EE -171 SIGNALS AND SYSTEMS
FINAL EXAM
December 17, 2009
8:00 a.m. - 11:00 a.m.

CLOSED BOOK

PLEASE WRITE YOUR SOLUTIONS ON THE SAME PAGE AS THE PROBLEM
ALSO, PLEASE UNDERLINE OR *BOX* YOUR ANSWER

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Formulae:

$$x(t) = \sum_{k=-\infty}^{+\infty} \tilde{c}_k e^{jk\omega_0 t}$$

$$\tilde{c}_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

$$y[n] = \sum_{m=-\infty}^{+\infty} x[m]h[n-m]$$

$$x[n] = \sum_{k=<N>} X[k]^{jk(\frac{2\pi}{N})n}$$

$$X[k] = \frac{1}{N}\sum_{n=<N>} x[n]e^{-jk(\frac{2\pi}{N})n}$$

$$e^{-at}u(t) = x(t) \Leftrightarrow X(\omega) = \frac{1}{(j\omega+a)}$$

$$tx(t) \Leftrightarrow \frac{1}{(j\omega+a)^2}$$

$$x(at) \Leftrightarrow \frac{1}{|a|}X(\frac{j\omega}{a})$$

$$1 \Leftrightarrow 2\pi\delta(\omega)$$

$$Cos\omega_0 t \Leftrightarrow \pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)$$

$$x(t-t_o) \Leftrightarrow e^{-j\omega t_o}X(j\omega)$$

$$x(t)e^{j\omega_o t} \Leftrightarrow X(j(\omega-\omega_o))$$

$$P_{ave} = Lim_{T\to\infty}\int_{-T}^{+T} |x(t)|^2 dt$$

$$Energy = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt$$

(Note: There are a total of 125 points.)

1. (Periodicity of signals)
(10 points)

Determine whether or not each of the following signals is periodic.
If a signal is periodic, determine its fundamental period and the fundamental frequency in radians per second.

(a) $x(t) = cos(2\pi 20t)$

(b) $x(t) = cos(2\pi 20t) - sin(2\pi 30t)$

(c) $x[n] = cos(2n)$

2. (Signal power and signal energy)
(10 points)

Determine the (average) signal *power* of the following signals:

(a) $x(t) = A$

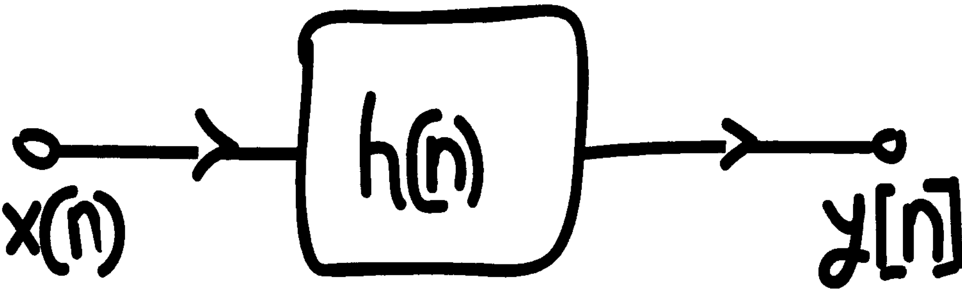
(b) $x(t) = cos(t)$. {Recall: $cos^2(t) = \frac{1}{2}(1 + cos2t)$}

Determine the *energy* of the following signal:

(a) $x(t) = A[u(t) - u(t - 10)]$.

3. (Discrete-time convolution)

(10 points)



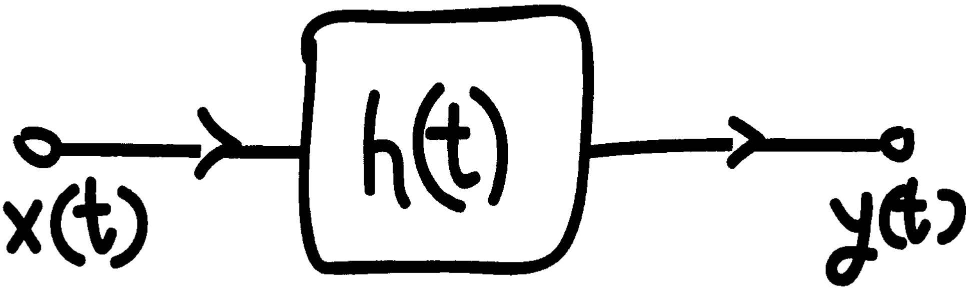
The discrete-time LTI system has input $x[n] = u[n]$. Impulse response $h[n] = u[n]$.

(a) Find output $y[n]$. Plot $y[n]$.

(b) Is the system stable? Explain.

4. (Continuous-time convolution)

(10 points)



Consider the LTI system whose impulse response $h(t)$ is as follows:

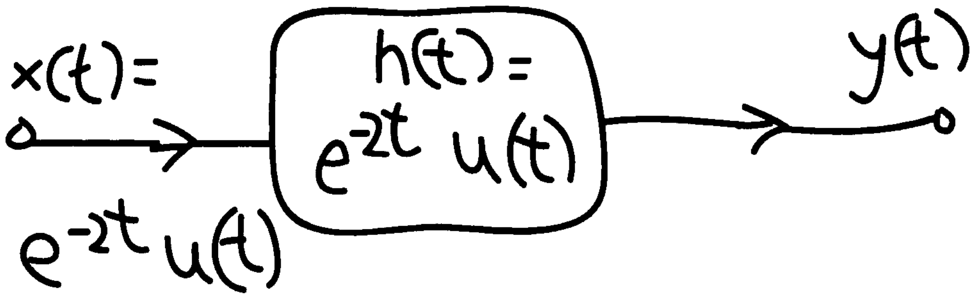
$$h(t) = e^{-2t}u(t)$$

Using convolution, determine the output $y(t)$ if input $x(t)$ is as follows:

$$x(t) = u(t)u(2-t)$$

5. (Continuous-time convolution)

(10 points)



Let the input to a LTI system be $x(t) = e^{-2t}u(t)$. Consider the situation when the impulse response has a "natural frequency" which is the same as that of the excitation. That is, $h(t) = e^{-2t}u(t)$.

Do the following:

1. What is the natural frequency?
2. Find output $y(t)$. (You can choose any method you wish to solve the problem).

6. Fourier series

(15 points)

Find the Fourier series coefficients $\tilde{c}_k$ of the following periodic signals and plot $\tilde{c}_k$.
For the plot, just show $\tilde{c}_k$. Separate magnitude and phase plots not needed.

(a) x(t) = Cos2π20 t

(b) x(t) = Sin2π30t

(c) x(t) = Cos2π20t + Sin2π30t

(d) x(t) a unit pulse of width 1 second, centered at the origin and with period $T = 10$.

7. Fourier Transform

(15 pts) *Calculate* the Fourier transform $X(j\omega)$ of the following signals:

(a) $x(t) = \delta(t)$.

(b) $x(t) = Cos 2\pi 20t$. (Determine this any way that you want.)

(c) $x(t)$ a unit pulse of width 1 second, centered at the origin. Plot the spectrum. Show points ω where the spectrum is zero.

8. Fourier Transform

(10 points)

Determine the inverse Fourier transform $x(t)$ of the following signal.

(a) $X(j\omega) = 2\pi\delta(\omega)$

(b) $X(j\omega) = 2\pi\delta(\omega - 200\pi)$

(c) $X(j\omega) = 5(2\pi\delta(\omega - 200\pi))$

(d) $X(\omega) = \frac{5}{2j}2\pi\delta(\omega - 200\pi) - \frac{5}{2j}2\pi\delta(\omega + 200\pi)$

9. (Discrete-time system)
(10 points)

Consider the discrete-time LTI system described by,

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

and suppose that

$$x[n] = (\frac{1}{3})^n u[n]$$

Do the following: Calculations are not necessary in (a), (b) (c).

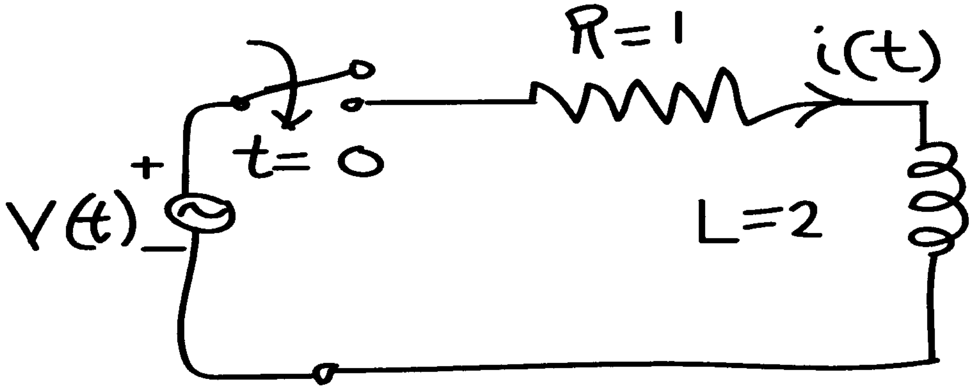
(a) Give the specific *form* of the transient solution.

(b) Give the specific *form* of the steady state solution.

(c) Give the specific *form* of the total response.

10.

(15 points)



For the circuit shown above, initially at rest (i.e. i(0-) = 0), do the following:

(a) Write the differential equation relating input $v(t)$ and output $i(t)$.

(b) Solve for the transient solution. What is the natural frequency?

(c) Determine the impulse response $h(t)$. That is, find output $i(t)$ when input $v(t) = \delta(t)$. Solve this in the time domain.

(d) Determine the impulse response from the differential equation using Laplace transforms.

(e) Apply Laplace transforms to the differential equation in part(a) and find the transfer function $T(s) = \frac{I(s)}{V(s)}$. How is the inverse Laplace transform of the transfer function related to the impulse response determined in part (d)?

11. (Laplace Transform
(10 points)

For the transfer functions $H(s)$ shown below, do the following:

(1.) Show the poles in the s-plane.

(2.) Obtain the associated inverse-Laplace transform $h(t)$. For Laplace transforms of order greater than 1, you should derive the inverse using partial fractions or any other method.

$H(s) = \frac{1}{s+2}, \qquad Re\{s\} > -2$

$H(s) = \frac{1}{s^2+2s+5}, \qquad Re\{s\} > -1$

$H(s) = \frac{1}{s-2}, \qquad Re\{s\} > 2$

$H(s) = \frac{1}{(s+2)^2}, \qquad Re\{s\} > -2$

$H(s) = \frac{1}{s^2+4}, \qquad Re\{s\} > 0$

$H(s) = \frac{1}{s^2-2s+5}, \qquad Re\{s\} > 1$