

Three-dimensional field structure in open unstable resonators Part I: Passive cavity results

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Abstract The three-dimensional field distribution of the diffractive cavity mode structure in a passive, open, unstable resonator is presented as a function of the equivalent Fresnel number of the cavity. The qualitative structure of this intracavity field distribution, including the central intensity core (or oscillator filament), is characterized in terms of the Fresnel zone structure that is defined over the cavity feedback aperture. Previous related research is reviewed.

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References and Links

- 1 A. E. Siegman, *Lasers* (University Science Books, 1986) Chapters 21-23.
- 2 A. E. Siegman and R. Arrathoon, "Modes in unstable optical resonators and lens waveguides," *IEEE J. Quant. Electron.* **QE-3**, 156-163 (1967).
- 3 K. E. Oughstun, "Unstable resonator modes," in *Progress in Optics*, vol. xxiv, E. Wolf, ed. (North-Holland, 1987) pp. 165-387.
- 4 E. A. Sziklas and A. E. Siegman, "Diffraction calculations using fast Fourier transform methods," *IEEE Proc.* **62**, 410-412 (1974).
- 5 Yu. A. Anan'ev, "Unstable resonators and their applications (Review)," *Sov. J. Quant. Electron.* **1**, 565-586 (1972).
- 6 Yu. A. Anan'ev, *Laser Resonators and the Beam Divergence Problem* (Adam Hilger, 1992).
- 7 W. H. Steier and G. L. McAllister, "A simplified method for predicting unstable resonator mode profiles," *IEEE J. Quant. Electron.* **QE-11**, 725-728 (1975).
- 8 K. E. Oughstun, "Passive cavity transverse mode stability and its influence on the active cavity mode properties for unstable optical resonators," in *Optical Resonators*, SPIE Proceedings vol. 1224, D. A. Holmes, ed. (SPIE, 1990) pp. 80-98.
- 9 Yu. A. Anan'ev, "Angular divergence of radiation of solid-state lasers," *Sov. Phys. Usp.* **14**, 197-215 (1971).
- 10 Yu. A. Anan'ev, "Establishment of oscillations in unstable resonators," *Sov. J. Quant. Electron.* **5**, 615-617 (1975).
- 11 C. C. Khamnei, *Open Unstable Optical Resonator Mode Field Theory* (Ph.D. dissertation, University of Vermont, 1998).

1. Introduction

The essential optical parameters for describing the transverse mode structure properties of an open, unstable cavity with a single, sharp-edge feedback aperture are the cavity magnification M and equivalent Fresnel number N_{eq} . These properties are embodied in the appropriate diffractive transverse mode eigenvalue equation [1-3]; for a cylindrical cavity, each azimuthal component radial mode satisfies the integral equation

$$\tilde{\gamma}_{nl} u_{nl}(r) = 2\pi e^{-i(l+1)\pi/2} \frac{N_c}{M} \int_0^1 u_{nl}(r_0) J_l(2\pi N_c r r_0 / M) \exp[i\pi N_c (r_0^2 + r^2 / M^2)] r_0 dr_0, \quad (1)$$

where $l = 0, \pm 1, \pm 2, \dots$ is the azimuthal mode index and $n = 1, 2, 3, \dots$ is the radial mode index for the total cavity mode field $u(r, \phi) = \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} u_{nl}(r) \exp(il\phi)$. The radial mode indices are chosen such that $\gamma_{nl} \geq \gamma_{n+1, l}$, where $\gamma_{nl} = |\tilde{\gamma}_{nl}|$ denotes the magnitude of the complex eigenvalue of the mode. The integration domain in Eq. (1) extends over the normalized transverse radial extent of the cavity feedback aperture. Here

$$N_c = \frac{Ma^2}{B\lambda} \quad (2)$$

is the collimated Fresnel number of the cavity [1], where $2a$ is the feedback aperture dimension, B is the equivalent collimated cavity length, and λ is the wavelength of the cavity wave field.

The cavity magnification M determines the geometric optical properties of the transverse mode structure, as described by the geometrical mode equation of Siegman and Arrathoon [2]

$$\tilde{\gamma}_{nl}^{(G)} u_{nl}^{(G)}(r) = \frac{f_l}{M} u_{nl}^{(G)}(r/|M|) \quad ; \quad r < |M| \quad (3)$$

for a cylindrical, circular aperture cavity, where $f_l = 1$ for a positive branch ($M > 1$) cavity, while $f_l = (-1)^{l+1}$ for a negative branch ($M < -1$) cavity. This relation expresses the conservation of energy in a single geometrical magnification of the cavity field and follows from the asymptotic behavior of the integral equation (1) in the limit as the collimated Fresnel number N_c approaches infinity. Superimposed on this geometrical optics contribution is the diffractive edge-scattered wave from the cavity feedback aperture whose first-order contribution modifies the geometric mode equation (3) into the form [3]

$$\begin{aligned} \tilde{\gamma}_{nl}^{(1)} u_{nl}^{(1)}(r) = & \frac{f_l}{M} u_{nl}^{(1)}(r/|M|) \\ & - \frac{e^{-i\pi l}}{\pi(MN_c r)^{3/2}} \frac{u_{nl}^{(1)}(1)}{1 - (r/M)^2} \exp\left[i\pi N_c \left(1 + (r/M)^2\right)\right] \\ & \left\{ \cos\left(2\pi N_c \frac{r}{M} - l\frac{\pi}{2} - \frac{\pi}{4}\right) - i \frac{r}{M} \sin\left(2\pi N_c \frac{r}{M} - l\frac{\pi}{2} - \frac{\pi}{4}\right) \right\} \quad (4) \end{aligned}$$

for $0 < r < |M|$ asymptotically as $N_c \rightarrow \infty$. The transverse mode structure for a large Fresnel number cavity is then seen to be dominated by the geometric optics mode solution plus the secondary edge diffracted wave that is a characteristic of the cavity magnification and collimated Fresnel number.

In the opposite extreme as the collimated Fresnel number N_c becomes small and approaches zero in the limit, the transverse mode equation (1) may be approximated as

$$\tilde{\gamma}_{nl} u_{nl}(r) \cong 2\pi e^{-i(l+1)\pi/2} \frac{N_c}{M} \exp\left(i\pi N_c \frac{r^2}{M^2}\right) \int_0^1 u_{nl}(r_0) J_l\left(2\pi \frac{N_c}{M} r r_0\right) r_0 dr_0 \quad , \quad (5)$$

which is proportional to the Hankel transform of the feedback aperture field. The transverse mode structure in a small Fresnel number unstable cavity is then dominated by the edge diffracted wave.

The collimated Fresnel number N_c of the unstable cavity represents the number of Fresnel half-period zones for a plane wave field filling the magnified feedback aperture when viewed from the center of that aperture at a distance of one equivalent collimated cavity length away [3] and describes the diffractive phenomena that occurs in a single round-trip propagation through the cavity. As such, it is the parameter upon which the sampling criteria for the numerical evaluation of the transverse mode structure are based [3, 4]. However, it does not describe the Fresnel zone structure of the cavity mode field distribution at the feedback aperture, as this is the result of repeated round-trip iterations through the optical cavity. That parameter must be defined in terms of the number of Fresnel half-period zones that are present in the geometrical optics mode phase front incident upon the feedback aperture plane as viewed from the center of the feedback aperture plane one iteration removed. As a consequence, the equivalent Fresnel number N_{eq} of an unstable cavity is defined such that the quantity $N_{eq}\lambda$ is equal to the sagittal distance between the expanding geometrical optics mode phase front and the corresponding converging wave front at the edge of the cavity feedback aperture [5, 6]. With this definition the general expression [1]

$$N_{eq} = \frac{M^2 - 1}{2M^2} N_c \quad (6)$$

is obtained within the paraxial approximation.

2. Fresnel Zone Structure of an Open Unstable Cavity

The diffractive mode properties of an unstable open cavity may be understood through a consideration of the edge diffraction effects introduced by the cavity feedback aperture [5, 6]. Diffraction at the cavity feedback aperture produces both a reflected feedback field and an edge scattered field whose virtual source is located at the edge of the feedback aperture, as described in Eq. (4). A portion of this edge-scattered field gives rise to a converging, demagnifying wave field which is nearly completely retained within the cavity, even after several round-trip propagations through the cavity, and only begins to diverge when the fundamental physical process of diffraction begins to dominate its geometric demagnification. By comparison, the flux of the magnifying geometrical mode field decreases by the factor $L_n = 1 - |\gamma_n^{(G)}|^2$ after each round-trip propagation through the cavity, while the field scattered along other directions rapidly escapes from the unstable cavity. As a consequence, the intensity of the converging wave, which is negligible near the edge of the cavity feedback aperture, is geometrically amplified as it propagates inward toward the cavity optical axis to such an extent that it has a significant influence on the entire structure of the cavity field [5]. It is precisely this mechanism of diffractive feedback, along with its coherent interaction with the magnifying cavity field that determines the diffractive mode properties of an unstable optical cavity. The coherent radiation in the diverging cavity field that is incident upon the edge of the feedback aperture and is then scattered into the converging wave travels the distance $N_{eq}\lambda$ between these two wavefronts. As a consequence, when the equivalent Fresnel number of the cavity is changed by unity, the phase shift between the diverging and converging waves changes by 2π and the coherent interaction between those two wave fields is essentially unchanged [5, 6]. The diffractive properties of an unstable resonator with a sharply defined feedback aperture are then quasiperiodic with respect to the equivalent Fresnel number of the cavity.

The diffractive structure of the cavity mode field may be explained in terms of the Fresnel zone structure that is defined over the feedback aperture [3, 7, 8]. For a cylindrically symmetric cavity, the sagittal distance between the magnifying and demagnifying geometrical mode phase fronts at the circular feedback aperture is, in the paraxial approximation, given by [3]

$$\Delta(r) \cong \frac{1}{2} \left(\frac{1}{r_+} - \frac{1}{r_-} \right) r^2 \quad (7)$$

for $r \leq a$, where a is the transverse radial extent of the cavity feedback aperture. Here $1/r_{\pm} = (1/B) \left[-(A-D)/2 \pm \sqrt{((A-D)/2)^2 - 1} \right]$, where the quantities A , B , and D are elements of the paraxial ray-transfer matrix $\mathbf{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ for a single round-trip propagation through the stable cavity, where the unimodular property $AD - BC = 1$ is related to the Lagrange invariant of the paraxial optical system. For a positive branch cavity, $(1/r_+) - (1/r_-) = (M^2 - 1)/(MB)$, so that

$$\Delta(r) \cong \frac{M^2 - 1}{2MB} r^2. \quad (8)$$

A radially dependent equivalent Fresnel number function may then be defined at the cavity feedback aperture as [8]

$$N_{eq}(r) \cong \frac{\Delta(r)}{\lambda} \cong N_{eq} \frac{r^2}{a^2} \quad (9)$$

for $r \leq a$. At the feedback mirror edge ($r = a$) this function is equal to the equivalent Fresnel number of the cavity. The associated cavity Fresnel zones over the circular feedback aperture are then defined by the set of concentric circles whose radii $r_n \leq a$ satisfy the condition

$$N_{eq}(r_n) = n + f \leq N_{eq}, \quad n = 0, 1, 2, 3, \dots, \quad (10)$$

where $0 \leq f < 1$ such that the fractional number f is set by the numerical value of N_{eq} . The radii of the cavity Fresnel zones at the feedback aperture are then given by

$$r_n = a \left(\frac{n + f}{N_{eq}} \right)^{1/2}, \quad n = 0, 1, 2, 3, \dots \quad (11)$$

The importance of the central Fresnel zone lies in the observation that it is from this central core region of the resonator that the cavity field propagates out from and constructs the remainder of the cavity field [5-7]. The resulting edge-diffracted wave component from the feedback aperture edge that gives rise to the converging wave propagating back into the cavity provides the feedback to the central core region and accounts for the Fresnel number dependence of the cavity mode structure.

Anan'ev [9] has pointed out that a laser with an unstable cavity corresponds to an optical system comprised of a driving generator and an amplifier with a matching telescope between them. The role of the generator is played by the central intensity core that is defined by the central Fresnel zone of the cavity and the role of the amplifier by the remaining peripheral zone of the cavity, with the edge-diffracted field at the feedback aperture edge providing the controlling feedback to the central intensity core. It is this mechanism of diffractive feedback into a converging wave field and its interaction with the magnifying or diverging cavity field that produces the central intensity core and determines the diffractive properties of the cavity mode structure. It is important to recognize that the converging and diverging cavity mode wave fields are intimately related to each other, as is evident in both the geometrical and diffractive wave theories [1-3]. As stated by Anan'ev [10]: "Both converging and diverging waves form two different complete systems of functions which can be used equally satisfactorily for expanding an arbitrary signal as a series. Expansions produced by these two methods naturally give identical final results." The explicit form of these expansions may be found in Ref. 3.

3. Three-Dimensional Field Structure in a Positive Branch Half-Symmetric Unstable Cavity

The Fresnel zone structure and associated central intensity core of an unstable resonator is best illustrated through a detailed consideration of the three-dimensional field structure of the dominant cavity mode. For this purpose, a positive branch half-symmetric unstable cavity geometry, illustrated in Fig. 1, was chosen. The cavity magnification was set at $M = 2$ and the intracavity field structure was numerically determined as a function of the equivalent Fresnel number of the cavity. The diffractive field calculations are based on the angular spectrum of plane waves representation utilizing the Fast Fourier Transform (FFT) as described, along with the associated sampling criteria, in Ref. 3. Once the dominant, azimuthally symmetric ($l = 0$) mode field structure incident upon the cavity feedback mirror was obtained in a Fox and Li type iteration procedure, the three dimensional intracavity field distribution was obtained by calculating the diffractive feedback field at 80 transverse planes evenly spaced through the unfolded cavity [11].

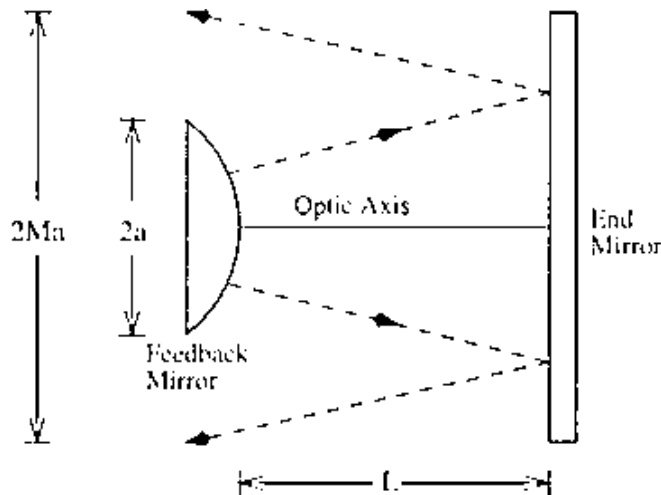


Fig. 1. Half-symmetric, positive branch unstable cavity geometry with magnification M .

Consider first the small equivalent Fresnel number series presented in Figs. 2-7. In Fig. 2, $N_{eq} = 0.5$ so that $n = 0$ and $f = 0.5$; the feedback mirror (which extends in the transverse dimension from $-a$ to $+a$ indicated at the left of this and subsequent figures) then encompasses one-half of the central Fresnel zone and a well-defined central intensity core is seen to emanate from this region into the central volume of the cavity with a transverse mode discrimination ratio $\gamma_{1,0}/\gamma_{2,0} = 2.34$ that is at (or very near to) the global maximum for this $M = 2$ cavity, as well as a near maximum eigenvalue magnitude $\gamma_{0,1} = 0.746$ and minimal outcoupling loss $L = 1 - \gamma_{0,1}^2 = 0.443$. At $N_{eq} = 0.75$, so that $f = 0.75$, the feedback mirror encompasses three-quarters of the central Fresnel zone and the transverse mode discrimination ratio has decreased in value to $\gamma_{1,0}/\gamma_{2,0} = 1.38$; the central intensity core illustrated in Fig. 3 is not as well-defined about the optic axis as that depicted in Fig. 2. When $N_{eq} = 1.0$, so that $n = 1$ and $f = 0$, the feedback mirror occupies a full Fresnel zone, the transverse mode discrimination ratio is near minimal at $\gamma_{1,0}/\gamma_{2,0} = 1.06$, and the central intensity core is now poorly defined about the optic axis due to destructive interference between the converging and diverging wave fields, as seen in Fig. 4. The eigenvalue magnitude $\gamma_{0,1} = 0.661$ of the dominant cavity mode is now very near to a local minimum with an associated locally maximum outcoupling loss of $L = 0.563$ due to the poorly defined central intensity core. This marks the beginning of the next cycle with $n = 1$. When $N_{eq} = 1.25$, as illustrated in Fig. 5, so that $n = 1$ and $f = 0.25$, the central Fresnel zone occupies (radially) the inner 20% of the feedback mirror and the definition of the central intensity core of the cavity mode field about the optic axis has increased from that depicted in Fig. 4. Locally optimal behavior is obtained when $N_{eq} = 1.5$, so that $n = 1$ and $f = 0.5$, as seen in Fig. 6. The eigenvalue magnitude $\gamma_{0,1} = 0.689$ is now at (or very near to) a local maximum, as is the transverse mode discrimination ratio with a value of 1.26, with an associated locally minimal outcoupling loss of $L = 0.525$ due to the well-defined central intensity core that extends from the feedback aperture past the end mirror of the cavity. At $N_{eq} = 1.75$, as illustrated in Fig. 7, so that $n = 1$ and $f = 0.75$, the central Fresnel zone occupies (radially) the inner 42.86% of the feedback mirror and the definition of the central intensity core of the cavity mode field about the optic axis has decreased from that depicted in Fig. 6, with an associated decrease in the eigenvalue magnitude.

The same behavior is obtained for larger equivalent Fresnel number cavities [11], as illustrated in Figs. 8-11. At the integer equivalent Fresnel number $N_{eq} = 6.0$, the central intensity core is weakly defined, as seen in Fig. 8, has improved definition at $N_{eq} = 6.25$, as seen in Fig. 9, achieves a local optimum at $N_{eq} = 6.5$, as seen in Fig. 10, and decreases in definition at $N_{eq} = 6.75$, as seen in Fig. 11.

The numerical results presented in Figs. 2-11 clearly illustrate the Fresnel zone structure of the passive cavity mode field distribution and the interrelationship between the central intensity core and the transverse mode discrimination ratio. A notable characteristic of each of these three-dimensional passive cavity mode field distributions is that each represents the decaying cavity field, the peak in the intensity structure appearing in the feedback field, the relative intensity decreasing as the field propagates away from the feedback mirror because of the cavity magnification. The opposite occurs for a laser with an unstable cavity since the gain medium compensates for both the geometric magnification and the resultant outcoupling loss from the cavity.

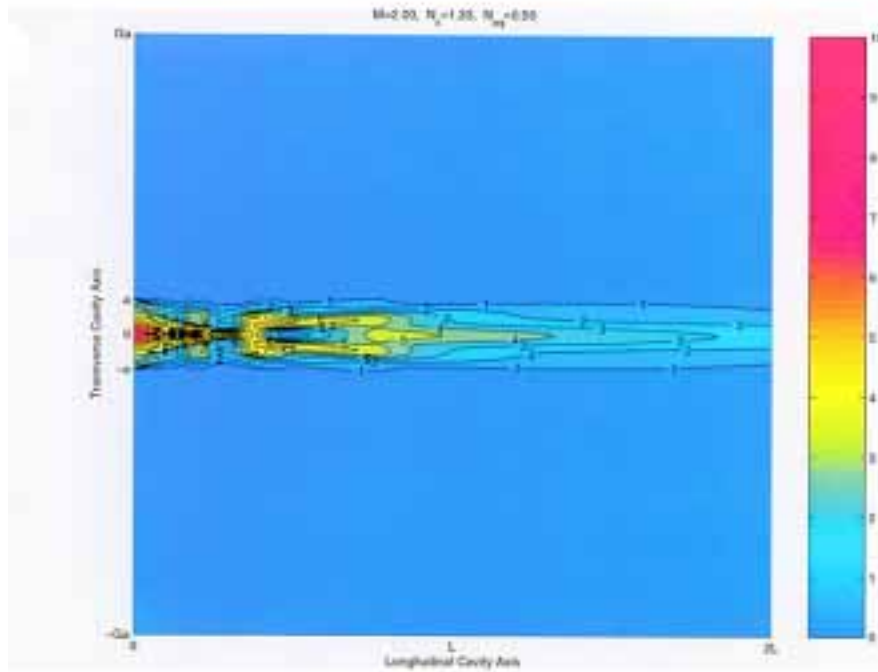


Fig. 1. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 0.5$.

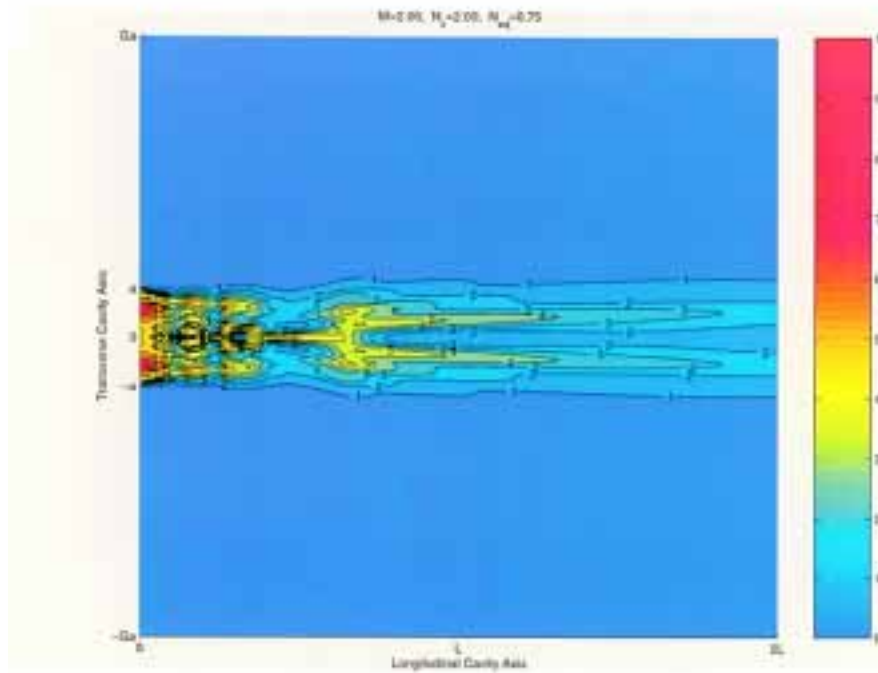


Fig. 2. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 0.75$.

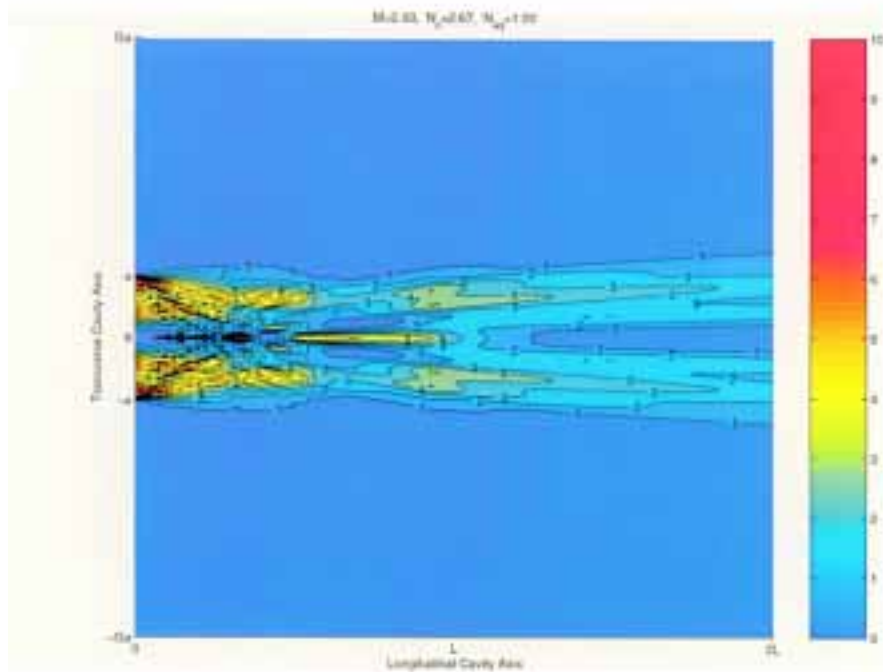


Fig. 3. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 1.0$.

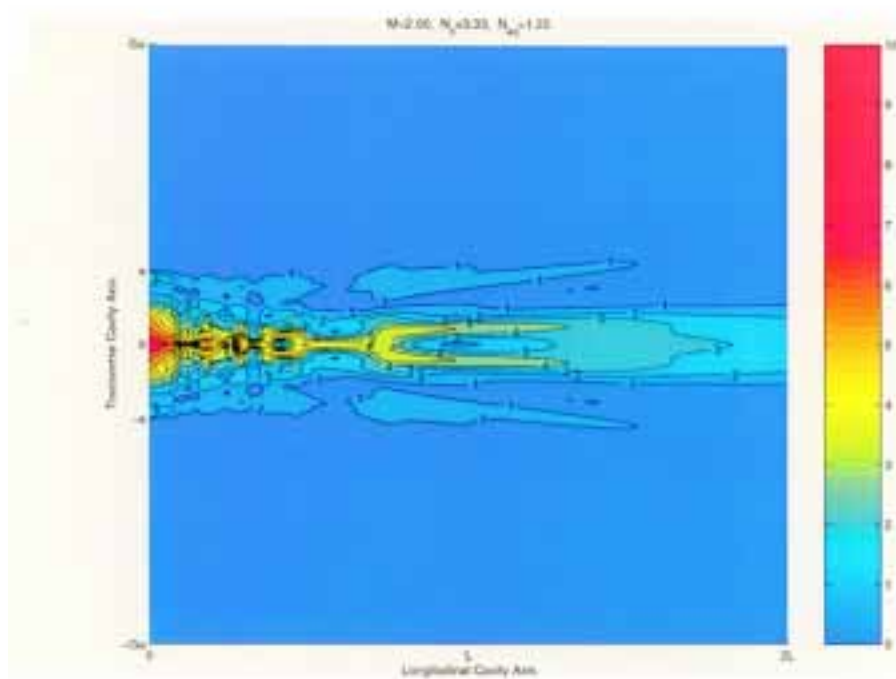


Fig. 4. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 1.25$.

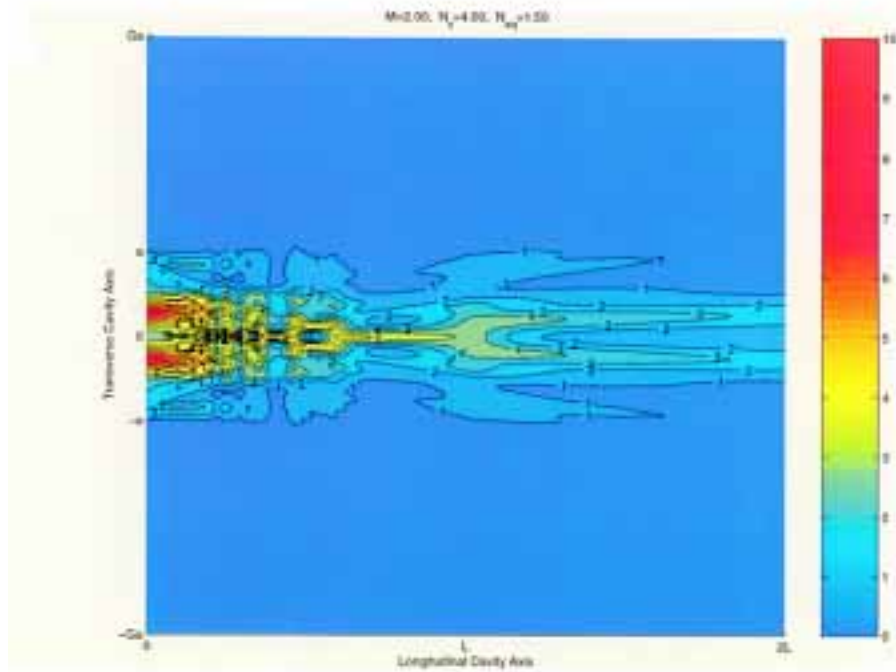


Fig. 5. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 1.5$.

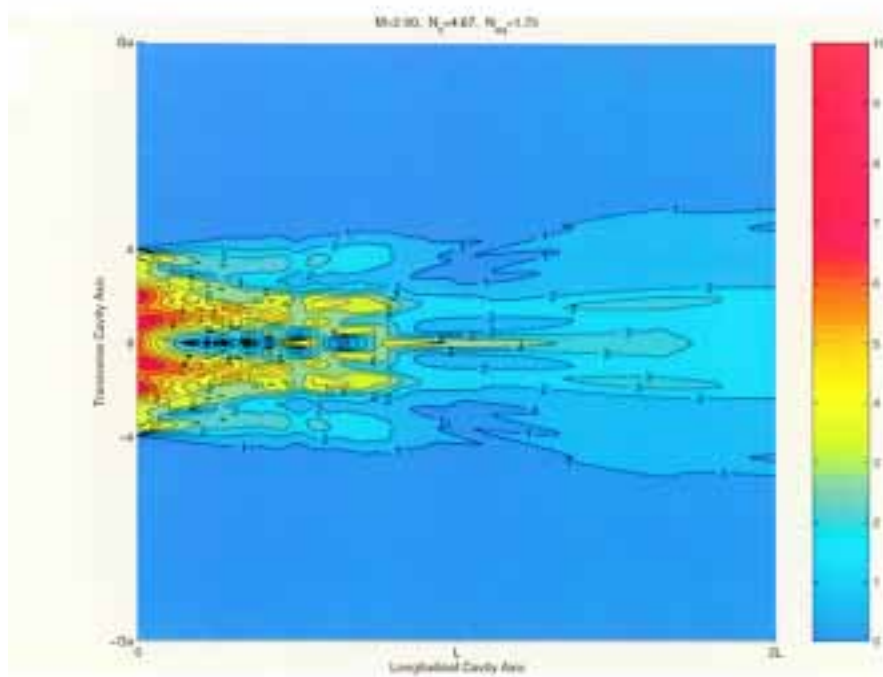


Fig. 6. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 1.75$.

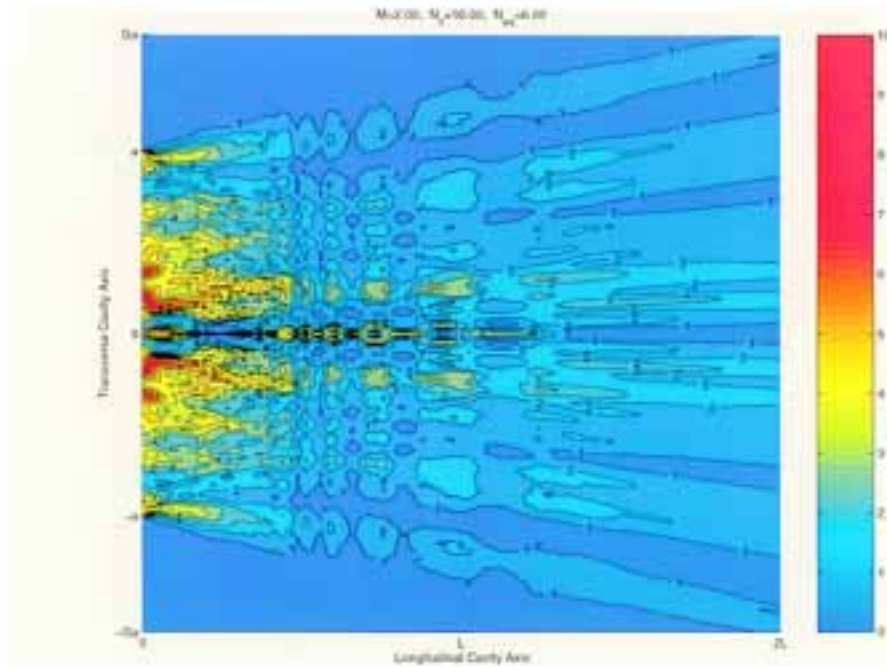


Fig. 7. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 6.0$.

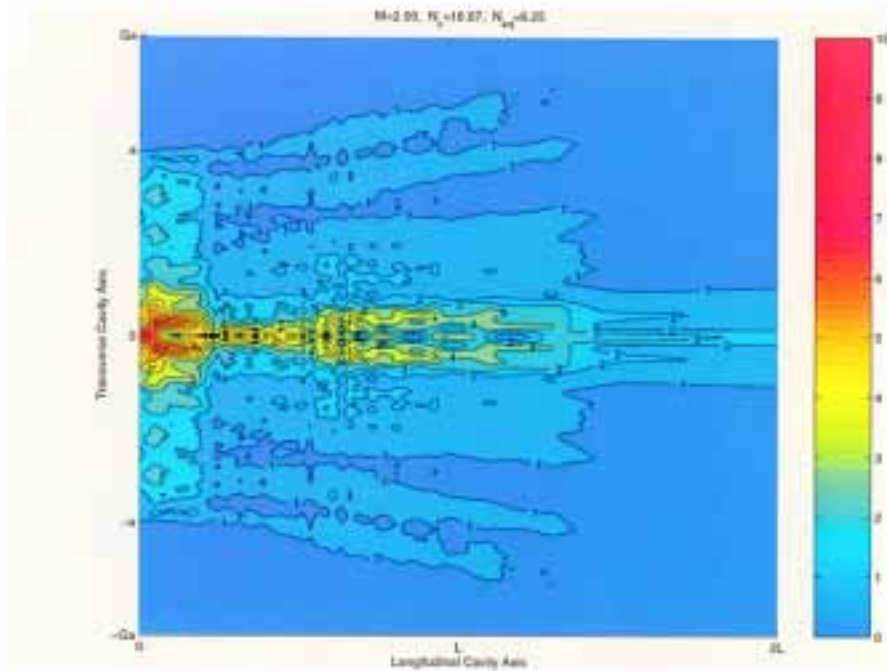


Fig. 8. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 6.25$.

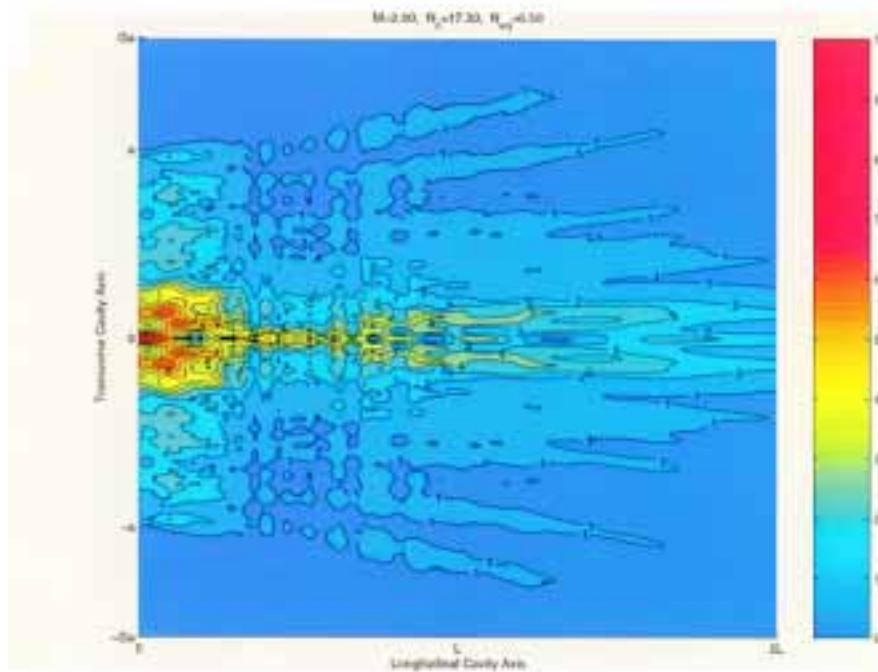


Fig. 9. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 6.5$.

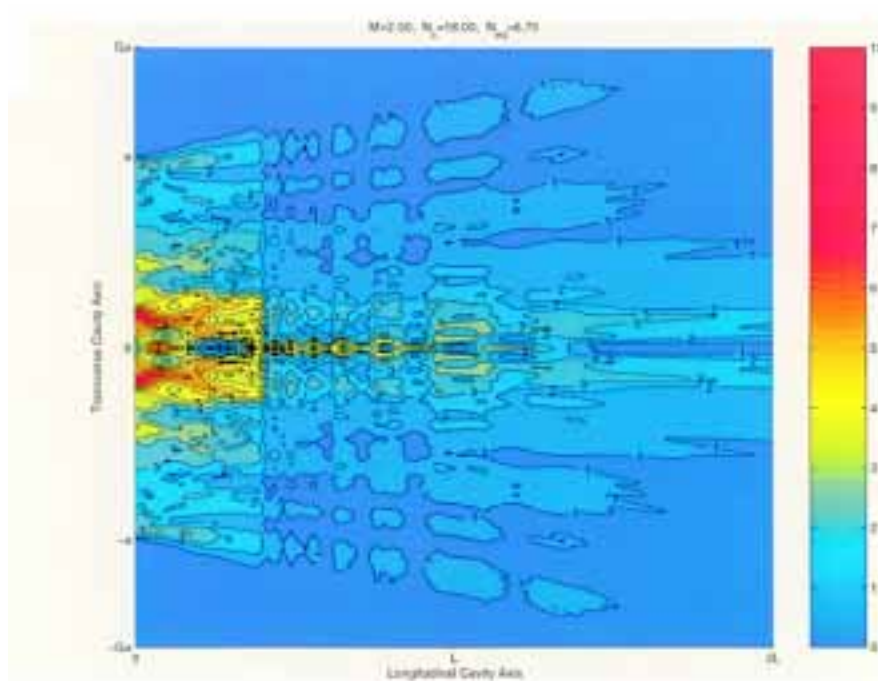


Fig. 10. Passive three-dimensional intracavity field distribution in an $M=2$ half-symmetric unstable cavity with $N_{eq} = 6.75$.

4. Discussion

These results clearly show the importance of the Fresnel zone structure on the intracavity mode structure properties of an unstable resonator in the passive (purely optical) cavity case. Anan'ev's analogy [9] that a laser with an unstable cavity corresponds to an optical system comprised of a driving generator and an amplifier with a matching telescope between them has been verified through these calculations for the passive (i.e. zero gain) case. The role of the generator is played by the central intensity core that is defined by the central Fresnel zone of the cavity and the role of the amplifier by the remaining peripheral zone of the cavity, with the edge-diffracted field at the feedback aperture edge providing the controlling feedback to the central intensity core. It is this mechanism of diffractive feedback into a converging wave field and its interaction with the magnifying or diverging cavity field that produces the central intensity core and determines the diffractive properties of the cavity mode structure.

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