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Sec. 12.2, Problem #7

$y' = e^{x-y}$, $y(0) = 0$

$y' = e^{x}e^{-y}$

$e^{y}y' = e^{x}$

$\int e^{y}\, dy = \int e^{x}\, dx$

$e^{y} = e^{x} + C$

$y = \ln\left(e^{x} + C\right)$

Find C

y = 0 when x = 0

$0 = \ln\left(e^{0} + C\right)$

$0 = \ln\left(1 + C\right)$

$1 + C = 1 \quad \left(\ln 1 = 0\right)$

$C = 0$

$y = \ln\left(e^{x} + 0\right)$

$y = \ln\left(e^{x}\right)$

$y = x$

Sec. 12.2, Problem #11

$y' = \dfrac{y}{x}$, $y\left(1\right) = 1$

$\dfrac{1}{y}y' = \dfrac{1}{x}$

$\int \dfrac{1}{y}\, dy = \int \dfrac{1}{x}\, dx$

$\ln y = \ln x + c$

$e^{\ln y} = e^{\ln x + c}$

$y = e^{\ln x}e^{c}$

$y = Cx$

Find C knowing that when x = 1, y = 1

1 = C(1) or C = 1

Answer: y = x

Section, 12.2, Problem 313

$$\frac{dy}{dx}=\frac{x+1}{y^2+1},\ y(0)=3$$

$$\left(y^2+1\right)\frac{dy}{dx}=x+1$$

$$\int\left(y^2+1\right)dy=\int\left(x+1\right)dx$$

$$\frac{1}{3}y^3+y=\frac{1}{2}x^2+x+c$$

$$x=0, y=3$$

$$\frac{1}{3}(3)^3+3=\frac{1}{2}(0)^2+0+c$$

$$9+3=c$$

$$c=12$$

$$\frac{1}{3}y^3+y=\frac{1}{2}x^2+x+12$$

Section 12.1, Problem #17

$$y_p=Ce^{x^3+x^2};\quad y'=\left(3x^2+2x\right)y;\ \ y(0)=100$$

Find the solution

Find C; let x = 0, y = 100

$$y_p=Ce^{x^3+x^2}$$

$$100=Ce^0$$

$$C=100$$

Solution

$$y_p=100e^{x^3+x^2}$$

Section 11.1, Problem #17

Random Variable x	-2	0	1	2	4
P(X = x)	0.1	0.3	0.1	0.2	0.3

find the mean

$$\mu=(-2)(0.1)+0(0.3)+1(0.1)+2(0.2)+4(0.3)=1.5$$

variance

$$(-2-1.5)^2(0.1)+(0-1.5)^2(0.3)+(1-1.5)^2(0.1)+(2-1.5)^2(0.2)+(4-1.5)^2(0.30)$$

$$=(-3.5)^2(0.1)+(-1.5)^2(0.3)+(-0.5)^2(0.1)+0.5^2(0.2)+2.5^2(0.3)$$

$$=3.85$$

standard deviation

$$\sqrt{3.85}=1.96$$

Section 11.2, Problem #21

f(x) = e^{-x}, x all real numbers
why is this not a pdf?
Look at $\int_{-\infty}^{\infty} e^{-x}\ dx$ diverges or is not finite

Section 11.2, Problem #25
f(x) = 6x(1 – x) on [0, 1]
Show that this is a pdf

1. $f(x) \geq 0, 0 \leq x \leq 1$ true
2. $\int_0^1 f(x)\, dx = 1$

Look at
$$\int_0^1 6x(1-x)\, dx = \int_0^1 (6x - 6x^2)\, dx = \left[3x^2 - 2x^3\right]_0^1 = \left[3(1)^2 - 2(1)^3\right] - \left[3(0)^2 - 2(0)^3\right] = 1$$

Section 11.2, Problem #33
pdf f(t) = $0.15e^{-0.15t}$,
exponential pdf
Find probability that life is at most 10 hours
$$P(T \leq 10) = \int_0^{10} 0.15e^{-0.15t}\ dt = \left[0.15\left(\frac{1}{-0.15}\right)e^{-.15t}\right]_0^{10} = \left[-e^{-0.15t}\right]_0^{10}$$
$$= \left[-e^{-0.15(10)}\right] - \left[-e^{-0.15(0)}\right] = -e^{-1.5} + 1$$

Probability of at least 10 hours
$$P(T \geq 10) = 1 - P(T \leq 10) = 1 - \left(1 - e^{-1.5}\right) = e^{-1.5}$$

$$P(20 < T < 40) = \int_{20}^{40} 0.15e^{-0.15t}\ dt = \left[-e^{-0.15}\right]_{20}^{40} = -e^{-0.15(40)} + e^{-.15(20)} = -e^{-6} + e^{-3}$$