Particular cases of circular, elliptic dots and tri-layer circular dots will be considered. The vortex eigenfrequencies are quantized due to the geometrical confinement and depend on the dot size and geometry. The observed low-frequency oscillations of the vortex core position are described as a gyrotropic motion of the magnetic vortices around the equilibrium positions induced by a gyroforce and a dynamic magnetostatic restoring force. The role of the vortex topological charges in the magnetization dynamics will be discussed. The topological charges such as vorticity and polarization determining the vortex gyrovectors are especially important for the case of dynamics of coupled vortices [2]. The high-frequency modes are described as radial or azimuthal spin waves over the vortex ground state with strong pinning on the dot circumference. The new mechanism of the vortex core dynamical reversal [3] via the creation/annihilation of vortex-antivortex pairs or vortex escape from the dot induced by in-plane variable magnetic field will be discussed. The origin of the vortex core polarization reversal will be considered. We also will present a soliton model of the domain walls in magnetic nanostripes, which explains their dynamics by the motions of a limited number of magnetic topological solitons such as vortex and antivortex. We predict the reduced wall velocity and critical field in the low-field regime, and increased wall-oscillation frequency in nanostripes, compared to the 1D Walker solution for bulk magnets.

References:

1. K.Y. Guslienko et al., Phys. Rev. B, 71, 144407(2005); Phys. Rev. Lett., 96, 067205 (2006); Phys. Rev. Lett. 100, 027203 (2008).
2. K. Buchanan et al., Nature Phys. 1, 172 (2005).
3. S.K. Choi et al., Phys. Rev. Lett., 98, 087205 (2007).

Bistable magnetization profiles in magnetic thin films

Ramaz Khomeriki[1], Jerome Leon[2], and Miguelle Manna[2]
[1]Department of Exact and Natural Sciences, Tbilisi State University, 128 Tbilisi, Georgia
Tel: 995-32-915681, email: khomeriki@hotmail.com
[2]Laboratoire de Physique Thorique et Astroparticules, CNRS-UMR 5207, Universit Montpellier II, 34095 Montpellier, France

Abstract:

An yttrium-iron-garnet magnetic thin film, driven by means of two antennas, produces a standing wave in-plane magnetization. When the driving frequency is chosen close to the upper edge of the passing band, it is shown by rigorous asymptotic multiple scale analysis that the governing model for the generated backward volume waves is the defocusing nonlinear Schr?dinger equation. Although being driven inside the allowed band, the nonlinear response of the system is discovered to allow for the formation of bistable magnetization profiles for a film width comparable with the wavelength of the driving radiation.

Bi-dimensional soliton propagation in a ferromagnetic slab in the short wave approximation

Herve Leblond[1] and Miguel Manna[2]
[1]Laboratoire POMA, CNRS FRE 2988, Universit d' Angers, 2 Bd Lavoisier, 49045 Angers Cedex 01, France
Tel: 33-241735386, email: herve.leblond@univ-angers.fr
[2]Laboratoire de Physique Thorique et Astroparticules, CNRS-UMR 5207, Universit Montpellier II, 34095 Montpellier, France

Abstract:

We investigate the propagation of volume polaritons in a ferromagnetic slab magnetized to saturation, by means of a short wave-type approximation. The set of equation which governs the wave behavior is shown to be (2+1)-dimensional generalization of the sine-Gordon equation. It depends on the direction of the applied magnetic field H_0: a term accounting for a transverse drift appears when H_0 is in the plane of the slab [1], which is absent if it is perpendicular to it. The derivation shows that neither the damping nor the demagnetizing field have an appreciable effect on the wave propagation. Line-solitons can propagate, the conditions for their transverse stability is established. The threshold value of the soliton parameter below which the soliton is stable depends on the direction of the applied field H_0. The unstable line soliton evolves into stable two-dimensional localized solitary waves, or lumps. These are described by means of both a numerical and a variational approach. Interactions between the lumps are also mentioned.

References:

1. H. Leblond and M. Manna, Phys. Rev. Lett. 99, 064102 (2007).

Dynamics of two coupled short spin wave envelope solitons

Yongshan Cheng[1], Mingzhong Wu[2], and Rongzhou Gong[3]

[1]Department of Physics, Hubei Normal University, Huangshi 435002, China
Tel: 01186-714-6571339, email: yong_shan@163.com

[2]Department of Physics, Colorado State University, Fort Collins, CO 80526, U.S.A.

[3]Department of Electronic Science and Technology, Huazhong University of Science and Technology, Wuhan 430074, China

Abstract:

The interaction and stability of two coupled spin wave envelope bright solitons in magnetic thin films were investigated analytically by means of the variational approach. The work was done within the framework of the higher-order nonlinear Schr?dinger equation which includes both the nonlinear dispersion terms and the third-order linear dispersion term. Evolution of the solitons is analyzed in the adiabatic approximation. The effects of the parameters of medium on the solitons are discussed.

Integrable models from coalgebras

Angel Ballesteros
Department of Physics, Facultad de Ciencias, University of Burgos, Burgos 09001, Spain
Tel: +34947258804, email: angelb@ubu.es

Abstract:

The construction of completely integrable systems from symplectic realizations of Poisson coalgebras with Casimir elements is reviewed [1]. Several examples of Hamiltonians with either undeformed or 'quantum' coalgebra symmetry are given, and this symmetry is shown to provide a universal prescription for the choice of the dynamical variables [2,3]. The quasi-maximal Liouville superintegrability of any coalgebra model is discussed and the essential role of symplectic realizations in this context is emphasized [4]. As a concrete application, sl(2)-coalgebra kinetic energy Hamiltonians describing geodesic motions are shown to generate "dynamically" a large family of N-dimensional curved spaces. Moreover, certain potentials on these ND spaces can be also introduced by making use of the coalgebra symmetry, in such a way that the integrability properties of the full system are preserved [5,6]. In particular the first example of a maximally superintegrable system on a ND space of non-constant curvature is presented [7].

References:

1. A. Ballesteros and O. Ragnisco, *J. Phys. A: Math. Gen.* **31**, 3791 (1998).
2. A. Ballesteros and F.J. Herranz, *J. Phys. A: Math. Gen.* **32**, 8851 (1999).
3. A. Ballesteros and O. Ragnisco, *J. Phys. A: Math. Gen.* **36**, 10505 (2003).
4. A. Ballesteros, F.J. Herranz, F. Musso and O. Ragnisco, in *Superintegrability in Classical and Quantum Systems* CRM Proceedings and Lecture Notes **37** ed P Tempesta *et al* (Providence, RI: AMS), 1 (2004).
5. A. Ballesteros, F.J. Herranz, *J. Phys. A: Math. Theor.* 40, F51 (2007).
6. A. Ballesteros, A. Enciso, F.J. Herranz and O. Ragnisco, *Phys. Lett.* **B652**, 376 (2007).
7. A. Ballesteros, A. Enciso, F.J. Herranz and O. Ragnisco, *Physica D*, to appear, (2008). arXiv:math-ph/0612080

The Multiscale method for discrete systems and Integrability

R. Hernández Heredero[†], D. Levi[‡] , M. Petrera[*‡] and C. Scimiterna[*‡]

[†]Depto. de Matemática Aplicada, Universidad Politécnica de Madrid,
EUIT de Telecomunicación, Ctra de Valencia Km. 7 28031, Madrid, Spain
e-mail: rafahh@euitt.upm.es

[‡]Dipartimento di Ingegneria Elettronica, Universit'a degli Studi Roma Tre
and Sezione INFN, Roma Tre,
Via della Vasca Navale 84, 00146 Roma, Italy
e-mail: levi@fis.uniroma3.it

[*]Dipartimento di Fisica,, Universit'a degli Studi Roma Tre
and Sezione INFN, Roma Tre,
Via della Vasca Navale 84, 00146 Roma, Italy
e-mail: petrera@fis.uniroma3.it
e-mail: scimiterna@fis.uniroma3.it

Abstract:

In the continuous, differential setting, multiscale techniques [1] have proved to be important tools for finding approximate solutions to many physical problems by reducing a given dispersive nonlinear PDE to a simpler equation. Furthermore, multiscale expansions are structurally strong and can be applied to determine integrability of a given system, like in [2].

In this talk we will present a precise formalism of multiscale perturbative expansion of discrete systems [3]. The formalism gives a system of "reduced" differential or difference equations corresponding to the different scales of the system. We will discuss how the properties of the reduced system are related to properties of the original system, specifically integrability, using a symmetry interpretation of the reduced equations.

References:

1. T. Taniuti, Reductive Perturbation Theory and Far Fields of Wave Equations, *Supplement of the Progress of Theoretical Physics* 55 (1974) 1–35.
2. F. Calogero, Why are Certain Nonlinear PDEs Both Widely Applicable and Integrable?, in *What is Integrability?* (Ed. V. E. Zakharov), Springer series in Nonlinear Dynamics 50 (1994) 1–61.
3. R. Hernández Heredero, D. Levi , M. Petrera and C. Scimiterna, *Jour. Phys. A* 40 (2007) F831F840.

Finite genus solution to the lattice sine-Gordon equation

Cewen CAO

Department of Mathematics, Zhengzhou University, Zhengzhou, China
email: cwcao@zzu.edu.cn

The purpose of the present paper is to investigate the lattice sine-Gordon equation (LSG):

$$\partial_x(\Psi_{n+1}-\Psi_n)+\beta(\sin\Psi_{n+1}+\sin\Psi_n)=0,$$

with the Lax pair $E\chi=U\chi,\ \partial_x\chi=V\chi$, where:

$$U=\frac{1}{\sqrt{\lambda^2-\beta^2}}\begin{pmatrix}\lambda-\beta\cos\Psi & -\beta\sin\Psi\\ -\beta\sin\Psi & \lambda+\beta\cos\Psi\end{pmatrix},\quad V=\begin{pmatrix}\frac{\lambda}{2} & w\\ -w & -\frac{\lambda}{2}\end{pmatrix},$$

with $w=-\frac{1}{2}(\Psi_x+\beta\sin\Psi)$. The discrete part $E\chi=U\chi$ comes from the Darboux transformation for the usual sine-Gordon equation. Under certain Bargmann's constraints:

$$(\cos\Psi,\sin\Psi)=f_S(p,q),\quad w=f_+(p,q),$$

the linear spectral problems in the Lax pair are nonlinearized into an integrable symplectic map S and an integrable Hamiltonian system (H_1), respectively, sharing the same integrals $\{H_1,\cdots,H_N\}$, involutive with each other and functionally independent in an open set of the phase space. The compatible solution $(p(n,x),q(n,x))$ of S and (H_1) is mapped by the Bargmann's map f_S into a special solution of LSG.

In the Jacobi variety $J(\Gamma)$ of the associated hyperelliptic curve $\Gamma:\xi^2=R(\lambda)$, both the discrete S-flow and the continuous H_k-flow are straightened out with constant evolution speeds:

$$\phi(n+1)-\phi(n)=\Omega_S,\quad \frac{d\phi}{d\tau_k}=\Omega_{2k-1},\quad (\mathrm{mod}\mathcal{J}),$$

where $\mathcal{J}$ is the periodic lattice of Γ. The discrete angular speed is expressed with the help of the basis $\omega=(\omega_1,\cdots,\omega_g)$ of normalized holomorphic differentials of Γ:

$$\Omega_S=\int_{\infty_1}^{P(\beta)}\omega+\int_{\infty_2}^{P(-\beta)}\omega,\quad (\mathrm{mod}\mathcal{J})$$

where $P(\beta),P(-\beta)$ is the branch points of Γ. The continuous angular speed comes from the coefficients in the expansion near ∞_l, $\omega=(-1)^{l-1}\sum_{j=1}^{\infty}\Omega_j z^{j-1}dz$, $l=1,2$. These yield the Abel-Jacobi solution to the LSG model:

$$\phi(x,n)=x\Omega_1+n\Omega_S+\phi_0,\quad (\mathrm{mod}\mathcal{J}),$$

which is transformed by the Jacobi inversion into the finite genus solution:

$$\sin\Psi(x,n)=\frac{1}{\beta}e^{\gamma x}(E-1)w(0,n)\frac{\Theta(x\Omega_1+n\Omega_S+\delta_{\infty_2})}{\Theta(x\Omega_1+n\Omega_S+\delta_{\infty_1})}\frac{\Theta(n\Omega_S+\delta_{\infty_1})}{\Theta(n\Omega_S+\delta_{\infty_2})}.$$

Discrete integrable systems and geometry

Adam Doliwa
Department of Mathematics and Computer Science, University of Warmia and Mazury
ul. Żołnierska 14, 10-561 Olsztyn, Poland
Tel: 48-89-5246071, email: doliwa@matman.uwm.edu.pl

Abstract:

The theory of quadrilateral lattices [1,2] provides unifying scheme for studying various distinguished integrable discrete equations, like the discrete Darboux system, the discrete BKP and CKP equations [3,4]. Starting from the Geometric Integrablity Scheme some selected new results on the quadrilateral lattice, its basic reductions and transformations will be revieved with emphasizing the fundamental role of incidence geometry configurations. Recent results on the noncommutative lattices [5] will be also discussed within this framework.

References:

1. Doliwa A. and Santini P. M., *Phys. Lett. A* **233** (1997) 365.
2. Doliwa A., Mañas M. and Santini P. M., *J. Math. Phys.* **41** (2000) 944.
3. Doliwa A., *J. Geom. Phys.* **57** (2007) 1171.
4. Doliwa A., `arXiv:0710.5820 [nlin.SI]`.
5. Doliwa A., `arXiv:0801.0512 [nlin.SI]`.

Solutions of the (2+1)-dimensional KP, SK and KK equations generated by gauge transformations from non-zero seeds

Jingsong He and Xiaodong Li
Department of Mathematics,University of Science and Technology of China, Hefei, 230026 Anhui, P.R.China
Tel: 0551-3600217, email: jshe@ustc.edu.cn

Abstract: In this talk, several new solutions of (2 + 1)-dimensional Kadomtsev-Petviashvili(KP), Kaup-Kuperschmidt(KK) and Sawada-Kotera(SK) equations are given from non-zero seeds by using gauge transformations. For each of the preceding equations, a Galilean type transformation between these solutions u_2 and the previously known solutions u'_2 generated from zero seed is given. We present several explicit formulas of the single-soliton solutions for u_2 and u'_2, and further point out the two main differences of them under the same value of parameters, i.e., height and location of peak line, which are demonstrated visibly in three figures.

Integrable generalizations of Volterra system

Oleg Bogoyavlenskij
Department of Mathematics, Queen's University, Kingston, K7L 3N6, Canada
Tel: 613 533 23 99, email: bogoyavl@mast.queensu.ca

Abstract:

In this talk, we present a new general construction of integrable Hamiltonian systems that generalize the well-known Volterra system [1,2]:

$$\dot{a}_i = a_i(a_{i+1} - a_{i-1}).$$

The Volterra system describes interaction between only neighboring particles a_i and $a_{i\pm1}$. The constructed new integrable systems include more complex interaction between distant particles. For example, we prove integrability of the system

$$\dot{a}_i = a_i(a_{i+1} - a_{i-1}), \quad i \leq \ell - 2,$$

$$\dot{a}_{\ell-1} = a_{\ell-1}(a_\ell + a_{\ell+1} - a_{\ell-2}), \quad \dot{a}_\ell = a_\ell(a_{\ell+1} + a_{\ell+2} - a_{\ell-1}),$$

$$\dot{a}_{\ell+1} = a_{\ell+1}(a_{\ell+2} + a_{\ell+3} - a_\ell - a_{\ell-1}),$$

$$\dot{a}_{\ell+2} = a_{\ell+2}(a_{\ell+3} - a_{\ell+1} - a_\ell), \quad \dot{a}_{\ell+3} = a_{\ell+3}(a_{\ell+4} - a_{\ell+2} - a_{\ell+1}),$$

$$\dot{a}_j = a_j(a_{j+1} - a_{j-1}), \quad j \geq \ell + 4.$$

The system contains Volterra system on the invariant submanifold $a_{\ell+1} = 0$.

The new method of construction gives many integrable systems with different kinds of interaction between particles. All the constructed systems are Hamiltonian and possess equivalent Lax representations. Exact solutions of soliton type are derived. The introduced integrable systems contain both infinite dimensional lattices and finite dimensional periodic systems.

New integrable infinite dimensional lattices are obtained that describe local interaction between several standard Volterra lattices. Their continual limits give integro-differential equations that possess an infinite number of first integrals.

References:

1. M. Kac and P. van Moerbeke, Adv. Math. 16, 160-169 (1975).
2. J. Moser, Adv. Math. 16, 197-220, (1975).
3. O.I. Bogoyavlenskij, Breaking Solitons, Moscow, Nauka, (1991).

A Hierarchy of Integrable Multi-Component Equations and Bi-Hamiltonian Structures

Lixin Tian and Lu Sun
Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, China
Tel: 0086-511-88791467, email: tianlx@UJS.EDU.CN

Abstract:

A new isospectral problems which has two potentials are introduced.Its associated multi-component equations and expanding multi-component models are derived. Bi-Hamiltonian structures are obtained through the trace identity. Finite-dimensional Hamiltonian systems are given by using the nonlinearization method. It is shown that the equations are completely integrable in the Liouville sense. Further, solutions of equations are possessed.

References:

1. G.Z.Tu,J.Math.Phys.40(1989)330.
2. W.X.Ma,R.G.Zhou,Chinese Ann.Math.Ser.B 23(2002)373.
3. W.X.Ma,R.G.Zhou,Phys.Lett.A,276(2000)73.
4. Takhtajan La,Phys.Lett.A.64(1977)235.
5. M.Jimbo, M. Kruskal, T.Mivva,Phys.Lett.A.92(1982)59.
6. S.Y.Lou, Phys.Lett.A.176(1993)96.
7. S.Q.Liu,Y.J.Zhang,J.Geom.Phys.54(2005)427.
8. M.Chen,S.Q.Liu,Y.J.Zhang,Lett.Math.Phys.75(2006)1.
9. C.W.Cao,Sci.Chin.Ser.A.33(1990)528.
10. Z.J.Qiao,J.Phys.A.26(1993)4407.
11. W.X.Ma,Phys.Lett.A.367(2007)473.

A new multi-component CKP hierarchy and solution

Yunbo Zeng
Department of Mathematical Sciences, Tsinghua University, Beijing 100084, China
Tel: 86-10-62787874, email: yzeng@math.tsinghua.edu.cn

Abstract:

We construct a new multi-component CKP hierarchy based on the eigenfunction symmetry constraint. It provides an effective way to find new two types of CKP equation with self-consistent sources (CKPSCS) and their Lax representations. Also it admits reductions to constrained CKP hierarchy and to a (1+1)-dimensional soliton hierarchy with self-consistent source, which gives rise to new two types of Kaup-Kuperschmidt equation with self-consistent sources (KKSCS) and of bi-directional Kaup-Kuperschmidt equation with self-consistent sources. By using the solutions of the CKP equation and KK equation and their corresponding eigenfunctions, N-soliton solution for the CKPSCS and KKSCS are constructed by means of the method of variation of constant, respectively.

References:

1. H. X. Wu, Y. B. Zeng and T. Y. Fan, A new multi-component CKP hierarchy and solutions, to be submitted.
2. X. J. Liu, Y. B. Zeng and R. L. Lin, A new multi-component KP hierarchy, to be submitted.

On quasi-triviality and integrability of a class of evolutionary PDEs

Youjin Zhang
Department of Mathematical Sciences, Tsinghua University, Beijing 100084, P.R. China
Tel: +86-10-62771380, email: youjin@mail.tsinghua.edu.cn

Abstract:

For certain class of perturbations of the hyperbolic equation $u_t = f(u)u_x$, we prove the existence of change of coordinates, called quasi-Miura transformations, that reduce these perturbed equations to the unperturbed ones. We also prove that if in addition the perturbed equations possess Hamiltonian structures of certain type, the same quasi-Miura transformations also reduce the Hamiltonian structures to their leading terms. As an application, we propose criteria for the existence of Hamiltonian structures and integrability for a class of scalar evolutionary PDEs.

References:

1. S.-Q. Liu, Y. Zhang, On quasi-triviality and integrability of a class of scalar evolutionary PDEs. J. Geom. Phys. **57** (2006), 101–119.
2. S.-Q. Liu, C.-Z. Wu, Y. Zhang, On properties of Hamiltonian structures for a class of evolutionary PDEs, arXiv:0711.2599.

Doubly periodic, simply periodic and rational solitary waves of partially integrable nonlinear PDEs

Robert Conte[1,2] and Micheline Musette[3]

1. Service de physique de l'état condensé (URA 2464),
CEA Saclay, F–91191 Gif-sur-Yvette Cedex, France.
Robert.Conte@cea.fr

2. Centre de mathématiques et de leurs applications (UMR 8536),
École normale supérieure de Cachan,
61, avenue du Président Wilson, F–94235 Cachan Cedex, France.

3. Dienst Theoretische Natuurkunde, Vrije Universiteit Brussel,
Pleinlaan 2, B–1050 Brussels, Belgium
MMusette@vub.ac.be

Abstract:

The solitary waves of autonomous nonlinear partial differential equations (PDE) present a high interest in physics. When the PDE is integrable, there exist powerful methods to find all these solitary waves explicitly, we will discard this case and concentrate on the partially integrable situation. The question is then, given an N-th order autonomous nonlinear ODE, to find a M-parameter particular solution in closed form, with M maximal and $M < N$.

There exist many methods to look for such solutions: "projective Riccati method", "tanh-method", "exponential method", "Jacobi expansion method", "new ...", etc. The common default to all these "truncation methods" is to only provide some solutions, not all of them. Example: any differential consequence of $2u'^2+(24u^2-3)u'+72u^4-17u^2+1=0$.

In fact, there exists a method [2,4] able to find all those solutions which are elliptic (doubly periodic), trigonometric (simply periodic) or rational, making obsolete all the truncation methods. Its only ingredients are: (1) two beautiful classical results of Briot and Bouquet [1], (ii) the local representation of the unknown solution as a Laurent series. The result is an algorithm having a *linear* complexity. A recent achievement is a new elliptic solitary wave [3] for the quintic complex Ginzburg-Landau equation.

References:

1. C. Briot et J.-C. Bouquet, *Théorie des fonctions elliptiques*, 1ère édition (Mallet-Bachelier, Paris, 1859); 2ième édition (Gauthier-Villars, Paris, 1875).
2. M. Musette and R. Conte, Analytic solitary waves of nonintegrable equations, Physica D **181** (2003) 70–79. http://arXiv.org/abs/nlin.PS/0302051
3. S.Yu. Vernov, Elliptic solutions of the quintic complex one-dimensional Ginzburg-Landau equation, J. Phys. A **40** 9833–9844 (2007). http://arXiv.org/abs/nlin.PS/0602060
4. R. Conte and M. Musette, *The Painlevé handbook*, about xx+256 pages (Springer, Berlin, 2008).

Reflection and reconnection interactions of resonant dromions

Xiao-yan Tang
Department of Physics, Shanghai Jiao Tong University, Shanghai, 200240, P. R. China
Tel: 021-54742326, email: xytang@sjtu.edu.cn

Abstract:

The interactions between dromions have still not yet been understood very well. In this talk, we will report two new types of dromion interactions, dromion reflection and reconnection, for the generalized Broer-Kaup system as a representative example. Dromion reflection is when a dromion behaves like a ball reflected by a wall. Essentially, it is a dromion interacting with an invisible ghost wall caused by a ghost line soliton. While dromion reconnection interaction is the scenario where the bounded dromions may be opened and then reconnected during the interaction. These two novel interaction phenomena are quite universal in high dimensions.

References:

1. K. Nishinari, K. Abe and J. Satsuma, J. Phys. Soc. Jpn. 62, 2021-2029 (1993).
2. K. Nishinari, K. Abe and J. Satsuma, Theor. Math. Phys. 99, 745-753 (1994).
3. Tang X. Y., Li J. M. and Lou S. Y., Phys. Scr. 75, 201-205 (2007).

Lump solutions in a 2+1-dim. NLS equation

Javier Villarroel and Julia Prada
Facultad de Ciencias (Matemáticas), Universidad de Salamanca.
Plaza Merced s/n, E-37008, Salamanca, Spain,
Tel: 34-923-294458 , email: javier@usal.es

Abstract:

We use the inverse scattering method to derive classes of solutions of the system

$$m_y + |u|^2 = 0, \quad iu_t + u_{xx} + 2um_x = 0 \tag{1}$$

that satisfy the boundary conditions

$$\lim_{r\to\infty} |u|(x,y) = 1, \quad \lim_{r\to\infty} u_x/u = 0, \quad m(x, y = 0, t) = 0 \tag{2}$$

where $r^2 \equiv x^2 + y^2$ and $u = u(x, y, t)$. This system is interesting since reduction to the manifold $x = y$ yields the Nonlinear Schrödinger equation (NLS) whereby Eq. (1) generalizes in a natural way NLS to 2+1 dimensions. Under different boundary conditions this equation was proposed first by Shulman [4] and embedded by Fokas [3] into a general class of integrable equations. See also [2] regarding the radiation spectrum .

We show that eqs. (1,2) posses classes of smooth, rationally "decaying" lump configurations with generically nontrivial dynamics and scattering. The physical behavior of these solutions is reminiscent to the non standard lumps for KPI and DSII, described in [1,5,6]. Solutions can be constructed which scatter in any desired angle. We find that the associated eigenfunctions are meromorphic with higher order pole singularities. Characterization of the discrete spectrum of this operator involves giving the pole multiplicity and, presumably, in the spirit of the ideas of [5] and [6], an associated integer winding number.

References:

1. Ablowitz M. J and Villarroel J., *Physical Review Letters* **78** (1997) 570.
2. Boiti M, J Leon and F Pempinelli, *Inverse Problems* **3**, 371, (1987).
3. Fokas A, *Inv. Prob.* **10**, L19-L22, (1994).
4. Shulman E. 1984, *Theor. Math. Phys*, **56** (1984) 720
5. Villarroel J. and Ablowitz M.J., Comm.Math.Phys. **207** (1999) 1.
6. Villarroel J. and Ablowitz M.J., *SIAM J. Math Anal.* **34**(2003), 1252-1277

Darboux transformations for some two dimensional affine Toda equations

Zi-Xiang Zhou
School of Mathematical Sciences, Fudan University, Shanghai 200433, China
E-mail: zxzhou@fudan.edu.cn

Abstract:

The Lax pairs for the two dimensional $A_{2l}^{(2)}$, $C_l^{(1)}$ and $D_{l+1}^{(2)}$ Toda equations have a reality symmetry, a cyclic symmetry and a unitary symmetry. The Darboux transformations for these systems are considered. These Darboux transformations should be of high degree. Exact solutions are written down by computing the Darboux transformations explicitly.

References:

1. Z. X. Zhou, J. Math. Phys. 46, 033515 (2005).
2. Z. X. Zhou, J. Phys. A39, 5727-5737 (2006).
3. Z. X. Zhou, Darboux transformations of lower degree for two dimensional $C_l^{(1)}$ and $D_{l+1}^{(2)}$ Toda equations, to appear, 2007.

Nonlinear PML

Saul Abarbanel
School of Mathematical Sciences, Tel Aviv University, Ramat Aviv, Tel Aviv 69978, Israel
Tel: +972-3-6408951 email: saula@post.tau.ac.il

Abstract:

In this paper we present a new set of nonlinear PML equations for the multi-dimensional Maxwell's equations and show that they are strongly well posed and temporally stable. The equations are given in vector form and so may be directly written in any ortogonal coordinate system. Another virtue of this set is that there is no need for nonphysical variables in the absorbing layer, thus greatly reducing the number of equations that one solves numerically (6 instead of 12 in the three dimensional case).

Numerical examples demonstrate the validity and efficacy of the new method.

Numerical Solution of the Nonlinear Helmholtz Equation

Guy Baruch
School of Mathematical Sciences, Tel Aviv University Tel Aviv 69978, Israel
Tel: 972-36406210, email: guybar@tau.ac.il

Abstract:

The nonlinear Helmholtz equation (NLH) models the propagation of intense laser light in Kerr dielectrics such as water, silica or air. The key physical phenomenon of interest is self-focusing of the incoming beam which occurs because the index of refraction of the medium increases when the intensity of the field increases. Mathematical analysis of the NLH requires solution of a quasi-linear elliptic boundary value problem with non-homogeneous and non-self-adjoint boundary conditions. This proves difficult for both theoretical study and numerical computation.

A commonly used simplified model is based on the forward scattering parabolic approximation of the NLH; it yields the nonlinear Schrödinger equation (NLS). The NLS is solved as a Cauchy problem, which is easier. However, it is known to render nonphysical predictions such as singularities in the solution developing at a finite propagation distance (self-focusing to a point). Therefore, we consider the question of whether or not there are regular solutions to the NLH in 3D under the conditions that lead to the blow-up of the NLS. In our work, this question is addressed numerically: we develop an advanced method and compute examples of such solutions.

One key issue in building a numerical scheme for the NLH is approximation of the non-self-adjoint radiation boundary conditions. This has to be done at a finite computational boundary, which leads to nonlocal operators (unlike the classical local Sommerfeld conditions that are set at infinity). Moreover, these boundary conditions have to be capable of specifying a given incoming wave that drives the problem along with making the boundary transparent for all the outgoing waves. This, for example, enables accurate modeling of backscattering in nonlinear self-focusing. The phenomenon of backscattering is very important because it may drastically affect the dynamics of the beam. It, however, has not been quantitatively analyzed before.

Another critical issue is that of choosing the solver for the system of nonlinear equations obtained from the discretization of the NLH on the grid. In our previous work we have employed a fixed-point iteration scheme based on freezing the nonlinearity and then using a sequence of Born approximations. This method, however, diverges for large nonlinearities. For that reason, a new solver was constructed based on Newton's method. Newton's linearization has some technical issues, because the Kerr nonlinearity for complex quantities is not differentiable in the sense of Frechét. The additional effort, however, pays off and in practice Newton's method enables numerical solution of the NLH for high nonlinearities, when solutions to the NLS collapse, and sometimes all the way up to the level of material breakdown.

In the talk, we will outline theoretical derivations and present the results of numerical simulations. This is joint work with Gadi Fibich and Semyon Tsynkov.

Adaptive Radial Basis Function Methods

John P. Boyd & Lei Wang
Department of Atmospheric, Oceanic and Space Science, University of Michigan, Ann Arbor MI 48109-2143, U.S.A.
Tel: (734) 764-3338, email: j.p.boyd@umich.edu

Abstract:

Radial basis functions are a meshless spectral method that in principle allows very irregular, adaptive grids in any number of dimensions. However, the lack of a fast transform similar to the Fast Fourier Transform and numerical ill-conditioning have retarded the use of this otherwise-promising method. We shall describe recent progress in overcoming these challenges on an unbounded interval and the surface of a sphere.

References:

1. T. A. Driscoll and A. R. H. Heryudono, Comput. Math. Applic. 53, 927–939 (2007).
2. J. P. Boyd and L. Wang, Appl. Numer. Math., submitted (2007).

A grid redistribution method for singular problems

Adi Ditkowski
School of Mathematical Sciences, Tel Aviv University, Tel Aviv 69978, Israe
Tel: 972-3-6405987, email: adid@post.tau.ac.il

Abstract:

Many physical phenomena develop singular or nearly singular behavior in localized regions, e.g., boundary layers or blowup solutions. Using uniform grids for the numerical solution of these problems require extremely fine grids for accurately resolving the solution in those small regions. Hence, the use of uniform grids becomes computationally prohibitive as the solution approaches singularity.

Ren and Wang developed a semi-static adaptive grid method [1] for the solution of these problems, known as the Iterative Grid Redistribution (IGR) method. In this talk we present a theoretical basis for semi-static adaptive grid method. Based on this theory, we had obtained the key result of our study - a methodology for designing robust weight functionals which ensures grid resolution in the singular region, as well as control of the maximal grid spacing in the outer region, for a wide range of singular behaviors. Using this methodology, we introduce a semi-static adaptive grid method, which does not involve an iterative procedure for grid redistribution, as in the IGR method. We demonstrate the efficacy of this method with a numerical example of a solution localized by more than 9 orders of magnitude.

Joint work with Nir Gavish.

References:

1. W. Ren and X.P. Wang, J. Comput. Phys. 159, 246-273 (2000).

Bloch-decomposition based method for wave propagation in periodic medium

Zhongyi Huang
Dept. of Mathematical Sciences, Tsinghua University, Beijing 100084, China
Tel: 8610-62796893, email: zhuang@math.tsinghua.edu.cn

Abstract:

In this talk, we conduct numerical simulations of (non)linear wave propagation in periodic medium. We provide an unconditionally stable numerical method to achieve the spectral convergence in space even when the periodic coefficients vary fast or/and are discontinuous. In the contrast, the traditional pseudo-spectral methods and finite difference/volume schemes have poor convergent rates especially when the coefficients are discontinuous and highly oscillatory. We also consider the wave propagation in disordered medium. We get the well-known *Anderson localization* in our numerical tests.

References:

1. P. W. Anderson, *The question of classical localization. A theory of white paint?* Philos. Mag. B, **52** (1985): 505–509.
2. A. Bensoussan, J. L. Lions, and G. Papanicolaou, *Asymptotic Analysis for Periodic Structures*, North-Holland Pub. Co., 1978.
3. Z. Huang, S. Jin, P. Markowich, and C. Sparber, *A Bloch decomposition based split-step pseudo spectral method for quantum dynamics with periodic potentials*, SIAM J. Sci. Comput. **29** (2007): 515–538.

Efficient Iteration Methods for Solitary Waves and Their Stability Spectra

Jianke Yang
Department of Mathematics and Statistics, University of Vermont, Burlington, VT 05401, U.S.A.
Tel: 802-656-4314, email: jyang@math.uvm.edu

Abstract:

In this talk, we present a few new efficient iteration methods for the computation of solitary waves and their linear-stability spectra for general solitary waves in arbitrary spatial dimensions. On the computation of solitary waves, we present (i) an accelerated imaginary-time evolution method with amplitude normalization [1]; (ii) a modified squared-operator iteration method [2]. We show that method (i) converges faster than the Petviashvili method (when both methods converge), while method (ii) converges for all solitary waves. On the computation of discrete eigenvalues in the linear-stability spectra of solitary waves, we also present two iteration methods: (a) an original-operator iteration method [3]; (b) a modified squared-operator iteration method [3]. It is shown that method (a) generally converges to the most unstable eigenvalue very quickly, while method (b) can converge to any discrete eigenvalue in the stability spectrum. All these methods described above are spectrally accurate, using very little computer memory, fast converging, and their matlab codes are only a couple of inches long (even for high spatial dimensions). We will demonstrate these numerical methods on a number of physically important and numerically-challenging examples such as lattice solitons in two spatial dimensions and Bose-Einstein condensates in three spatial dimensions, and determine solitary waves as well as their stability spectra in these physical systems.

References:

1. J. Yang and T.I. Lakoba, "Accelerated Imaginary-time Evolution Methods for the Computation of Solitary Waves", Stud. Appl. Math. 120, 265-292 (2008).
2. J. Yang and T.I. Lakoba, "Universally-convergent squared-operator iteration methods for solitary waves in general nonlinear wave equations", Stud. Appl. Math. 118, 153-197 (2007).
3. J. Yang, "Iteration Methods for Stability Spectra of Solitary Waves", to appear in J. Comp. Phys.

Long Range Numerical Simulation of Short Waves as Nonlinear Solitary Waves

John Steinhoff, Subhashini Chitta
University of Tennessee Space Institute, Tullahoma, TN 37388, U.S.A.
Tel: 931-3937467, email: jsteinho@utsi.edu

Abstract:

A new numerical method has been developed to propagate short wave equation pulses over indefinite distances. The method, Wave Confinement, utilizes a newly developed nonlinear partial differential equation that propagates basis function according to the wave equation. The discretized equation can be solved without any numerical dissipation. The method can also be used to solve for harmonic waves in the high frequency (Eikonal) limit, including multiple arrivals. The solution involves discretizing the wave equation on a uniform Eulerian grid and adding a simple nonlinear Confinement term. This term does not change the amplitude (integrated through each point on the pulse surface) or the propagation velocity, or arrival time, and yet results in capturing the waves as thin surfaces that propagate as nonlinear solitary waves and remain 2-3 grid cells in thickness with no numerical spreading. With the method, only a simple discretized equation is solved each time step at each grid node. The method can be contrasted to Lagrangian Ray Tracing: it is an Eulerian based method that captures the waves directly on the computational grid, where the basic objects are codimension 1 surfaces (in the fine grid limit), rather than rays. In this way, the complex logic of current ray tracing methods, which involve allocation of markers to each surface and interpolation as the markers separate, is avoided. With the new method, even though the surfaces can pass through each other and involve a nonlinear term, there is no interaction effect from this term on the variables of interest, allowing the linear wave equation to be accurately simulated. This absence of interaction is due to the form of the difference operators of the added term.The new method should be important in areas of wave propagation, from radar scattering and long distance communications to cell phone transmission.

References:

1. J. Steinhoff, M. Fan and L. Wang,J. Steinhoff, M. Fan and L. Wang, Computation of Short Wave Equation Pulses Using Nonlinear Solitary Waves, Computer Modeling in Engineering & Sciences, 5, 373-382(2004).

Conservative Discretizations of Periodic Hamiltonian Evolution Equations

Brenton LeMesurier
Department of Mathematics, College of Charleston, Charleston, SC 29424, U.S.A.
Tel: 843-953-5917, email: lemesurierb@cofc.edu

Abstract:

This talk describes a systematic approach to generating discrete time stepping methods from energy functions (Hamiltonians) which exactly conserve the energy, and often other conserved quantities of the corresponding continuous evolution equations, through respecting continuous and discrete symmetries of the Hamiltonian. These serve as numerical methods for Hamiltonian DE's but are also of intrinsic interest as totally discrete conservative dynamical systems with symmetries such as shifts around a spatial period. In particular, this is applied to the case of the completely integrable focusing cubic NLS with periodic boundary conditions, via spatial semi-discretizations including the Ablowitz-Ladik system.

The resulting discrete systems are compared to some symplectic integrators, which can respect many conserved quantities, but not the energy itself.

Numerical simulations of 3D overturning waves

Philippe Guyenne
Department of Mathematical Sciences, University of Delaware, Newark, DE 19716-2553, U.S.A.
Tel: 302-8318664, email: guyenne@math.udel.edu

We present numerical simulations of 3D overturning waves over bottom topography. The numerical model solves the fully nonlinear potential flow equations using a 3D high-order boundary element method combined with an explicit time integration scheme, expressed in a mixed Eulerian-Lagrangian formulation. Results on wave profiles and kinematics will be presented. Comparisons with 2D results will also be shown.

This is joint work with S. grilli (University of Rhode Island).

Numerical method for one-dimensional linear and nonlinear Klein-Gordon equations on unbounded domain

Zhiwen Zhang

Department of Mathematics, Tsinghua University, BeiJing,100084, P.R.China.
Tel: +86(0)10 62796907-8006, email: zhangzhiwen02@mails.tsinghua.edu.cn

In this talk, I will report the numerical method for one-dimensional Klein-Gordon equations on unbounded domain, which appears in various application areas. The talk consists of two parts.

In the first part, the numerical solution for one-dimensional linear Klein-Gordon equation on unbounded domain is analyzed. Two transparent artificial boundary conditions are obtained to reduce the original problem to an initial boundary value problem on a bounded computational domain, which is discretized by an explicit difference scheme. The stability and convergence of the scheme are analyzed by the energy method. A fast algorithm is obtained to reduce the computational cost and a discrete artificial boundary condition(DABC) is derived by the Z-transform approach. Finally, several numerical examples are given to illustrate the efficiency of the proposed method.

In the second part, the numerical solution for one-dimensional nonlinear Klein-Gordon equation on unbounded domain is studied. Split local absorbing boundary (SLAB) conditions are obtained by the operator splitting method, then the original problem is reduced to an initial boundary value problem on a bounded computational domain, which can be solved by the finite difference method. Several numerical examples are provided to show the advantages and effectivity of the given method, and some interesting collision behaviors are also observed.

This work is supervised by Prof. Houde Han, Department of Mathematics, Tsinghua University, BeiJing, 100084, P.R.China.

References:

1. H. Han and C. X. Zheng, Chinese J.Comp.Phy. 22(2), 95-107,(2005).
1. X.Wu and Z.Sun, Appl.Num.Math. 50,261-277(2004).
3. A. Arnold, VLSI Design,6, 313-319(1998).
4. Z.Xu and H.Han, Phys.Rev.E,74,037704(2006).
5. W.Z.Bao, S.Jin, and P.A.Markowich, J.Comp.Phys. 175,487-524,(2002).

Collisions between co-axial three-dimensional Ginzburg-Landau vortex solitons

Dumitru Mihalache
Horia Hulubei National Institute for Physics and Nuclear Engineering (IFIN-HH),
407 Atomistilor, Magurele-Bucharest, 077125, Romania
Tel: +40-21-4042300, ext. 3422, email: Dumitru.Mihalache@nipne.ro

In this talk, we report on generic outcomes of collisions between stable three-dimensional solitons with vorticities $S = 1$ and 2 in the complex cubic-quintic Ginzburg-Landau equation, in the axially symmetric configuration. For the sake of comparison, results are also included for fundamental (vorticityless) three-dimensional solitons. Depending on the collision momentum, χ, three generic outcomes are identified: merger of the solitons into a single one, at small χ; quasi-elastic interaction, at large χ; and creation of an extra soliton ("soliton birth"), in an intermediate region.

References:

1. D. Mihalache, D. Mazilu, F. Lederer, H. Leblond, and B. A. Malomed, Phys. Rev. A **75**, 033811 (2007); *ibid.* **76**, 045803 (2007).
2. N. Akhmediev, J. M. Soto-Crespo, and P. Grelu, Chaos **17**, 037112 (2007).
3. D. Mihalache, D. Mazilu, F. Lederer, H. Leblond, and B. A. Malomed, Phys. Rev. A (submitted).

Interactions between two solitons with arbitrary propagation directions in two-dimensional plasma

Xin Jiang, Xiu-yun Gao, Sheng-chang Li, Yu-ren Shi, Wen-shan Duan
Department of Physics, Northwest Normal University, Lanzhou, 730070, China
Tel: 0086 931 7972035, email: duanws@nwnu.edu.cn

The nonlinear waves in two-dimensional plasma are studied. Solitary waves propagating in two different directions are firstly investigated in this paper. Although the interaction among solitons in plasma physics has been studied previously, all the works are focused on the one-dimensional system. The equation of motion of nonlinear wave in the different directions are obtained. The phase shifts after the interaction for multi-solitons in different directions are given in the present work.

References:

1. N. N. Rao, P. K. Shukla , M. Y. Yu. Planet Space Sci., 38 543 (1990).
2. P. K. Shukla and A. A. Mamun, *Introduction to Dusty Plasma Physics* (Institute of Physics, Bristol, 2002).
3. I. Kourakis, P. K. Shukla, Physics Letters A 351, 122 (2006).
4. N. D'Angelo. Planet Space Sci., 38 1143 (1990).
5. A. Barkan, R. L. Merlino, N. D'Angelo. Phys. Plasmas, 2 3563 (1995).
6. L. Bringol-Barge and T. W. Hyde, Adv. Space Res.29 (9) 1283-1288 (2002).
7. L. J. Hou, Y. N. Wang, Z. L. Miskovic, Phys. Rev. E 64 046406 (2001)
8. G. X. Huang, M. G. Velarde, and V. A. Makarov, Phys. Rev. A. 64 013617 (2001).
9. J. X. Ma and J. Y. Liu, Phys. Plasmas 4, 253 (1997).
10. B. S. Xie, K. F. He, Z. Q. Huang, Phys. Lett. A 247 403 (1998).
11. W. S. Duan, Phys. Plasmas 8, 3583 (2001).
12. W. S. Duan, H. J. Yang, Y. R. Shi, and K. P. Lv, Physics Letters A 361, 368 (2007).
13. W. S. Duan, J. Parkes, Phys. Rev. E, 68 067402 2003.
13. W. S. Duan, Y. R. Shi, X. R. Hong, Phys. Lett. A 323 89-94 2004

Dynamics of Nonlinear Traveling Waves in Synaptically Coupled Neuronal Networks

Linghai Zhang
Department of Mathematics, Lehigh University
14 East Packer Avenue, Bethlehem, Pennsylvania 18015 USA
Telephone: +1-610-758-4116. Fax: +1-610-758-3767. Email: liz5@lehigh.edu

In this paper, by using a singularly perturbed system of integral differential equations as a reasonable realistic model, we show how a nonlinear nonlocal neuronal network can generate several stable traveling waves, including homoclinic orbits and heteroclinic orbits. All of these orbits serve as nontrivial local attractors of the dynamical systems.

Furthermore, we establish relation between speed index functions and stability index functions whose zeros are necessary and sufficient to determine the stability of the brain waves.

References:

1. David H. Terman, G. Bard Ermentrout and Alice C. Yew, Propagating activity patterns in thalamic neuronal networks. *SIAM Journal on Applied Mathematics*, **61**(2001), 1578-1604.
2. Linghai Zhang, On stability of traveling wave solutions in synaptically coupled neuronal networks. *Differential and Integral Equations*, **16**(2003), 513-536.
3. Linghai Zhang, Traveling waves of a singularly perturbed system of integral-differential equations arising from neuronal networks. *Journal of Dynamics and Differential Equations*, **17**(2005), 489-522.

Asymptotic analysis of an input-shaping scheme for suppressing motion-induced residual vibration of nonlinear mechanical systems

Tian-Shiang Yang
Department of Mechanical Engineering, National Cheng Kung University,
Tainan, Taiwan 701, R.O.C.
Tel: +886-6-2757575 ext 62112, email: tsyang@mail.ncku.edu.tw

Residual vibrations generally are excited when lightly damped mechanical structures move, and can be suppressed by properly administering the actuating forces (or, using the language of control systems engineering, *shaping* the *input* commands) applied on the dynamical system in question. In this talk, we shall consider the simple model of a one-degree-of-freedom spring–mass system, where the restoring spring force depends nonlinearly with the displacement of the point mass from its equilibrium position. For the ideal case where the elastic potential energy associated with the spring force is known with infinite accuracy, a class of input-shaping schemes—which split the input force into a series of consecutively administered step forces of constant magnitude—can be readily derived by analyzing how the external work input is partitioned into potential and kinetic energies. In particular, a two-step input force will be derived, which can be applied to bring the point mass in our model system from initial rest to a new equilibrium position without exciting any residual vibration. From a practical standpoint, however, the elastic potential energy usually can only be estimated to a certain accuracy, and may even vary slowly with time due to system aging. For such cases, the resulting residual vibration amplitude is calculated asymptotically, assuming that the nominal (i.e., estimated) elastic potential energy only has a small error. It will also be demonstrated that, by iteratively comparing the analytic result of the asymptotic analysis with the residual vibrations observed in experiments, one may improve the accuracy of the estimated elastic potential energy. The connections between the present problem and the amplitude control of nonlinear forced waves will be discussed in this talk as well.

References:

1. H.-T. Kao, I. Hu and T.-S. Yang, preprint (2008).
2. T.-S. Yang, K.-S. Chen, C.-C. Lee and I. Hu, J. Eng. Math. 57, 365–379 (2007).
3. T.-S. Yang, K.-S. Chen, C.-C. Lee and J.-F. Yin, J. Eng. Math. 54, 1–15 (2006).
4. K.-S. Chen, T.-S. Yang and J.-F. Yin, ASME J. Vibr. Acoust. 128, 778–789 (2006).
5. T.-S. Yang and W.-L. Liang, Wave Motion 37, 101–117 (2003).

Anomalous interaction of spatial gap solitons in optically induced photonic lattices

Sheng Liu, Peng Zhang, Fajun Xiao, Xuetao Gan and Jianlin Zhao
Institute of Optical Information Science and Technology and Shaanxi Key Laboratory of Optical Information Technology, School of Science, Northwestern Polytechnical University, Xi'an 710072, China
Tel: 0086-029-88495724, email:jlzhao@nwpu.edu.cn

We study the coherent and incoherent interactions of spatial gap solitons (GSs) in one-dimensional optically induced photonic lattices. We show that the π-staggered phase structures of GSs will lead to anomalous interactions. For mutually coherent two GSs, a transition between attractive and repulsive interaction forces can be obtained solely by changing the initial separations. And the direction of energy transfer during the interaction between two GSs with phase difference of $\pi/2$ also can be changed merely by altering their initial separations. While mutually incoherent GS pairs always attract each other. Surprisingly, the interactions between GSs bifurcating from the bottom of the 1st band will be influenced by negative refraction. In addition, oscillation phenomena are also observed during the interactions of GSs, which represents similar dynamics to Tamm oscillations.

References:

1. D. N. Christodoulides, et al., Nature 424, 817 (2003).
2. J. W. Fleischer, et al., Nature 422, 147 (2003).
3. D. Mandelik, et al., Phys. Rev. Lett. 92, 093904 (2004).
4. J. Meier, et al., Phys. Rev. Lett. 93, 093903 (2004).
5. M. Stepić, et al., Phys. Rev. E. 74, 046614 (2006).
6. F. J. Xiao, et al., Chin. Phys. Lett. 24, 3435 (2007).

Determination of the growth rate curve for transverse instabilities of plane soliton solutions of the extended Zakharov-Kuznetsov equation

Michael A. Allen[1], Jeerawan Uppaman[2], and George Rowlands[3]
[1]Physics Department, Mahidol University, Rama 6 Road, Bangkok 10400 Thailand
[2]Mathematics Department, Mahidol University, Rama 6 Road, Bangkok 10400 Thailand
[3]Physics Department, University of Warwick, Coventry, CV4 7AL, UK
[1]Tel: +662 201 5754, email: frmaa @ mahidol.ac.th

The extended Zakharov-Kuznetsov (eZK) equation in reduced variables takes the form

$$u_t + (u + bu^2)u_x + \nabla^2 u_x = 0$$

in which the subscripts denote partial derivatives. It has applications in plasma physics and transmission line theory [1,2]. The eZK equation is a generalization of the Zakharov-Kuznetsov equation (which it reduces to when $b = 0$) [3]. Like its simpler cousin, the eZK equation admits cylindrically and spherically symmetric quasi-soliton solutions as well as plane solitons. The plane solitons, themselves solutions of the extended KdV equation, are unstable with respect to transverse perturbations whose wavenumber k is less than a critical value, k_c. The instabilities develop into higher-dimensional quasi-solitons with a size of the order of the fastest growing mode. It is therefore of interest to determine the instability growth rate as a function of perturbation wavelength (the growth rate curve). For the eZK equation an exact expression for the growth rate cannot be obtained analytically. However, we can obtain a good approximate formula by first finding the growth rate for small k, k close to k_c, and from the value of k_c itself. The results are then combined using a Padé approximant. Obtaining the small-k growth rate is relatively straightforward for an equation of this form [4]. However, for an equation with two nonlinear terms such as this, finding an approximate expression for k_c (as a function of b) and the growth rate close to it is more involved – we use a technique introduced in Ref. 5 and also review an alternative variational approach. The Padé approximant for the growth rate we obtain agrees with the numerically derived growth rate curves (obtained in Ref. 2 and by ourselves) to within 0.4%.

References:

1. F. Verheest, R. L. Mace, S. R. Pillay, and M. A. Hellberg, J. Phys. A: Math. Gen. 35, 795 (2002).
2. W.-S. Duan, Europhys. Lett. 66, 192 (2004).
3. V. E. Zakharov and E. A. Kuznetsov, Soviet Phys. JETP 39, 285 (1974).
4. N. Hongsit, M. A. Allen, and G. Rowlands, *Phys. Lett. A* (in press).
5. M. A. Allen, S. Phibanchon, and G. Rowlands, J. Plasma Phys. 73, 215 (2007).

Discrete systems free of the Peierls-Nabarro potential

S.V. Dmitriev
Institute for Problems of Superplasticity of Metals, RAS,
Khalturina St. 39, Ufa 450001, Russia
Email: dmitriev.sergey.v@gmail.com

In the past few years, a variety of physical applications have stimulated an enormous growth in the study of nonlinear discrete models and the differential-difference equations that describe them. These models have a two-fold role and importance. On the one hand, they serve as discretizations of the corresponding continuum field theories; however, on the other hand, they may also be important physical models in their own right, e.g. in the context of crystal lattices. Taking into account the discreteness of media at the molecular and atomic level is becoming increasingly important for nanotechnologies.

We offer a wide class of discrete nonlinear systems that possess remarkable properties: (i) they are translationally invariant (TI), i.e., their equilibrium static (stationary) solutions can be placed anywhere with respect to the lattice, which is associated with the absence of the Peierls-Nabarro potential; (ii) static (stationary) versions of the discrete equations are exactly solvable.

We discuss the discrete equations that, in the continuum limit, reduce to the Klein-Gordon field or to the nonlinear Schrödinger equation, with a number of applications in the condensed matter physics and materials science.

Solitary waves in the discrete systems free of the Peierls-Nabarro potential are not trapped by the lattice. As a result, they can propagate with a small velocity and can be accelerated by even weak external field, which is impossible in the usual discrete systems. Since the solitons are highly mobile in the TI lattices, and since they can carry mass, energy, momentum, electric charge, information, etc., one can say that such lattices have better transport properties. Spectra of the small-amplitude vibrations calculated for the TI lattices hosting an equilibrium soliton, always contain zero eigenvalues, corresponding to the translational invariance (Goldstone mode).

The discretization method we offer uses the discretized first integral of the static (stationary) version of the corresponding continuum equation and is called the discretized first integral (DFI) approach [1]. We thus generalize the TI models constructed in the pioneering works [2].

References:

1. Dmitriev S V, Kevrekidis P G, Yoshikawa N and Frantzeskakis D J 2006, *Phys. Rev. E* **74** 046609; Dmitriev S V, Kevrekidis P G, Yoshikawa N and Frantzeskakis D J 2007 *J. Phys. A: Math. Theor.* **40** 1727; 99. Dmitriev S V, Kevrekidis P G, Khare A and Saxena A 2007 *J. Phys. A: Math. Theor.* **40** 6267; Dmitriev S V Kevrekidis P G, Sukhorukov A A, Yoshikawa N and Takeno S 2006 *Phys. Lett. A* **356** 324
2. Speight J M 1999 *Nonlinearity* **12** 1373; Kevrekidis P G 2003 *Physica D* **183** 68

Solitons of the derivative nonlinear Schrödinger equation with nonvanishing boundary conditions

Xiang-Jun Chen
Department of Physics, Jinan University, Guangzhou 510632, China
Tel: 8620-85224385 ext. 310, email: tcxj@jnu.edu.cn

The derivative nonlinear Shrödinger(DNLS) equation is an integrable equation with many applications in physics, describing some nonlinear Alfvén waves in space plasma, ultra short pules in nonlinear optics, polarized electromagnetic waves in some dielectric and magnetic systems. Its quantum version also plays a role in some Luttinger liquids in condensed matter physics.

Both of the vanishing and the nonvanishing boundary conditions for the DNLS equation are physically significant. Soliton solutions for the vanishing boundary conditions(VBC) were found in 1970s. Behaviors of solitons for the nonvanishing boundary conditions(NVBC) were basically known in the same period. The general soliton solutions are breather-like bright-dark soliton pairs, which approach VBC solutions as amplitudes of the boundary conditions vanish and degenerate to pure bright or dark solitons as the discrete spectral parameters become purely imaginary. However, explicit solutions in closed form for the NVBC problem have not yet been found until our recent works exploiting the affine parameter technique.

In this talk, the inverse scattering transform on the affine parameter space and a special technique to represent the N-soliton solutions in series are reviewed, behaviors of the DNLS solitons with NVBC are demonstrated with explicit expressions in [1]. Collisions between pure solitons, including bright-bright, dark-dark, and bright-dark soliton collisions, are also demonstrated using explicit pure N-soliton solutions in [2]. An interesting phase problem in conservation laws is discussed, as an erratum to [3].

References:

1. X.-J. Chen and W.K. Lam, Phys. Rev. E, **69**, 066604(2004)
2. X.-J. Chen, J. Yang and W.K. Lam, J. Phys. A, **39**, 3263(2006).
3. X.-J. Chen, L.-J. Hou and W.K. Lam, Chin. Phys. Lett., **22**, 830(2005)

Integrable models for few-cycle optical pulses

Hervé Leblond,[a] I. V. Mel'nikov,[b,c] D. Mihalache,[d] and F. Sanchez[a]

a) Laboratoire POMA, CNRS FRE 2988, Université d'Angers, 2 Bd Lavoisier, 49045 Angers Cedex 01, France

Tel: (33) 241735386, e-mail: herve.leblond@univ-angers.fr

b) High Q Laboratories Inc., 2 Gledhill Crescent, Hamilton, Ontario, Canada L9C 6H4

c) Optolink Ltd, bldg 5, proezd 4806, Zelenograd, 124498 Moscow, Russian Federation

d) Horia Hulubei National Institute for Physics and Nuclear Engineering (IFIN-HH), 407 Atomistilor, Magurele-Bucharest, 077125, Romania

Using the reductive perturbation method, we show that the propagation of optical few-cycle pulses (FCP) can be described by models of the modified Korteweg-de Vries (mKdV) or sine-Gordon (sG) type. The former equation is derived in the long wave approximation, the latter in the short wave one, starting from the Maxwell-Bloch equations [1].

A more general model of the mKdV-sG type is also derived, which is integrable under certain conditions. Optical FCP solitons are described by the breather solutions of these equations [2].

After a presentation of these models, including the assumptions that allow their derivation, we will discuss new results about interactions of FCP solitons.

Indeed, the four-soliton solution to the integrable mKdV-sG equation [3] allows to study analytically the interactions of the optical FCP solitons. First we obtain the envelopes of the FCPs before and after the interaction. Then the corresponding shifts in phase and location are explicitly computed as functions of the frequency and duration of the FCP solitons.

References:

1. H. Leblond and F. Sanchez, Phys. Rev. A. 67, 013804 (2003).
2. H. Leblond, S.V. Sazonov, I.V. Mel'nikov, D. Mihalache, and F. Sanchez, Phys Rev. A 74, 063815 (2006).
3. D.-Y. Chen, D.-J. Zhang and S.-F. Deng, J. Phys. Soc. Jpn 71, 658-659 (2002).

Limit Solutions for Soliton Equations

Da-jun Zhang
Department of Mathematics, Shanghai University, Shanghai 200444, P.R. China
Tel: 86-21-66135655, email: djzhang@staff.shu.edu.cn

This talk focuses on limit solutions of soliton equations. In general, from the viewpoint of the Inverse Scattering Transform, N solitons are identified by N distinct eigenvalues or in other words, N distinct simple poles $\{k_j\}$ of the transmission coefficient $\frac{1}{a(k)}$, (or N simple zeros of Jost solution $a(k)$). When $\{k_j\}$ are multiple poles, the related multiple-pole solution can be obtained by a limit procedure like $k_2 \to k_1$ from simple-pole solution. In the talk we will make a survey on limit solutions from viewpoints of the Inverse Scattering Transform, Darboux transformation, Bäcklund transformation, Hirota's method and Wronskian technique. We discuss simple and compact expressions for the limit solutions, and also describe an exact limit procedure to show how each parameter in a limit solution is exactly derived through the procedure. This procedure is also helpful for investigating dynamics of limit solutions. Besides, we consider new symmetries which is derived also based on a kind of limit ideas and investigate the related new group-invariant solutions.

References:

1. M. Wadati and K. Ohkuma, J. Phys. Soc. Jpn. 51, 2029-2035 (1982).
2. V.B. Matveev, Phys. Lett. A 166 205-208, 209-212 (1992).
3. D.Y. Chen, D.J. Zhang and S.F. Deng, J. Phys. Soc. Jpn. 71, 2072-2073 (2002).
4. V.S. Shchesnovich and J. Yang, J. Math. Phys. 44, 4604-4639 (2003).
5. D.J. Zhang, Notes on solutions in Wronskian form to soliton equation: KdV-type, arXiv:nlin.SI/0603008 v3, (2006).
6. Q.F. Xuan, M.Y. Ou and D.J. Zhang, Wronskian solutions to the KdV equation via Bäcklund transformation, arXiv:nlin.SI/0706.3487, (2007).

Evolution of higher-order gray Hirota solitary waves

S.M. Hoseini† and T.R. Marchant‡
† Mathematics Department, Vali-e-Asr University, Rafsanjan, Iran.
†‡ School of Mathematics and Applied Statistics, University of Wollongong, Wollongong, 2522 NSW, Australia.
email: hoseini@uow.edu.au, tim_marchant@uow.edu.au

The defocusing Hirota equation has dark and gray soliton solutions which are stable on a background of periodic waves of constant amplitude. In this research, gray solitary wave evolution for a higher-order Hirota equation is examined. A direct analysis is used to identify families of higher-order embedded gray solitary waves. Soliton perturbation theory is then used to determine the detailed behavior of an evolving higher-order gray Hirota solitary wave; an integral expression for the first-order correction to the wave is found and analytical expressions for the solitary wave tail are derived. A subtle and complex picture of the development of solitary wave tails emerges. It is shown that the higher-order gray one-soliton solution can only be derived if one of the two algebraic relationships is satisfied. The first condition, relating the higher-order coefficients, implies that a three-parameter family of higher-order asymptotic solitary waves exists. The second condition represents a relationship between the three free parameters. Physically the second condition implies that the soliton velocity is the same as the phase velocity of the background wave and implies the existence an additional two-parameter family of higher-order asymptotic solitary waves. It is also found that solitary wave tails develop for two reasons, one is decay of the solitary wave caused by resonance, the second is corrections at first-order to the background wave. Strong agreement is found between the theoretical predictions of the perturbation theory of and numerical solutions.

Higher-dimensional integrable hierarchies with non-isospectral parameters

Kouichi Toda
Department of Mathematical Physics, Toyama Prefectural University,
Kurokawa 5180, Imizu, Toyama, 939-0398, JAPAN
Tel: 81-766-56-7500, email: kouichi@yukawa.kyoto-u.ac.jp

We would like to give formulations of Lax representations with non-isospectral parameters[1−7] and associated higher-dimensional integrable hierarchies such as the Ablowitz-Kaup-Newell-Segur, the Kaup-Newell and so on.

References:

1. F. Calogero: *Lett. Nuovo Cim.*, **Vol. 14**, 443 (1975).
2. S. P. Burtsev, V. E. Zakharov and A. V. Mikhairov: *Theor. Math. Phys.* **Vol. 70**, 227 (1987).
3. O. I. Bogoyavlenskii: *Russian Math. Surveys*, **Vol. 45**, 1 (1990).
4. P. A. Clarkson, P. R. Gordoa and A. Pickering: *Inverse Problems*, **Vol. 13**, 1463 (1997).
5. P. R. Gordoa and A. Pickering: *J. Math. Phys.*, **Vol. 40**, 5749 (1999).
6. P. G. Estévez: *Inverse Problems*, **Vol. 17**, 1043 (2001).
7. T. Kobayashi and K. Toda: *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, **Vol. E88-A**, 2548 (2005).

Exploring the possible 2×2 partial difference equation Lax pairs

Mike Hay
School of Mathematics and Statistics, University of Sydney, NSW, 2006, Australia
Tel: 02 9351 5726, email: michaelh@maths.usyd.edu.au

Despite the existence of a Lax pair for a given equation often being used as the definition of its integrability, there have been few studies that sought to find or categorize nonlinear equations that used Lax pairs as their starting point. Of those studies that did begin with Lax pairs, most chose *a priori* a form of the Lax pair, *i.e.* its dependence on the spectral parameter, thus limiting the possible results.

The present study begins with general Lax pairs that are 2×2, where each entry of the Lax matrices contains only one term. We then examine all of the combinations of terms that can arise via the compatibility condition. Once a system of equations is extracted from the compatibility condition, it is solved in a way that preserves its generality, up to a point where nonlinear evolution equations are apparent, or the system is shown to be trivial.

In fact, of all the potential Lax pairs identified by this method, only two lead to interesting evolution equations. These are new, higher order varieties of the lattice sine-Gordon (LSG) and the lattice modified KdV (LMKDV) equations. The remaining systems are shown to be trivial, overdetermined or underdetermined. As noted in [1], this suggests a connection between the existence of a Lax pair and the singularity confinement method of identifying integrable equations.

As we do not make any assumptions about the explicit dependence on the spectral parameter, we show that a particular nonlinear equation may have many Lax pairs, all depending on the spectral parameter in different ways. The effect that this freedom has on the process of inverse scattering is, as yet, unclear.

References:

1. R. Sahadevan, O.G. Resin, and P.E. Hydon, J. Math. Anal. Appl. 331 (1) 712–726

Exploring vortex solutions of Faddeev model

Chang-Guang Shi and Minoru Hirayama
Department of Mathematics and Physics, Shanghai University of Electric Power, Shanghai 200090, China
Tel: 021-6430410, email: shichangguang@shiep.edu.cn

Faddeev model is a nonlinear field theory proposed to obtain higher-dimensional soliton solutions. The numerical analysis of the model have shown that it possesses solitons with various types of knot structures. In this talk, a method to reduce the second order partial differential equation of the Faddeev model to a set of first order ones is presented.

Especially, solutions of vortex-type are explored by an Ansatz specifying the azimuthal-angle dependence of the solution. The field equation of the model is converted to an algebraic ordinary differential equation containing an integer quantum number. An approximate analytic expression of the vortex solution is sought so that the energy per unit vortex length becomes as small as possible. It is observed that the minimum energy of vortex is approximately proportional to the above quantum number which specifies the solution.

References:

1. C-G Shi and M. Hirayama, Inter. Jour. Mod. Phys. A, in press (2008).
2. M. Hirayama and C-G Shi, Phys. Lett. B, 652, 384-387 (2007).

Transverse instability of the modified nonlinear Schrodinger equation

Sarun Phibanchon
Faculty of Science and Art, Burapha University
57 Moo.1 Thamai, Chanthaburi, 22170, Thailand
Tel: +66 39 310000, email: sarunp@buu.ac.th

The ion-acoustic waves in the non-isothermal electron distribution is derived from the Schamel-Zakharov-Kuznetsov equation. The first order of the growth rate can be determined from the long-wavelength perturbation.

References:

1. H. Schamel, J.Plasma. Phys. 9, 377-87 (1973).
2. G. Rowlands, Nonlinear phenomena in science and engineering, Ellis Horwood, London (1990).
3. P. A. E. M. Janssen, J. J. Rasmussen, Phys. Fluids., 26, 1279-87 (1982).

Exact solutions to the 2×2 associated linear problem of the second and third q-discrete Painlevé equations

Nalini Joshi and Yang Shi
School of Mathematics and Statistics, University of Sydney, Sydney, NSW 2006, Australia
Tel: 9351 2172, email: nalini@maths.usyd.edu.au
Tel: 9351 5790, email: yangshi@maths.usyd.edu.au

Abstract:

We study the 2×2 associated linear system (Lax pair) of the q-analogue of the Second and Third Painlevé equation (qPII and qPIII). We show that in the cases of rational special solutions for qPII and qPIII, the corresponding associated linear problem can be solved exactly in terms of q-Gamma functions. We discuss what this means for the problem of completeness.

References:

1. Hay, M. and Hietarinta, J. and Joshi, N and Nijhoff,F. "A Lax pair for a lattice modified KdV equation, reductions to q-Painlevé equations and associated Lax pairs" J. Phys. A, Math.Theor 40 (2007) F61–F73
2. Birkhoff, G.D.,"The Generalised Riemann Problem..." Proc. Am. Acad., 49,521-568 (1913)

Slow Light and Fast Light Solitons in Resonant Multi-Level Atomic Systems

Guoxiang Huang
State Key Laboratory of Precision Spectroscopy and Department of Physics, East China Normal University, Shanghai 200062, China
Tel: 86-21-62233944, email: gxhuang@phy.ecnu.edu.cn

In recent years, much attention has been paid to the study of slow light and fast light in resonant multi-level atomic systems [1,2]. A stable pulse propagation of slow and fast lights has not only fundamental theoretical interest but also many important applications in optical information processing and engineering. This talk will report our recent progress on the ultraslow optical solitons via electromagnetically induced transparency (EIT) and the superluminal optical solitons in an active Raman gain medium. We show that stable temporal and spatial optical solitons with extremely low light intensity and very slow propagating velocity can be generated in an EIT-based resonant system through quantum interference and the balance between dispersion and nonlinearity of the system. Furthermore, ultraslow vector optical solitons are also investigated in a double lambda system and their possible application in soliton logic gates is also described [3-10]. In addition, the possibility of superluminal optical solitons with very low light intensity in an active Raman gain system are also predicted [11].

References:

1. R. W. Boyd and D. J. Gauthier, *Progress in Optics* (Elsevier Science, Amsterdam, 2002), Vol. 43, Chap. 6, p. 275.
2. M. Fleischhauer, A. Imamoğlu, and J. P. Marangos, Rev. Mod. Phys. 77, 633 (2005), and references therein.
3. Y. Wu and L. Deng, Phys. Rev. Lett. **93**, 143904 (2004).
4. G. Huang, L. Deng, and M. G. Payne, Phys. Rev. E 72, 016617 (2005).
5. L. Deng, M. G. Payne, G. Huang, and E. W. Hagley, Phys. Rev. E 72, 055601 (Rapid communication) (2005).
6. G. Huang, K. Jiang, M. G. Payne, and L. Deng, Phys. Rev. E 73, 056606 (2006).
7. C. Hang, G. Huang, and L. Deng, Phys. Rev. E 73 (2006) 036607.
8. J. Wang, C. Hang, and G. Huang, Phys. Lett. A 366 (2007) 528.
9. C. Hang, G. Huang, and L. Deng, Phys. Rev. E 74 (2006) 046601.
10. C. Hang and G. Huang, Phys. Lett. A, in press (Available online Jan 16) (2008).
11. G. Huang, C. Hang, and L. Deng, Phys. Rev. A 77, in press (Rapid communication) (2008).

The dispersion-managed Ginzburg-Landau equation and its application to femtosecond lasers

Gino Biondini

State University of New York at Buffalo, Department of Mathematics, Buffalo, NY 14260 U.S.A.; biondini@buffalo.edu

The complex Ginzburg-Landau equation is a universal model which has been used extensively to study various non-equilibrium phenomena. In the context of lasers, it describes the evolution of a pulse by averaging over the changes which take place in a single roundtrip in the cavity. Pulses produced by Ti:sapphire femtosecond lasers, however, undergo significant dynamical changes in different parts of the cavity during each roundtrip, and are not adequately described by an average model which does not take these changes into account. In this talk we first derive the dispersion-managed Ginzburg-Landau equation (DMGLE) in a general context as an average model that describes the long-term dynamics of systems characterized by rapid variations of dispersion, nonlinearity and gain, and we study the properties of the equation. We then explain how in particular the DMGLE arises for Ti:sapphire femtosecond lasers and we characterize its solutions.

Nonlinear analysis of modulational instability in a layered optical medium

Jun Yu and Yi Yang
Department of Mathematics and Statistics, University of Vermont
Burlington, VT05401, U.S.A.
Tel: 802-6568539, email: jun@cems.uvm.edu

As one of the principal mechanisms leading to the emergence of localized coherent nonlinear structure and the formation of solitary waves, the modulational instability is important in understanding dynamic behaviors in many areas. Recently there is a growing interest in studying the modulational instability in a layered optical medium periodic in the evolution variable. Experimental investigation, numerical simulation and linear analysis have shown that there are multiple instability regions for the layered medium, rather than just one as for a uniform medium. In this talk, we report our investigation on nonlinear evolution of a slightly supercritical wave in the layered medium. A weakly nonlinear analysis is proposed to describe the dynamical behaviors of the system for long time. In particular, we study the wave-wave interaction, stabilization of the linearly unstable mode, as well as the formation of coherent pattern. Comparison of experimental data, numerical solutions and results from our nonlinear analysis is also presented.

References:

1. M. Centurion et al., Phys. Rev. A 75, 063804 (2007).
2. J. Yu and Y. Yang, Stud. Appl. Math. 111: 1-27. (2003)

Modulational instability and spatial solitons in one-dimensional lattices with resonant nonlinearity

Milutin Stepić[1], Aleksandra Maluckov[2], Detlef Kip[3], and Ljupčo Hadžievski[1]
[1]Vinča Institute of Nuclear Sciences, P.O.B. 522, 11001 Belgrade, Serbia
Tel: +381-11-2455-272, email: mstepic@vin.bg.ac.yu, ljupcoh@vin.bg.ac.yu
[2]Faculty of Sciences, University of Niš, P.O.B. 224, 18001 Niš, Serbia
Tel: +381-18-533-015/ext. 44, email: sandra@pmf.ni.ac.yu
[3]Institute of Physics and Physical Technologies, Clausthal University of Technology, 38678 Clausthal-Zellerfeld, Germany
Tel: +49-5323-72-3700, email: d.kip@pe.tu-clausthal.de

In this talk our analytical and numerical results on both modulational instability (MI) and discrete soliton formation in one-dimensional (1D) defect-free lattices with resonant self-focusing nonlinearity [1,2] will be presented. In 1D photonic lattices MI occurs under the combined influence of diffraction and nonlinear response of the system [3, 4]. One possible outcome of this universal process in 1D lattices is the formation of trains of discrete solitons. These intrinsically localized modes have been suggested by Christodoulides and Joseph two decades ago [5] and observed in 1D AlGaAs waveguide arrays ten years later [6]. Here we propose a novel 1D model within the tight-binding approximation, identify linearly unstable region for a homogeneous solution, calculate the corresponding conserved quantities of the system, find approximate analytical solutions for different types of discrete resonant solitons and investigate their stability and steering properties.

References:

1. M. Chauvet, S. A. Hawkins, G. J. Salamo, M. Segev, D. F. Bliss, and G. Bryant, Opt. Lett. 21, 1333 (1996).
2. R. Uzdin, M. Segev, and G. J. Salamo, Opt. Lett. 26, 1547 (2001).
3. J. Meier, G. I. Stegeman, D. N. Christodoulides, Y. Silberberg, R. Morandotti, H. Yang, G. Salamo, M. Sorel, and J. S. Aitchinson, Phys. Rev. Lett. 92, 163902 (2004).
4. M. Stepić, C. Wirth, C. E. Rüter, and D. Kip, Opt. Lett. 31, 247 (2006).
5. D. N. Christodoulides and R. I. Joseph, Opt. Lett. 13, 784 (1988).
6. H. S. Eisenberg, Y. Silberberg, R. Morandotti, R. Boyd, and J. S. Aitchinson, Phys. Rev. Lett. 81, 3383 (1998).

Necklace Solitons and Ring Solitons in Bessel Optical Lattices

Liangwei Dong, Hui Wang, Weidong Zhou
Department of Physics, Zhejiang Normal University, Jinhua, China, 321004
Tel: +8657982298833, email: donglw@zjnu.cn

In this talk, we will put forward the existence of localized necklace solitons and ring solitons in a defocusing cubic nonlinear medium with an imprinted Bessel optical lattice. Novel families of necklace solitons are found and their unique properties, including multistable states are revealed. We show that both necklace solitons and ring solitons could reside on any ring of the Bessel lattices. They are dynamically stable provided that the lattices is modulated deep enough. Necklace solitons with the uneven "pearl" distributions were shown to exist and they are dynamically stable under appropriate conditions. We also check the propagation of the necklace solitons with a global angular momentum and find that the phase twists do not destroy their stability. The ring solitons with larger dark cores can be supported by the Bessel lattices. The existence and stability domains of such ring solitons are presented. They also can be regarded as special states of necklace solitons supported by the same model. The uncovered phenomena may open a new way for soliton control and manipulation.

References:

1 M. Soljačić, S. Sears, and M. Segev, Phys. Rev. Lett. 81, 4851 (1998).
2 A. Desyatnikov and Y. Kivshar,Phys. Rev. Lett. 87, 033901 (2001).
3 M. Soljačić, and M. Segev, Phys. Rev. Lett. 86, 420 (2001).
4 J. Yang, I. Makasyuk, P. Kevrekidis, B. Martin, H. Malomed, D. Frantzeskakis, and Z. Chen, Phys. Rev. Lett. 94, 113902 (2005).

2D Surface Gap Solitons at a Nonlinearity Interface

Tomáš Dohnal
Department of Mathematics, Univ. of Karlsruhe, Englerstr. 2, 76131 Karlsruhe, Germany
Tel: +49 721 6087670, email: dohnal@math.uka.de

We present the study of solitary waves localized at the interface of two nonlinear periodic media with different coefficients of the cubic nonlinearity in the two-dimensional Gross-Pitaevsky equation

$$\begin{aligned} &iu_t + \triangle u - V(x,y)u + \Gamma(x,y)|u|^2 u = 0, \\ &V(x+2\pi,y) = V(x,y+2\pi) = V(x,y), \quad \Gamma((x,y)\in\mathbb{R}^2_-) = \Gamma_-,\ \Gamma((x,y)\in\mathbb{R}^2_+) = \Gamma_+, \end{aligned}$$

where $\Gamma_+ \neq \Gamma_-$ are real constants and $\mathbb{R}^2_\mp$ are the half planes on each side of the material interface Π, which is a curve in the $(x,y)-$plane. The model is applicable in the field of nonlinear photonic crystals as well as in Bose-Einstein Condensates (BECs). In photonics applications such a structure corresponds to a cubically nonlinear 2D photonic crystal with different values of the nonlinear refractive index on each side of the interface Π and in BECs it describes a condensate with different $s-$wave scattering lengths on each side of Π.

We call solutions $u(x,y,t) = e^{-i\omega t}\phi(x,y)$ that are exponentially localized in space surface gap solitons (SGSs) [1,2] as they are inherent to the interface surface and because their propagation constant (or frequency) ω lies in the band gaps of the corresponding linear operator $-\triangle + V(x,y)$. In our construction standard gap solitons (GSs), including Townes-type solitons, $\pi/2$-phase delay solitons and vortices, are first found for $\Gamma_+ = \Gamma_-$ near band edges via asymptotics as solutions of envelope approximations, so called coupled mode equations. These are then continued numerically into GS families covering the whole band gaps in a similar fashion to [3,4].

We then study bifurcations of SGSs from these standard GSs and determine numerically the maximal jump of the nonlinearity coefficient allowing for the SGS existence. We show via asymptotics that the maximal jump vanishes near the thresholds of bifurcations of gap solitons while far from these thresholds SGSs at a true focusing/defocusing interface exist, similarly to the case of 1D SGSs [5]. We also investigate secondary bifurcations of SGS families via variational methods.

References:

1. Y.V. Kartashov, A.A. Egorov, V.A. Vysloukh and L. Torner, Opt. Express 14, 4049–4057 (2006).
2. S. Suntsov, K. G. Makris, D. N. Christodoulides, G. I. Stegeman, A. Hache, R. Morandotti, H. Yang, G. Salamo and M. Sorel, Phys. Rev. Lett. 96, 063901 (2006).
3. Z. Shi and J. Yang, Phys. Rev. E 75, 056602 (2007).
4. T. Dohnal, D. Pelinovsky and G. Schneider "Coupled-mode equations and gap solitons in a two-dimensional nonlinear elliptic problem with a separable periodic potential," submitted to J. Nonlin. Sci. (2007).
5. T. Dohnal and D. Pelinovsky, "Surface gap solitons at a nonlinearity interface," SIADS, in print (2008).

Band-gap engineering of two-dimenional photonic lattices with reconfigurable refractive index potential

Peng Zhang[1,2], Sheng Liu[1], Cibo Lou[2], Jianlin Zhao[1], Jingjun Xu[2] and Zhigang Chen[2,3]

1 Institute of Optical Information Science and Technology and Shaanxi Key Laboratory of Optical Information Technology, School of Science, Northwestern Polytechnical University, Xi'an 710072, China

2 The Key Laboratory of Weak-Light Nonlinear Photonics, Ministry of Education and TEDA Applied Physics School, Nankai University, Tianjin 300457, China

3 Department of Physics and Astronomy, San Francisco State University, San Francisco, California 94132, USA

Tel: 0086-029-88495724; email: jlzhao@nwpu.edu.cn

We study the band-gap properties of two-dimensional photonic lattices with different refractive index potentials. It is shown that, at an identical potential depth, the photonic band-gaps intensively depend on the lattice structures. For a quantum-dot lattice with an array of index maxima on flat background, by varying the ellipticity and orientation of the lattice dot, the size and location of the photonic band-gaps can be changed. While for an egg-crate lattice with alternating index maxima and minima, the Bragg reflection from the 1st and higher photonic band-gaps can be removed. The results encourage us to perform band-gap engineering and light steering with reconfigurable photonic lattices. By employing light-induction method, photonic lattices with different index potentials are successfully created in an electrically biased photorefractive crystal. The Brillouin zone spectra and the experimentally observed light steering are in good agreement with our theoretical predications.

References:

1. E. Yablonovitch, Phys. Rev. Lett. 58, 2059 (1987).
2. S. John, Phys. Rev. Lett. 58, 2486 (1987).
3. D.N. Christodoulides et al., Nature 424, 817 (2003).
4. N. K. Efremidis et al., Phys. Rev. E 66, 046602 (2002).
5. J. W. Fleischer et al., Nature 422, 147 (2003).
6. D. Neshev et al., Opt. Lett. 28, 710 (2003).
7. H. Martin et al., Phys. Rev. Lett. 92, 123902 (2004).
8. P. Zhang et al., Opt. Express 15, 536 (2007).

Linear and Nonlinear Landau-Zener Tunneling in Optical Directional Coupler

Ramaz Khomeriki,
Department of Physics, Tbilisi State University, Tbilisi 0128, Republic of Georgia.
Tel: 99532-915681, email: khomeriki@hotmail.com

Abstract:

In this talk I investigate theoretically a stationary distribution of light intensity along two coupled waveguides when their cross sections varies along the waveguides direction (at one end the cross section of one waveguide is larger then one of the second waveguide, while at the other end the cross section of the same waveguide is smaller). As numerical simulations show, in the linear limit injection of the beam in each waveguide leads to the light tunneling to the neighboring waveguide, while increasing the intensity of the injected beam this symmetry breaks down: the light injected in the waveguide with the smaller cross section successfully tunnels to the other waveguide like in the linear case, while the light injected into the other waveguide is trapped. It is shown that in the suggested setup observed effects could be identified as Landau-Zener tunneling. This nontrivial effect could be likely applied in producing all optical diodes and ultra fast optical switches.

Evolution of optical vortices in light-induced photonic lattices under nonconventional bias conditions

Xuetao Gan, Peng Zhang, Sheng Liu, Fajun Xiao and Jianlin Zhao

Institute of Optical Information Science and Technology and Shaanxi Key Laboratory of Optical Information Technology, School of Science, Northwestern Polytechnical University, Xi'an 710072, China

Tel: 0086-029-88495724, email:jlzhao@nwpu.edu.cn

Abstract:

We study the nonlinear evolution of optical vortices in light-induced photonic lattices under nonconventional bias conditions based on an anisotropic model. It is revealed that, in the presence of anisotropy, the vortex beams can evolve into asymmetric discrete vortex soliton states with four-lobe structures. Generally, the intensities of different lobes become nonuniform, but the diagonal two lobes possess an identical intensity. By tuning the lattice beam intensity and the external bias field, the vortex beam could evolve into a stationary dipole structure. Remarkably, the asymmetric soliton states with four lobes forming rhomboid configurations are observed under nonconventional bias cases. It is surprising that the topological sign of the input vortices plays a nontrivial role in the evolution dynamics of the vortex beams. In addition, the topological transformations including the charge flipping and orbit angular momentum inversion can also be observed in the nonconventionally biased conditions. All the above phenomena can be interpreted by the interplay among the vortex orbit angular momentum, the lattice periodicity and the anisotropic nonlinearity.

References:

1. P. Zhang, et al., Opt. Express 15, 536-544 (2007).
2. D. N. Neshev, et al., Phys. Rev. Lett. 92, 123903 (2004).
3. J. W. Fleischer, et al., Phys. Rev. Lett. 92, 123904 (2004).
4. T. J. Alexander, et al., Phys. Rev. Lett. 93, 063901 (2004).
5. J. Yang, et al., Phys. Rev. Lett. 94, 113902 (2005).
6. A. Bezryadina, et al., Opt. Express 14, 8317-8327 (2006).

Excitation of chaotic spin waves through four-wave parametric processes

Mingzhong Wu[1], Boris A. Kalinikos[1,2], Thomas Stemler[3], and Hartmut Benner[3]
[1]Department of Physics, Colorado State University, Fort Collins, CO 80523, U.S.A.
Tel: 001-970-4916312, email: mwu@lamar.colostate.edu
[2]St. Petersburg Electrotechnical University, 197376, St. Petersburg, Russia
[3]Institute for Solid State Physics, Darmstadt University of Technology, D-64289 Darmstadt, Germany

Abstract:

There are two main focus areas in the study of nonlinear spin waves in magnetic systems: (1) modulational instability and stationary spin wave envelope solitons and (2) chaotic spin wave dynamics. The processes responsible for these phenomena involve three-wave or four-wave parametric interactions. For (1), the work was done at relatively high frequencies where the three-wave processes were prohibited and the four-wave processes facilitate the modulational instability and the formation of solitons. For (2), the work was done at relatively low frequencies where both the three-wave and four-wave processes were allowed. The concurrence of these processes results in the excitation of spin wave chaos. This work reports for the first time the excitation of chaotic spin waves at relatively high frequencies where the three-wave processes are not allowed and the four-wave interactions are the only responsible process. The experiments were done with a magnetic film-based feedback ring. The ring consists of an yttrium iron garnet film strip with input and output microstrip transducers, a linear microwave amplifier, and an adjustable microwave attenuator for ring gain control. A static magnetic field was applied in the film plane and perpendicular to the long direction of the film strip. This film/field configuration corresponds to the propagation of magnetostatic surface waves. The chaotic signals were analyzed in both the time and frequency domains. The chaotic nature of the signals was confirmed by broadband power-frequency spectra and positive Lyapunov exponents.

Nonlinear spin waves in in-plane confined geometries

Mikhail Kostylev
School of Physics M013, University of Western Australia, Crawley 6009, WA, Australia
Tel: 61-8-64882723, email: kostylev@cyllene.uwa.edu.au

Abstract:

Magnetostatic spin waves in confined geometries have attracted a big deal of attention, since spin wave excitations naturally occur during switching of direction of magnetization in elements of magnetic memory and of magnetic logic schemes. Generally this spin wave dynamics is highly nonlinear and largely affected by element's shape and sizes. In this work we studied the effect of in-plane shape of magnetic elements on the nonlinear spin wave pattern formation and the thresholds of parametric spin wave processes.
We show by calculation and by comparison with experimental results that effective dipole coupling of magnetization at the edges of magnetic elements considerably lowers thresholds for transverse parametric instability (self-focusing) of backward volume spin waves pulses travelling along ferromagnetic stripes of rectangular cross-section [1]. It also results in a prominent kink structure on threshold curves for parallel pump instability ("butterfly curves") of spin waves on the stripes [2].
We also consider nonlinear spin-wave standing-wave pattern formation on in-plane magnetized square magnetic dots under effect of external driving field. We show that similar to the magnetic stripes dipole pinning of dynamic magnetisation at the dot edges has major effect on the nonlinear spin wave dynamics. However, this is not the only effect important in this case. As comparison with experimental results shows, in contrast to travelling spin wave pulses, the nonlinear pattern of standing spin wave oscillations is highly affected by nonlinear damping which changes character of the pattern from "attractive" to "repulsive" [3].

References:

1. A. A. Serga et al., arXiv:0704.0024.
2. M. Kostylev et al., Appl. Phys. Lett., 90, 012507 (2007).
3. M. Kostylev et al., Phys. Rev. B 76, 224414 (2007).

Brillouin light scattering observations of thermal modes in yttrium iron garnet films

Timo Neumann, Thomas Schneider, Alexander A. Serha, Burkhard Hillebrands
Fachbereich Physik, TU Kaiserslautern, 67663 Kaiserslautern Germany.
Tel: +49-631-205-4203, email: neumannt@physik.uni-kl.de

Abstract:

We report on optical observations of a thermally excited band of modes above the dipolar spectrum limits detected in yttrium iron garnet (YIG) films of different thicknesses. The modes were successfully amplified by means of longitudinal parametric pumping. Time-resolved investigations of the amplified signal revealed a self-oscillatory behavior.

Brillouin Light Scattering (BLS) spectroscopical observations [1] of the thermal spectrum of YIG-films with micrometer-thickness show three distinct peaks. The most pronounced is identified at the ferromagnetic resonance frequency while a second one is attributed to the bottom of the spin-wave spectrum. Besides these two signals a third one is observed far above the limit of surface magnetostatic spin waves in the Damon-Eshbach approximation neglecting the exchange contribution. The peak is observed for in-plane magnetization over a wide range of magnetic field from below 100 Oe to 4300 Oe. Its frequency f_{th} displays a linear behavior with respect to the magnetic field of the form $f_{th} = \gamma \cdot H + c_0$ with $\gamma = 2.8\ MHz/Oe$ denoting the gyromagnetic ratio and c_0 a film-dependent constant of the order of 7 GHz.

By using a micro-strip resonator effective longitudinal parametric amplification [2] is realized. Since this requires the pumping frequency f_P to be exactly equal to twice the mode frequency f_{th}, a precise determination of the latter becomes possible. In particular, the spectral width of the amplification region is determined to be as wide as 280 MHz indicating the existence of not only a single mode but of a whole band of modes. Moreover, the region of existence of the band is identified with a kink in the so-called butterfly curve relating the threshold power of parametric excitation to the applied bias magnetic field.

Time-resolved Brillouin light scattering observation of the pulse-amplified mode disclose an oscillatory intermediate stage passed before equilibrium is reached. The frequency spectrum of the oscillations consists of several peaks in the $1 - 10\ MHz$ range, whose frequency is increasing with increasing pumping power. This is shown to be consistent with the expected behavior of auto-oscillations as predicted in the framework of L'vov's S-theory [3].

Support by the MATCOR "Graduate Class of Excellence" is acknowledged gratefully.

References:

1. B. Hillebrands, *Progress in multipass tandem Fabry-Perot interferometry: I. A fully automated, easy to use, self-aligning spectrometer with increased stability and flexibility*, Rev. Sci. Instr. 70, 1589 (1999)
2. G.A. Melkov, A.A. Serga, V.S. Tiberkevich, Yu.V. Kobljanskij, A.N. Slavin, *Parametric interaction of magnetostatic waves with a nonstationary local pump*, JETP 89, 6 (1999)
3. V.S. L'vov, *Wave turbulence under parametric excitation*, Springer-Verlag (1994)

Negative transitional damping of spin waves at the bottom of magnon spectrum

Alexander A. Serga[1], Timo Neumann[1], Andrii Chumak[1,2], Burkard Hillebrands[1], Gennadiy A. Melkov[2], and Andrei N. Slavin[3]

[1] Fachbereich Physik, Kaiserslautern University of Technology, Kaiserslautern, Germany
Tel: 49-631-2053112, email: serga@physik.uni-kl.de

[2] Department of Radiophysics,Taras Schevchenko National University of Kiev, Ukraine

[3] Department of Physics, Oakland University, Rochester, Michigan 48309, U.S.A.

We present experimental results on the nonlinear non-stationary interaction between a few spin-wave (SW) groups under strong parametric pumping of one of them. We demonstrate the effect of negative transitional damping. It manifests itself through the increasing of the magnon density at the bottom of the spectrum above the equilibrium value *after* the external pumping source is switched off.

The experiment was performed on the magnon gas in an in-plane magnetized yttrium-iron garnet (YIG) film. The density of magnons was controlled in a wide range using the technique of pulsed microwave parallel pumping [1]. The pumping magnetic field was created by a 25 μm-wide microstrip resonator placed on the backside of a 5 μm thick YIG film. The dynamics of the gas was detected by means of time-resolved Brillouin light scattering (BLS) spectroscopy [2].

Two spectral peaks of scattered light were detected. The first peak at 7 GHz is caused by the spin waves excited directly through the microwave field of the pumping resonator at half the pumping frequency. By adjusting the magnetizing field to 1735 Oe this frequency was near the frequency of ferromagnetic resonance. The second peak, observed near 5 GHz, corresponds to spin waves concentrated at the bottom of the magnon spectrum in the process of thermalization of the first SW group via multistage four-magnon scattering processes. The temporal profiles of both SW groups were measured. After the pumping is switched off the intensity of the directly excited spin waves decreases exponentially with a decay time of a few nanoseconds. It is a result of a strong four-magnon scattering inside of this dense magnon group. The behavior of the bottom spin waves is completely different. Their intensity first drastically increases before it slowly decays with relaxation time of two hundred nanoseconds.

The described phenomenon is interpreted as a result of the stop of a destructive influence of the directly pumped magnons. The latter interact with the magnons from the bottom of the spectrum and scatter them over energetically higher SW states. Thus, the density of the bottom magnons depends on the competition of this additional nonlinear relaxation process and lattice relaxation with the flow of magnons to the minimal-energy state [3]. As after the pumping is switched off the density of the directly pumped magnons decays very fast the additional nonlinear relaxation channel is closed instantly and the bottom magnons experience net negative damping.

References:

1. A.A. Serga et al., Appl. Phys. Lett. 90, 022502 (2007).
2. S.O. Demokritov et al., Phys. Rep. 348, 441-489 (2001).
3. S.O. Demokritov et al., Nature 443, 430-433 (2006).

Theoretical and experimental studies on the evolution of nonlinear water waves

Arnaud Goullet & Wooyoung Choi
Dept. of Mathematical Sciences, New Jersey Institute of Technology (NJIT)
Newark, NJ 07102, U.S.A.
Tel: 973-642-7979, email: abg3@njit.edu, wychoi@njit.edu

Abstract:

Accurate modeling of surface wave dynamics in the ocean is a difficult task due to the complex nonlinear interaction between different wave components and a lack of understanding of various physical processes such as wave breaking and wind-wave interaction. Here we study a set of nonlinear evolution equations for the surface elevation and velocity potential fields derived using an asymptotic expansion technique and solve the system numerically using a pseudo-spectral method. Both regular and irregular surface wave fields are considered and our numerical solutions are validated with laboratory experiments.

References:

1. J. Smith, Phys. Rev. Lett. 10, 022222 (2001).
2. M. Jackson and D. Johnson, Stud. Appl. Math. 100, 200-300 (2002).

Modulated surface gravity waves in a shallow-water resonator

Alexander Ezersky[(1,2)], Alexei Slunyaev[(2)], Dominique Mouazé[(1)]
(1) UMR CNRS 6143 Morphodynamique Continentale et Côtière (M2C) Université de Caen-Basse Normandie 2-4 rue des Tilleuls, 14000 Caen, France
Tel: 33231565712, email: Alexander.Ezersky@unicaen.fr
(2) Institute of Applied Physics, Russian Academy of Sciences, 46 Ulyanov Street 603950 Nizhny Novgorod, Russia

Abstract:

We investigate experimentally and numerically arising of modulations of nonlinear shallow water waves in a resonator with a periodic external excitation. Experiments were carried out in a hydrodynamical channel. The length of the channel was L=17m, its width was d=0,5 m, and the depth of the liquid layer was h=0,17m. A piston type wavemaker consisting of a vertical plate set in motion by DC engine excited waves with frequencies 0,34 Hz $< F <$0,48 Hz (parameter 0,28 $< kh <$ 0,42, k is for wave number). It was shown in laboratory experiments that the modulation instability of a standing wave occured when the wave amplitude exceeded a certain threshold. The instability led to the excitation of the resonator modes. It was revealed that at the initial stage of instability, the neighboring modes of the initial standing wave are the most unstable. The instability causes generation of gravity waves of large amplitude tha! t may break. Three qualitative regimes of the wave dynamics may be distinguished on the "external force amplitude - external force frequency plane"; these are: (1) a standing periodic wave, (2) modulated waves without breaking, and (3) breaking modulated waves. The modulation growth starts at the minimum external force amplitude when the excitation frequency lies equidistantly between the frequencies of the nearest modes. The regimes without and with modulations are reproduced in numerical simulations by means of the strongly nonlinear Dommermuth and Yue code [1] with external local periodic forcing. The important role of the wave damping for different types of wave dynamics is shown. The dynamics of free weakly perturbed standing periodic waves without dissipation is studied. The quadratic nonlinearity is shown to be responsible for the leading order of wave modulation growth. The observed instability is evidently explained by the resonance between the carrier wave spectral satellites and the second harmonic of the initial strongly nonlinear cnoidial standing waves. The observed effect may lead to the formation of unexpectedly large breaking waves in semi-close basins such as bays and harbors.

References:

1. D.G.Dommermuth and D.K.P. Yeu, J.Fluid Mech. 184, 267-288 (1987)

Modulational instability in two-component discrete media with cubic-quintic nonlinearity

B. A. Umarov[1,2] B. B. Baizakov[2], A. Bouketir[1], and A. Messikh[1]

[1] Department of Computational and Theoretical Sciences, Faculty of Science, International Islamic University Malaysia, Jalan Istana, Bandar Indera Mahkota, 25200 Kuantan, Pahang, Malaysia

[2] Physical-Technical Institute of the Uzbek Academy of Sciences, G. Mavlyanov str. 2b, 100084, Tashkent, Uzbekistan

Tel: (609) 571 6754, email: bakhram@iiu.edu.my

Abstract:

Modulational instability (MI) is a generic phenomenon in nonlinear physics responsible for spontaneous pattern formation in uniform media [1]. It occurs under combined effects of nonlinearity and dispersion (in temporal domain) or diffraction (in spatial domain), when the constant background wave (CW) becomes unstable against small amplitude spatially periodic perturbations in a specific range of wavenumbers. MI is considered as a precursor to formation of localized nonlinear excitations, or soliton trains. The objective of this work consists in studying the MI in a discrete two-component NLS equation that may arise due to the effect of cubic-quintic nonlinearity, and associated inter-component couplings with applications in optics and BEC [2]. The coupled discrete NLS with cubic-quintic nonlinearity, on which further analysis will be based, has the following form

$$i\frac{dAn}{dz} + ca(An+1+An-1) + \lambda(|An|^2 + \beta|Bn|^2)An + \gamma(|An|^4 + 2\alpha|An|^2|Bn|^2 + \alpha|Bn|^4)An = 0,$$

$$i\frac{dBn}{dz} + cb(Bn+1+Bn-1) + \lambda(|Bn|^2 + \beta|An|^2)Bn + \gamma(|Bn|^4 + 2\alpha|Bn|^2|An|^2 + \alpha|An|^4)Bn = 0,$$

where $An(z)$, $Bn(z)$ are the complex probability amplitudes of components at lattice site n, ca, cb are the linear coupling constants. The variable z denotes propagation coordinate in optics applications, and time in BEC applications. The real parameters β and α are the cubic and quintic inter-component (XPM) nonlinear coupling coefficients respectively. Self-interaction (SPM) coefficients are re-scaled to be one. The coefficients λ and γ, which although can be re-scaled to one, are retained for convenience to explore the effect of only cubic ($\gamma = 0$), only quintic ($\lambda = 0$), or weighted contribution of these two nonlinear terms ($\gamma \neq 0, \lambda \neq 0$) to overall MI. We investigate the stability of stationary plane wave solution of the given above equations with respect to small modulations of the amplitude and phase. We have revealed the following peculiarities of MI induced by cubic-quintic nonlinearity and associated inter-component couplings in a two-component discrete media governed by DNLS equation: (i) For the same set of initial parameters (amplitudes of plane waves) and inter-component coupling coefficients, quintic nonlinearity gives rise to stronger MI than the cubic one. (ii) MI induced by cubic-quintic nonlinearity leads to more complete phase separation of a two-component repulsive BEC (when inter-component interaction is also repulsive), than in the case when only cubic nonlinearity is present. (iii) Among different types of localized states emerging from the MI in two-component discrete media with cubic-quintic nonlinearity, Page-like modes more frequently appear.

References:

1. F. Kh. Abdullaev, S. A. Darmanyan, J. Garnier, Prog. Opt. **44**, 303 (2002).
2. B. Wu and Q. Niu, Phys. Rev. A **64**, 061603(R) (2001)

An analytic approach to Faddeev model

Minoru Hirayama[1] and Chang-Guang Shi [2]
Department of Mathematics and Physics, Shanghai University of Electric Power,
Pinglian Road 2103, Shanghai 200090, China
Tel: 021-65430410,
email:[1]hirayama@shiep.edu.cn,
[2]shichangguang@shiep.edu.cn (correspondig person)

Abstract:

Faddeev model is an effective field theory which is expected to describe the low energy behavior of the $SU(2)$ gauge field. Because of the high nonlinearity of the model, however, its analytic analysis does not seem to have shown much progress. Here, a method to reduce the static field equation of the Faddeev model to a set of partial differential equations of first order is presented.

The model can be described by a complex field $u = Re^{i\Phi}$. We find that the field equation can be rewritten as

$$\nabla \cdot \alpha + i\beta \cdot \alpha = 0, \tag{3}$$

where α and β are defined by

$$\alpha = \frac{\nabla u^*}{1+R^2} + \frac{\nabla u^* \times (\nabla u \times \nabla u^*)}{(1+R^2)^3}, \quad \beta = \frac{2R^2}{1+R^2}\nabla\Phi. \tag{4}$$

Observing that eq.(1) is solved by

$$\alpha = \nabla\Phi \times \nabla\mu + (\nabla R \times \nabla\rho)\exp\left(-\frac{2i\Phi R^2}{1+R^2}\right) \tag{5}$$

with arbitrary complex functions ρ and μ, we are left with the conditions for the existence of ρ and μ. They are summarized as a set of first order partial differential equations.Thus, the static field equation of the Faddeev model is reduced to a set of first order partial differential equations.

As an example, the case of the vortex solution is discussed.

References:

1. Minoru Hirayama and Chang-Guang Shi, Phys.Lett.B 652,384-387(2007).

Nonclassical solitons in DNA dynamics .

Maximo Aguero (a), Ma. de Lourdes Najera(b)
(a) Facultad de Ciencias, (b) Plantel Nezahuaycoyolt, Universidad Autonoma del Estado de Mexico, Instituto Literario 100, Toluca, CP 50000 , Mexico
Tel: +52 722 2965556 ext 128, Fax: +52 722 2103235 email: mag@uaemex.mx, makxim@gmail.com

Abstract:

Non classic solitonic structures in the continuum approximation of DNA lattice model are investigated. The improved Peyrard-Bishop model with inharmonic quartic potential in the optical part of the Hamiltonian, gives rise to compact and peak soliton like structures. The two DNA's strands are linked together by hydrogen bonds, that are modeled by the Morse Potential. The non classic solitonic structures i.e. the compact cusp and the anti-peakon obtained in this work would enlarge the list of possible nonlinear structures that could be responsible for the appearance of open states along the DNA. Put the title of the talk here Nonlinear waves in periodic media
This work was partially supported by 61401 CONACYT research project

Ishimori-I Equation with Self-Consistent Sources

Juan Hu[1,2][1] Xing-Biao Hu[1][2] and Hon-Wah Tam[3][3]

[1] Institute of Computational Mathematics and Scientific Engineering Computing, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, P.O. Box 2719, Beijing 100080, P.R. CHINA

[2] Graduate School of the Chinese Academy of Sciences, Beijing, P.R. CHINA

[3] Department of Computer Science, Hong Kong Baptist university, Kowloon Tong, Hong Kong, P.R. CHINA

Abstract:

In this paper, grammian solutions of Ishimori-I (Ish-I) equation are firstly obtained by Hirota's direct method. Then the source generation procedure is utilized to generate the Ishimori-I equation with self-consistent sources (Ish-I ESCS) and its grammian solutions are derived. As a simple example, the (1, 1) dromion solution is examined.

[1]Electronic mail:hujuan@lsec.cc.ac.cn

[2]Electronic mail:hxb@lsec.cc.ac.cn

[3]Electronic mail:tam@comp.hkbu.edu.hk

A connection between HH3 and KdV with one source

Jun-xiao Zhao[1,2], Robert Conte[1,3]

[1]Service de physique de l'état condensé (URA 2464) CEA–Saclay, F–91191 Gif-sur-Yvette Cedex, France

[2]School of Mathematical Sciences, Graduate University of Chinese Academy of Sciences, Bejing, P.R.China

[3]Centre de mathématiques et de leurs applications (UMR 8536) École normale supérieure de Cachan, 61, avenue du Président Wilson, F–94235 Cachan Cedex, France.

E-mail: jxzhao@gucas.ac.cn, Robert.Conte@cea.fr

Abstract:

In this talk, we will show a connection between one of the three integrable cases of the cubic Hénon-Heiles Hamiltonian system and the KdV with one source system. In the system made of KdV with one source, we first show by applying the Painlevé test that the two components of the source must have the same potential. Then, we show how the computation of the Lax pair naturally introduces an additional term proportional to the inverse of the squared eigenfunction. This allows us to prove the identity between the travelling wave reduction and one of the three integrable cases of the cubic Hénon-Heiles Hamiltonian system.

References:

1. S. Baker, V. Z. Enol'skii, and A. P. Fordy, Phys. Lett. A 201, 167-174 (1995).
2. M. Błaszak, J. Math. Phys. 35, 1693–1709, (1994).
3. T. Bountis, H. Segur and F. Vivaldi, Phys. Rev. A 25, 1257-1264 (1982).
4. Y.F. Chang , M. Tabor, and J. Weiss, J. Math. Phys. 23, 531-538 (1982).
5. R. Conte, Phys. Lett. A 140, 383–390 (1989).
6. R. Conte and M. Musette, *The Painlevé handbook*, to appear (Springer, Berlin, 2008).
7. R. Conte, M. Musette, and C. Verhoeven, J. Nonlin.Math. Phys. 12 Supp. 1, 212-227 (2005).
8. R. Conte, M. Musette, and C. Verhoeven, Theor. Math. Phys. 144, 888-898 (2005). arXiv:nlin/0507011v1.
9. C. M. Cosgrove, Stud. Appl. Math. 104, 1-65 (2000).
10. J. Drach, C. R. Acad. Sc. Paris 168, 337-340 (1919).
11. A. P. Fordy, Physica D 52, 204–210 (1991).
12. B. Grammaticos, B. Dorizzi, and R. Padjen, Phys. Lett. A 89, 111-113 (1982).
13. M. Hénon and C. Heiles, Astron. J. 69, 73-79 (1964).
14. Ma Wen-xiu, Chaos, solitons and fractals 26, 1452-1458 (2005).
15. V.K. Mel'nikov, Lett. Math. Phys. 7, 129-136 (1983).
16. V.K. Mel'nikov, Phys. Lett. A 133, 493-496 (1988).
17. P. Painlevé, Bull. Soc. Math. France 28, 201-261 (1900).
18. V.E. Zakharov and E.A. Kuznetsov, Physica D 18, 455-463 (1986).
19. Y. Zeng, W. Ma and Y. Shao, J. Math. Phys. 42, 2113-2128 (2001).

Soliton solutions for two nonlinear partial differential equations using a Darboux transformation of the Lax pairs

Ji Lin [14], Bo Ren [1], Hua-mei Li[1], Yi-shen Li [2]
[1] *Department of Physics, Zhejiang Normal University, Jinhua 321004, China*
[2] *Department of Mathematics, University of Science and Technology of China, Hefei, 230026, China*

Abstract:
Two Darboux transformations of the (1+1)-dimensional Wu-Zhang (W-Z)equation and the 2-component Camassa-Holm (2-CH) system with the reciprocal transformation are obtained. A loop soliton, two-loop soliton solutions and multi-soliton (or -soliton like) solutions of the 2-CH system are given out by using the Darboux transformations and selecting the different seed solutions of the corresponding equations respectively. The bidirectional soliton solutions of the (1+1)-dimensional W-Z equation are obtained. The interactions of two-soliton head-on and overtaking collisions for the W-Z equation and the evolutions of the two-soliton (-soliton like) solutions for the 2-CH system are studied.

PACS: 46.40.Cd; 05.45.Yv; 02.30.Jr; 02.29.+p

Abelian functions associated with a cyclotomic tetragonal curve of genus six

Matthew England
Mathematics, School of Mathematical and Computer Sciences (MACS),
Colin Maclaurin Building, Heriot-Watt University, Riccarton, Edinburgh, EH14 4AS, UK
Tel: +44 (0)131 451 3439, email: mce3@hw.ac.uk

Abstract:
We consider specific classes of algebraic curves and define a set of abelian function upon their Jacobian. We do this with the aim of generalising the theory of the Weierstrass $\wp$-function and the associated theory of elliptic curves. A variety of methods are employed to find addition formula, PDEs, power series expansions and other properties that these functions satisfy. This research lies within the field of algebraic geometry, and has applications in non-linear PDEs and Soliton theory.

Choose (n, s) coprime with $n < s$. Then define an (n, s)-*curve* as the algebraic curve

$$y^{\mathbf{n}} = x^{\mathbf{s}} + \lambda_{s-1}x^{s-1} + ... + \lambda_1 x + \lambda_0.$$

which defines a surface with genus $g = \frac{1}{2}(n-1)(s-1)$. The simplest such curve is the (2,3)-curve (the classic elliptic curve) defining a genus 1 surface. The Weierstrass theory for this curve and the associated $\wp$ and σ-functions is well known.

$$\wp(u) = -\frac{d^2}{du^2}\ln[\sigma(u)], \qquad \frac{\sigma(u+v)\sigma(u-v)}{\sigma(v)^2\sigma(u^2)} = \wp(u) - \wp(v)$$
$$\wp'(u)^2 = 4\wp(u)^3 - g_2\wp(u) - g_3, \qquad \wp''(u) = 6\wp(u)^2 - \tfrac{1}{2}g_2$$

We define the Kleinian functions associated to an (n, s)-curve using the higher genus sigma function in analogy to the first equation above. (Here $\wp(u) \equiv \wp_{11}(\mathbf{u})$.)

$$\wp_{i..m} = -\frac{\partial}{\partial u_i}\frac{\partial}{\partial u_j}\cdots\frac{\partial}{\partial u_m}\ln\big[\sigma(\mathbf{u})\big], \qquad i \le j \le ... \le m \in \{1, ..., g\}$$

We consider the (4,5)-curve ($g = 6$) and modify the methods used in [1.] and [2.] to demonstrate how properties of the (4,5) Kleinian functions can be derived. For example, to generalise the equation for $\wp''$ we derive a large set of PDEs that express the 4-index $\wp$-functions using 2-index $\wp$-functions of order at most 2.

$$\{ \quad \wp_{6666} = 6\wp_{66}^2 - 3\wp_{55} + 4\wp_{46}, \qquad \wp_{5666} = 6\wp_{56}\wp_{66} - 2\wp_{45}, \quad \dots \quad \}$$

Kleinian functions have applications within the theory of non-linear waves. For example, differentiate the equation for $\wp_{6666}$ twice with respect to u_6 and set $\{\wp_{66} = w, u_6 = x, u_5 = y, u_4 = t\}$ to obtain a scaled version of the KP equation.

$$\big[w_{xxx} - 12ww_x - 4w_t\big]_x + 3w_{yy} = 0$$

References:
1. J.C. Eilbeck, V.Z. Enolski, S. Matsutani, Y. Onishi, and E. Previato. *Abelian Functions for Trigonal Curves of Genus Three*. Int. Math Res. Notices (2007)
2. S. Baldwin, J.C. Eilbeck, J. Gibbons, and Y. Onishi. *Abelian Functions for Purely Trigonal Curves of Genus Four.* To appear in J. Geom. Phys. (2008)

Tri-stability in a Pendula Chain

Ramaz Khomeriki,
Department of Physics, Tbilisi State University, Tbilisi 0128, Republic of Georgia.
Tel: 99532-915681, email: khomeriki@hotmail.com
Jerome Leon, Dominique Chevriaux
Laboratoire de Physique Theorique et Astroparticules CNRS-IN2P3-UMR5207, Universite Montpellier 2, 34095 Montpellier, France

Abstract:

In this poster different Stationary states of boundary driven coupled pendula chain has been investigated experimentally, numerically and analytically. The analytical treatment is based on the boundary driven sine-Gordon equation and it is found that for the same driving force three completely different regimes coexist: Besides two regimes discovered earlier [1-4] (first one is the small oscillations with exponentially decaying amplitude and the second one is a breather like solution which has an oscillation amplitude maximum at the end of the chain) there exists a third regime corresponding to the kink motion in the restricted geometry of the chain. The analytical solutions are compared with the numerical simulations on the associated Frenkel-Kontorova model and finally confirmed by direct laboratory experiments. It is proposed to extend these studies to the realistic physical systems which are governed by the same sine-Gordon equation.

References:

1. R. Khomeriki, J. Leon, Phys. Rev. E, 71, 056620 (2005).
2. R. Khomeriki, D. Chevriaux, J.Leon, European Physical Journal B, 49, 213 (2006).
3. D. Chevriaux, R. Khomeriki, J.Leon, Phys. Rev. B, 73, 214516 (2006).

A perturbative change of the nature of invariant varities of periodic points

Noriko SAITOH
Department of Applied Mathematics,
Yokohama National University,
Hodogaya-ku Yokohama 240-8501, Japan
phone: 81-45-335-4206
email: nsaitoh@ynu.ac.jp

Abstract:

We have shown in our recent papers [1] that periodic points of some higher dimensional integrable maps form a variety specific for each period if the maps have sufficient number of invariants. An invariant variety of periodic points (IVPP) is a discrete map analogue of an invariant torus in the continuous time Hamilton systems. In this contribution we study effects of a small perturbation on the varieties and show explicitly how the transition to nonintegrable regime takes place on the varieties.

[1] S.Saito and N.Saitoh: J. Phys. Soc. Jpn. **76** (2007) 024006,
[2] S.Saito and N.Saitoh: SIGMA **2** (2006) 098,
[3] S.Saito and N.Saitoh: J. Phys. A: Math. Theor. **40** (2007) 12775-12787,
[4] S.Saito and N.Saitoh: J. Phys. Soc. Jpn. **77** (2008) 024001.

Initial-boundary-value problems for linear differential-difference equations

Danhua Wang and Gino Biondini
Department of Mathematics, State University of New York at Buffalo, Buffalo, NY 14260, U.S.A.
Tel: 716-6456284 ext 164, 148, email: dwang9@buffalo.edu, biondini@buffalo.edu

Abstract:

A transform method for solving initial-boundary-value problems (IBVPs) for evolution linear partial differential equations was recently developed by A. S. Fokas. Here we show that an analogue of Fokas' method can be formulated to solve IBVPs for evolution linear differential-difference equations. We first show that, as in the continuum case, any such equation can be written as the compatibility condition of a discrete Lax pair, namely, an overdetermined linear system of differential-difference equations containing a spectral parameter. We demonstrate the method by solving explicitly the IBVP on the natural numbers for the discrete analogue of both the linear Schrödinger equation and the linearized Korteweg-de Vries equation. We then discuss a generic differential-difference equation. As in the continuum case, the method for the solution of the IBVP employs the simultaneous spectral analysis of both parts of the Lax pair. A key role is also played by the symmetries of the equation and by a relation, called the global algebraic relation, which couples all known and unknown boundary values. The method is simple to implement and yet it yields integral representations which are especially convenient for computing the long-time asymptotic of the solution.

References:

1. A. S. Fokas, *A new transform method for evolution partial differential equations*, IMA J. Appl. Math. 67, 559–590 (2002).
2. A. S. Fokas, *On the integrability of certain linear and nonlinear partial differential equations*, J. Math. Phys. 41, 4188-4237 (2000).
3. G. Biondini and G. Hwang, *Initial-boundary value problems for differential-difference evolution equations: discrete linear and integrable nonlinear Schrödinger equations*, submitted to *Inv. Probl.*
4. G. Biondini and D. Wang, *Initial-boundary-value problems for linear differential-difference equations*, in preparation.

On a new coupled KP equation and its soliton solution

Hong-Yan Wang
School of Information, Remin University of China Beijing, China
email: wanghy@lsec.cc.ac.cn

Abstract:

Source generation procedure is applied to reconstruct a new type of KP equation with self-consistent sources (KPESCS). Soliton solutions of this kind of systems include some arbitrary function of two independent variables, and this point is different from the ordinary SESCSs which soliton solutions contains arbitrary functions with respect to only one independent variable. More importantly, this SESCSs have a characteristic that it can reduce to two different simple SESCS, and it can be viewed as some kind of combination of two SESCSs. So we call this kind of SESCS a hybrid type of SESCS.

A Bilinear Backlund Transformation and Lax Pair for a (1+1)-Dimensional Differential-Difference sine-Gordon Equation

Xianmin Qian

Department of Physics, Shaoxing University, Zhejiang, China

email:xmqian@zscas.edu.cn

Abstract:

We obtain a (1+1)-dimensional integrable differential-difference model for the sine-Gordon equation by Hirota's discretization method. A bilinear Backlund transformation and the associated Lax pair are also proposed for this model.

Optical Bloch oscillation stimulated by interaction between discrete solitons

Fajun Xiao, Peng Zhang, Sheng Liu, Xuetao Gan and Jianlin Zhao
Institute of Optical Information Science and Technology and Shaanxi Key Laboratory of Optical Information Technology, School of Science, Northwestern Polytechnical University, Xi'an 710072, China
Tel: 029-88495724, email: jlzhao@nwpu.edu.cn

Abstract:

We show that optical Bloch oscillation (BO) can be stimulated by coherent and incoherent interactions between discrete solitons in one-dimensional optically induced photonic lattices. Interestingly, such BO can be observed in the uniform lattices without transverse index gradient. It is revealed that the BO can only exist for certain soliton power range. And the power range can be tuned by changing the external bias field and the intensity of the lattice beam. During the interactions between discrete solitons at different powers, we can trigger BO merely on the soliton whose power resides in the power range. Simultaneously, the BO can be switched by adjusting the phase relationship, resulting from the energy transfer during soliton interactions at phase difference $\pi/2$ or $-\pi/2$. Furthermore, the Bloch period can be also tuned by varying the soliton power and the external bias field. In addition, a direct observation of the BO evolution with the variation of period has been performed.

References:

1. J. W. Fleischer et al., Nature 422, 147 (2003).
2. F. Bloch, Z. Phys. 52, 555 (1928).
3. U. Peschel et al., Opt. Lett. 23, 1701 (1998).
4. R. Morandotti et al., Phys. Rev. Lett. 83, 4756 (1999).
5. T. Pertsch et al., Phys. Rev. Lett. 83, 4752 (1999).
6. H. Trompeter et al., Phys. Rev. Lett. 96, 053903 (2006).
7. M. Stepić et al., Phys. Rev. E 74, 046614 (2006).
8. H. Martin et al., Phys. Rev. Lett. 92, 123902 (2004).
9. F. J. Xiao, et al., Chin. Phys. Lett. 24, 3435 (2007).

Speaker Index

A

B

C

D

E

F

G

H

I

K

L

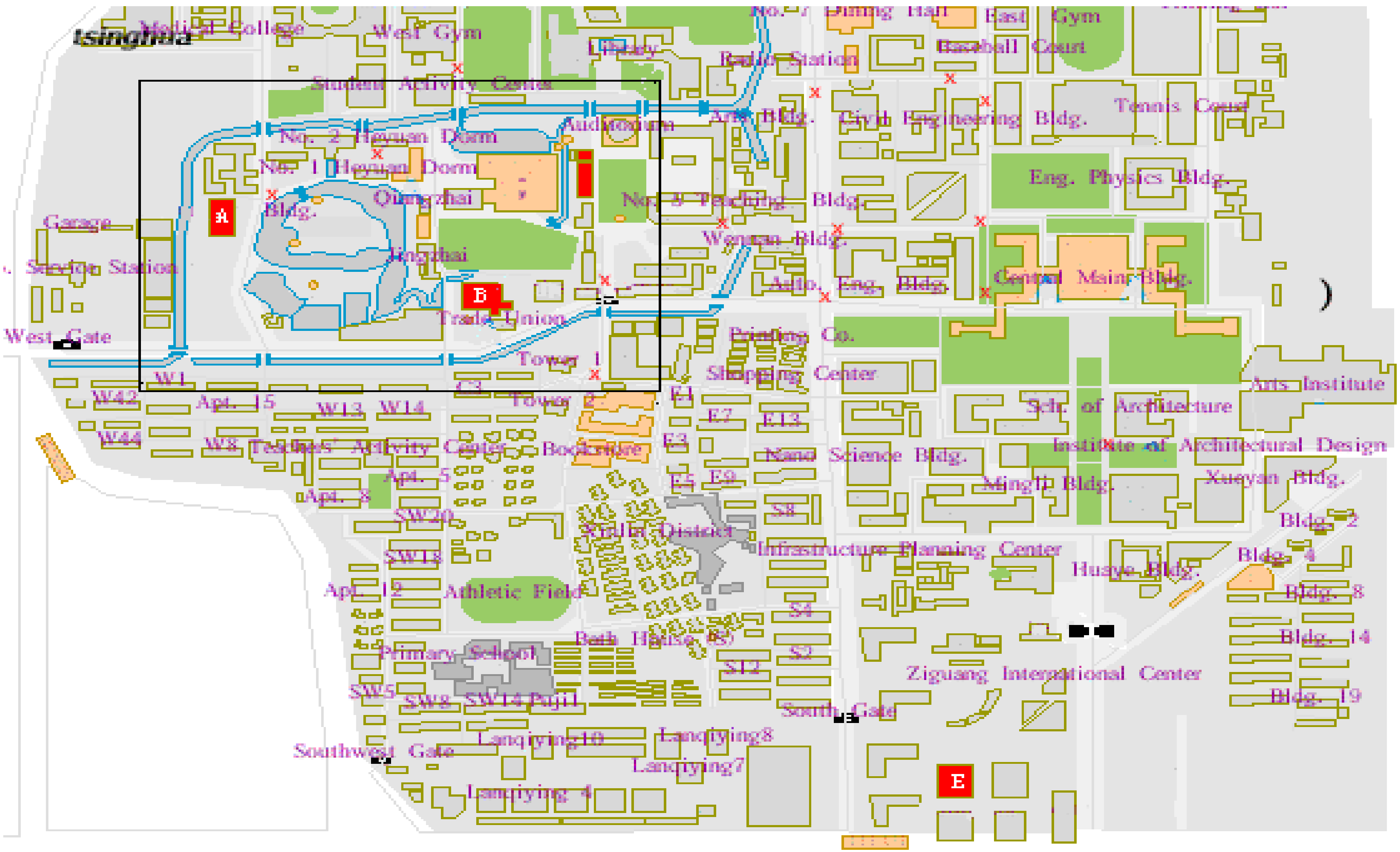
M

A: Jinchunyuan Hotel

B: Jiasuo Hotel

C: Science Building (*parallel sessions*)

D: West Auditorium (*Open ceremony and keynote talks*)



E: Quanjude Peking Duck House (Conference Reception)