

# A SIMPLE AND EFFICIENT WAVELET-BASED DENOISING ALGORITHM USING JOINT INTER- AND INTRASCALE STATISTICS ADAPTIVELY

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## ABSTRACT

We propose a simple and efficient image denoising algorithm in the wavelet domain. The algorithm adaptively weighs the joint inter- and intrascale statistics of detail coefficients. Direct correlation of detail coefficients across scales is used to select the significant coefficients. Intrascale statistics are used to adaptively modify the coefficients, using a new homogeneity measure. Unlike existing algorithm using parametric models, prior knowledge and estimation of parameters are not needed. New justification is provided for the choice of the ‘most regular’ wavelet derived from B-splines. The implementation is simple and efficient, with a performance comparable to results by state-of-art methods.

## 1. INTRODUCTION

Over the last decade, wavelet-based methods for the removal of noise without blurring the important features in signals have received a great deal of attention. Typically, important features are characterized by large wavelet coefficients across the scales, while most of the noise power is confined to several fine scales, thus facilitating the separation of feature-related coefficients and noise-related coefficients. Donoho and Johnstone [3] pioneered the use of thresholding in signal denoising. The general procedure for wavelet-based denoising algorithms consists of the following: *Forward wavelet transform, Modification of detail coefficients, Inverse wavelet transform*. The choice of the wavelet and the so-called shrinkage or modification of the coefficients are two of the critical components in this process.

Mallat and Zhong [6] first introduced the complete multiscale edge representation of signals using quadratic spline wavelets. They showed that multiscale edges can be detected and characterized from the local maxima of the wavelet transform. That idea was adopted by Xu *et al.* [14] in their simplified algorithm to remove additive white Gaussian noise (AWGN) from signals. Instead of calculating the Lipschitz exponents [6] to identify edges, Xu *et al.* proposed using correlation of wavelet coefficients across adjacent scales to distinguish significant edges from noise.

Recently, Faghih and Smith [4] have combined this scale-space method with statistical spatial domain analysis to achieve better edge localization and noise removal.

Statistical modeling of wavelet coefficients is another important process in methods for noise removal [2] [9] [7]. All such methods provide good denoising in a mean-squared-error (MSE) sense, mostly due to a Wiener filtering process in the wavelet domain. However, parameter estimation, requiring the solution of numerical equations, is computationally expensive. Since accurate estimation is necessary for good denoising, that computation is not easily avoidable. That then creates the bottleneck for real-time applications.

In this paper, the choice of the wavelet representation and the method used in modification of wavelet coefficients are re-examined. We provide new justification for our representation which uses wavelets derived from B-splines. A new method based on joint inter- and intrascale statistics is proposed for modifying the detail coefficients. First and second statistics of detail coefficients are used without resorting to parametric models. Since no computation is needed for parameter estimation, the computational burden is highly reduced. We see that the performance obtained compares very favorably with that of the state-of-art algorithms [9] [7].

## 2. THE APPROPRIATE REPRESENTATION

The choice of an appropriate representation of images can be fundamental in the solution of specific task. For the class of denoising algorithms discussed here, orthogonal wavelet representation is inappropriate due to the fact that no real-valued wavelets can have orthogonality and symmetry properties simultaneously. The requirement of isotropy for operators is important also for most gradient-based edge detection methods used in edge-preserving denoising algorithms. In addition, as realized by many researchers [2], overcomplete representations are better suited than orthogonal representations in applications such as denoising, etc. Our proposed method utilizes cubic B-splines wavelets to maintain the isotropy. Oversampling is used to obtain the necessary coefficient correlation within and across scales.

## 2.1. Wavelets Derived using B-Splines

In the setting of Mallat and Zhong [6], the scaling function is a smooth function  $\theta(x)$ , chosen as the cubic spline. The wavelet is the first order derivative of  $\theta(x)$ . We emphasize the fact that this choice is coincident with the property that the solution of regularized numerical differentiation problems is the cubic spline [10]. We recall the key equations in the derivation of wavelets based on B-splines [13]: First, the B-spline of order  $n$  is generated by  $n + 1$ -fold convolution of the B-spline of order 0,  $\beta^n(x) = \beta^0 * \beta^0 * \dots * \beta^0(x)$ , where  $\beta^0(x)$  is the box function with the support  $[-\frac{1}{2}, \frac{1}{2}]$ . It is known that B-splines are  $m$ -refineable. That is, for  $m = 2$ , we have the two-scale equation,

$$\beta^n(x) = \sum_{k=-\infty}^{+\infty} 2B_2^n(k)\beta^n(2x - k), x \in R,$$

where  $B_2^n$  is the discrete B-spline of order  $n$  with expansion factor 2 [11]. Therefore,  $\beta^n(x)$  qualifies as a scaling function. The corresponding wavelet is obtained through differentiation

$$\psi^n(x) = \frac{d\beta^{n+1}(2x)}{dx} = 2[\beta^n(2x + \frac{1}{2}) - \beta^n(2x - \frac{1}{2})],$$

and is referred to as the *Canny-operator-like wavelet function* [13]. It is easily verified that the associated high-pass filter is the well-known first-order difference operator with a half unit shift.

The choice of the synthesis subspaces is not unique. For simplicity, the synthesis subspace  $\tilde{V}$  is chosen the same as the analysis subspace  $V$ . Therefore, the synthesis lowpass filter  $\tilde{h}$  is the same as the analysis lowpass filter  $h$ . The synthesis highpass filter for the subspace  $\tilde{W}$  can be found using the perfect reconstruction constraint [13]. In our analysis we use cubic B-spline and its derivative to generate the lowpass filter  $h$  and the highpass filter  $g$ . We note however that the justification provided below for using B-splines is valid for any order.

## 2.2. Justification for the B-Spline Choice

Fig. 1 shows the spectra of  $h_1 * g_2$  where  $h_1$  and  $g_2$  are the first level lowpass and second level highpass analysis filters in a *trous* algorithm [6]. We observe that the spectrum of  $h_1 * g_2$ , (as also the detail spectrum stemming from all other lowpass levels), behaves like the optimal Farid-Simoncelli numerical differentiator [5]. That is, the spectrum is linear in the lowpass region with a sharp decay in the high frequency part. This again provides verification for the use of wavelet representation based on B-splines: the first derivative of cubic B-splines are optimal for regularized differentiation for edge detection in noisy signals [10].

As further justification for denoising using wavelets based on B-splines, we consider image restoration using different wavelets. Ideally, the goal is to recover the detail bands without error. That is, the final low pass image

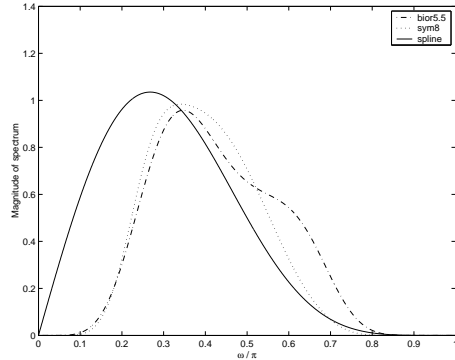


Figure 1: Spectra of  $h_1 * g_2$  (see text) for typical wavelets

should be devoid of all detail. We measure this effect experimentally by considering a 4-level decomposition. The approximation band of the noisy signal at level 4 and the 4 details bands from the original signal are used for reconstruction. We use the 512 x 512 *Lena* image with AWGN with  $\sigma = 20$ . We obtain a PSNR of 46.99 dB, 47.21 dB and 54.56 dB for *sym8*, *bior5.5* and *B-spline* wavelets respectively. Not surprisingly, the wavelet representation using B-splines gives the best result. Similar analysis at other levels of decomposition yield the same conclusion. Yet further insight on the B-spline choice can be found in the factorization of scaling functions as reported in [12]. From a new viewpoint, Unser and Blu represent a scaling function as the convolution of a B-spline (the regular part) and a distribution (the irregular part). If the distribution part is chosen as the delta function, we will find a set of 'most regular' scaling function consisting of only the regular part. In other words, the 'irregular parts' of the signal are left in the detail bands to be used in the shrinkage operation of the coefficients.

Wavelet	level 1	level 2	level 3	level 4
sym8	28.92	34.90	40.97	46.99
bior5.5	29.09	35.12	41.20	47.21
B-spline	35.04	41.98	48.29	54.56

Table 1. Experimental bounds (PSNR in dB) (see text)

## 3. SHRINKAGE OPERATION

Xu *et al.* use direct correlation of wavelets coefficients at adjacent scales to distinguish important edges from noise. As reported, they noticed that their mask filters worked well mainly for images with sharp boundaries, such as MR images. They encountered difficulty when treating smooth features in images [14]. We argue that the reason for that effect was their dependence on only interscale correlation for the selection of detail coefficients. Intrascale correlation was ignored. Since significant coefficients are also corrupted by noise, it would appear necessary to modify them as appropriate. We note here that the Xu *et al.*, procedure is similar to Donoho-Johnstone's hard thresholding method in that undesired detail coefficients are removed.

Also 'hard thresholding' is uniform with respect to the whole scale, not adapted to the local behavior. Since, in most natural images, the sharpness of edges vary in different regions, a uniform threshold is not appropriate. Hence, spatially adaptive thresholding techniques have been proposed [2] [9] [4] [7].

We propose to use the intrascale correlation of detail coefficients, that is, the estimated derivative of the smoothed signal, to enhance the identification of edges. Since each feature corresponds to a block of coefficients, we work with blocks, which is natural in the sense of preserving the features. We use the homogeneity measure [1]

$$h[m, n] = \frac{(C_{xx}[m, n] - C_{yy}[m, n])^2 + 4C_{xy}^2[m, n]}{(C_{xx}[m, n] + C_{yy}[m, n])^2},$$

where  $C_{xy} = \sum_{m,n} \{W^x I[m, n]\} \{W^y I[m, n]\}$ .  $I[m, n]$  denotes the image and  $WI[m, n]$  denotes the detail coefficients. (In our experiments, the window sizes for 256 x 256 images are chosen as {8, 4, 3} for three-level decomposition.) It is easy to verify that this homogeneity measure is dimensionless and normalized. Experimental results show that this measure reaches 1 when a pixel is near edges and reaches 0 for edge-free regions. In our algorithm, the homogeneity measure  $h$  is used adaptively to generate the new mask,  $mask_{new} = mask_{old} \times (1 - r) \times h + mask_{old} \times r$ , where  $mask_{old}$  is the mask used by Xu *et al.* to select the coefficients, and  $r = e^{-\sigma_n/10}$  is an empirical measure of the ratio of the use of interscale and intrascale statistics. If the noise level  $\sigma_n$  is large,  $r$  approaches 0; thus the use of intrascale statistics dominates, and the modification is 'hard'. While the noise level is small,  $r$  approaches to 1; then, the interscale statistics dominates, and the modification is 'soft'. Significant PSNR gains are achieved using this adaptive modification. In an independent study [8], intrascale correlation is used in a so-called *multiscale structure tensor*. That method is computationally intensive requiring the determination of eigenvalues. Even so, the PSNR performance (30.0 dB) is not as good as that achieved (31.1 dB) by our adaptive modification scheme, for the noisy Lena image (input PSNR = 22.1 dB).

In summary, the new denoising algorithm is as follows:

- Forward transform. Calculate homogeneity measures  $h_j[m, n]$  for each scale.
- Use the correlation of two consecutive scales to select significant detail coefficients; Use the homogeneity measure adaptively to modify the selected coefficients.
- Inverse transform.

#### 4. EXPERIMENTAL RESULTS

The new technique was applied to some standard images (same as [7]) with AWGN at different levels. Since slightly different versions of these images exist and lacking completeness in the reported experimental data, we compare our results with the following established denoising methods: *wiener2* function implemented in MATLAB, SWT De-noising with soft-thresholding in MATLAB, Strela's Wiener

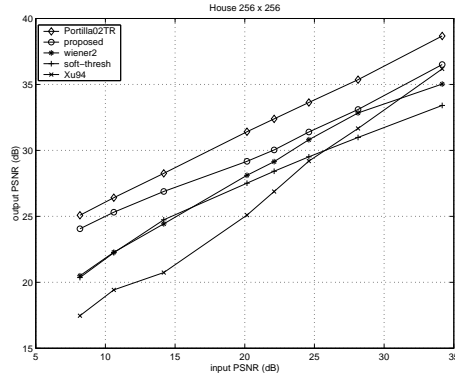


Figure 2: objective comparison (house)

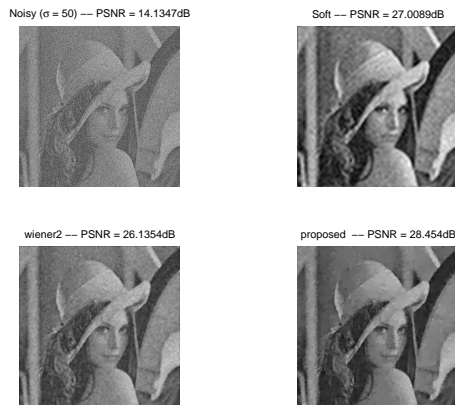


Figure 3: subjective comparison (Lena)

wavelet method [9], and the method proposed by Portilla *et al* [7]. *wiener2* is based on adaptive least squares smoothing. It utilizes the first and second order statistics estimated using a local window with a predefined size for each pixel. In our experiments, we choose the optimal window size  $w$  from the set  $3 \leq w \leq 11$ , based on PSNR values ( $PSNR = 20 \log_{10} \frac{255}{\sigma_{mse}}$ ). For the soft-thresholding method, we choose 'sym8' for decomposition. Results from other experiments are directly adopted from reports [7] [9].

In Fig. 2, the house image (256 x 256) with different noise levels is used for comparison. The proposed technique shows a greater PSNR over *soft-thresholding*, Xu *et al.* method, and *wiener2* at all noise levels, while it is still comparable to the results obtained by Portilla *et al.*, which are the best results reported in the literature. The reason the proposed method fares a little worse than the latter is that the texture parts in the image are not easily separated from noise in edge-related denoising algorithms. Conversely, more complex statistical models are better equipped to solve such problems. Table 2 compares the results for different images for two fixed noise levels. We also observe a similar advantage in a subjective comparison. In Figure 3 we see that the reconstructed image using the proposed method preserves significant features and is visually pleasant in comparison with *wiener2* and

*soft-thresholding* .

Methods	Lena	Barb	Boats	Peppers
$\sigma_n$	25	25	50	50
noisy	20.16	20.15	14.17	14.16
wiener2	28.81	24.28	23.75	23.51
Strela [9]	29.53	25.19	—	—
proposed	29.51	25.08	25.72	24.27
Portilla [7]	31.71	29.13	26.36	25.96

Table 2. PSNR (in dB) for different images

The computational cost of the overcomplete wavelet transform is  $O(MN_xN_y)$ , where  $M$  is the number of scales and  $(N_x, N_y)$  are the dimension of the image. Our new modification method also has this low computational cost. In our MATLAB implementation, on a Dell PC with 1.7GHz Intel P4 CPU, the run time (about half spent in the analysis and synthesis steps) are less than 10 CPU seconds for 256 x 256 images (less than 40 CPU seconds for 512 x 512 images).

## 5. CONCLUSION

Traditional wavelet-based image denoising algorithms identify and zero out the image wavelet coefficients due to the noise. We specifically address the issue of wavelet selection and the efficient modification of wavelet coefficients. As has been generally accepted in denoising problems, we too use the oversampled representation. We give new justification for the choice of the 'most regular' wavelet. The success of our method benefits from not only the proper representation but also the utilization of joint inter- and intrascale statistics of detail coefficients. The utilization of a simple homogeneity measure brings significant gain in performance. Implementation of the new algorithm is simple, results are visually pleasant and comparable to that of state-of-art algorithms.

## 6. REFERENCES

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